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Derivation of Deterministic Design Data from Stochastic Analysis in the Aircraft Design Process

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Abstract

The application of uncertainty and sensitivity analysis techniques to the aircraft design process is currently a high profile research area and of key strategic interest within aerospace industry.

Due to the computational cost of simulating aircraft structural dynamics, the implementation of an integrated modelling framework that can both generate accurate metamodels of aircraft dynamic responses and use them to perform trend analysis and aircraft non-specific design optimisation is particularly valuable in order to reduce the time required to complete aircraft dynamic analysis.

In this paper a metamodelling framework based on polynomial chaos expansion is presented and used to model the shear force acting on an aircraft fuselage numerical model during landing. A search algorithm based on particle swarm optimisation is also presented and used to explore the generated metamodels.

Keywords: industrial optimisation, metamodel, polynomial chaos expansion, sensitivity analysis, particle swarm optimisation, dimensionality reduction.

1 Introduction

The application of uncertainty management techniques to the aircraft design process is currently a high profile research area and of key strategic interest within aerospace industry. Within the aircraft design process there is always a difficult balance between non specific and specific design steps for configuration and design maturity versus the overall project lead time. This leads to either an immature design that causes delays of the entry into service or significant re-design loops within the aircraft development project again resulting in a significant cost penalty. The ability to quantify uncertainties in the design enables the application of more robust optimisation approaches to balance the quantitative risks of design evolution against the aircraft performance implications (e.g. aircraft weight) and specific design lead time.

Although the application of stochastic analysis is a powerful way of making informed design decisions, its integration into the standard design process requires the generation of deterministic design data. In other words, designers need to associate the uncertainty information corresponding to specific target levels of the aircraft model responses under study to the specific values of the aircraft model input variables that produce those target levels. Unfortunately, the computational cost of aircraft structural analysis excludes the possibility to use direct computer simulations to search for the desired input configurations.

In this paper the use of metamodels (or surrogate models) in lieu of numerical simulations is envisaged as a strategical solution to reduce the cost of metamodel exploration, expression used to refer to the search for the set of input configurations producing a desired level of the aircraft model response under analysis. A metamodelling framework based on polynomial chaos expansion (PCE) is described. Polynomial chaos expansion was selected for the specific properties of its metamodels: they are global and explicit, and sensitivity information related to their input variables can be extracted analytically from their coefficients in the form of Sobol indices. The variable ranking resulting from Sobol indices allows for straightforward dimensionality reduction of a metamodel through variable selection.

Furthermore, a search algorithm based on the particle swarm optimisation (PSO) paradigm is presented. The algorithm was used to explore the PCE metamodels in order to find the design points producing a specified level of the modelled response, in this case the shear force on an aircraft fuselage computer model. Although less efficient than deterministic (gradient-based) algorithms, population based algorithms like the PSO perform a robust exploration of the design space and the solutions they provide do not depend on the guesses provided at the start of the search. The generally high computational cost of population-based algorithms is made feasible by metamodels' inexpensive and immediate evaluation.

2 Metamodelling framework

Metamodelling is a cyclic process that leads to the generation of an approximate relationship $\tilde{f}$, the metamodel, between a set of input variables x and a response y:

$$y = \tilde{f}(\boldsymbol{x}) + \varepsilon(\boldsymbol{x}) \quad (1)$$

where $\boldsymbol{x} = (x_{1,\dots,}x_n)$ is a design point defined in some design space or domain $S \subset \mathbb{R}^n$ and $y \in \mathbb{R}$ is the actual response of the system, for example generated by numerical simulations or physical experiments. The term $\varepsilon(\boldsymbol{x})$ represents the error introduced by the metamodel (errors due to measurements or numerical noise are here not considered).

Metamodelling main stages can be identified as [1, 2]:

1. data generation,

2. metamodel generation or development,
3. problem analysis and reduction.

Data generation consists of all the operations involved in gathering, filtering and selecting the data that are used to generate the model. The amount of information that can be acquired through sampling can be maximised using techniques called design of experiments (DoE) [1, 2]. The gathered data are then processed by a metamodelling technique and a metamodel $\tilde{f}$ is generated. The problem analysis stage aims at extracting information from the generated metamodel to better understand the relative importance of each variable on the model response. More specifically, sensitivity analysis can be described as the set of operations that allow to identify the most influential input variables on the response produced by a metamodel. Based on the information acquired through sensitivity analysis, the whole metamodelling process can be repeated using a reduced set of the most important input variables to improve the accuracy of the model.

The diagram in Fig. 2.1 (based on images found in Reference [1] and [2]) represents the cyclic nature of this process: data generation, model development and sensitivity analysis are repeated to progressively refine the metamodel until an acceptable quality is reached.

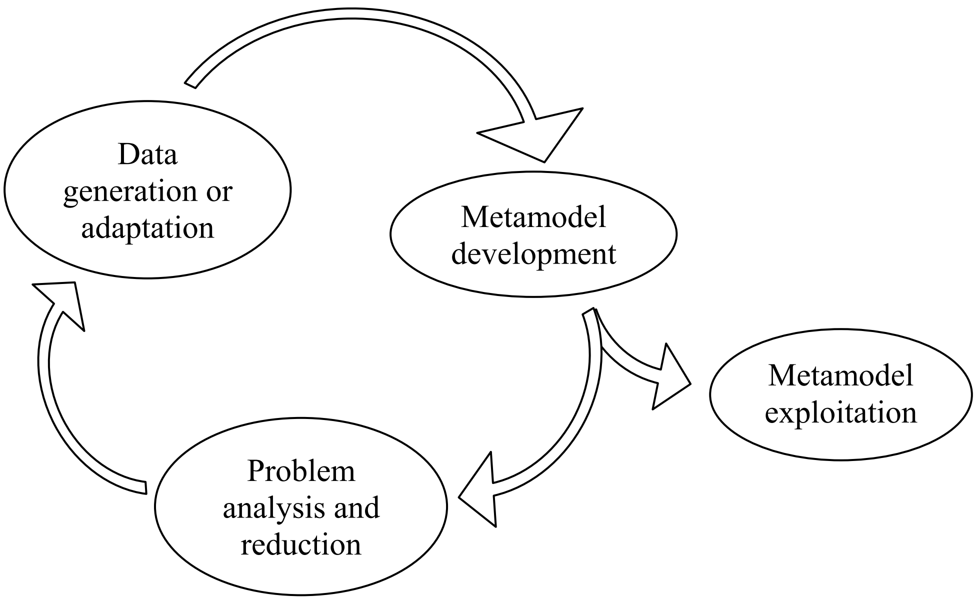


Figure 1: Metamodelling stages

2.1 Polynomial chaos expansion

The metamodelling framework used to produce the metamodels presented and analysed in this paper is based on polynomial chaos expansion (PCE) [3].

PCE is a metamodelling technique that uses a linear combination of polynomials of increasing dimensionality to approximate a given response y:

$$y = a_0 B_0 + \sum_{i_1=1}^{\infty} a_{i_1} B_1(\xi_1) + \sum_{i_1=1}^{\infty}\sum_{i_2=1}^{i_1} a_{i_1 i_2} B_2(\xi_{i_1}, \xi_{i_2}) + \sum_{i_1=1}^{\infty}\sum_{i_2=1}^{i_1}\sum_{i_3=1}^{i_2} a_{i_1 i_2 i_3} B_3(\xi_{i_1}, \xi_{i_2}, \xi_{i_3}) + \cdots \quad (2)$$

where ξ_{i_k} are input variables modelled by given statistical distributions, B_{i_k} are multivariate polynomials of increasing dimensionality and a_{i_k} are coefficients that have to be tuned on a building or training DoE provided by the user. The expression in (2) can be reformulated [3] as:

$$y = \sum_{j=0}^{\infty} \alpha_j \Psi_j(\boldsymbol{\xi}) \quad (3)$$

where $\Psi_j(\boldsymbol{\xi})$ stand for multivariate polynomials of increasing dimensionality and the coefficients α_j replace the a_{i_k} in Equation (2).

As the mathematical structure of a PCE metamodel is imposed by the sum of the multivariate polynomials B_{i_k} (see Equation (2)) or, equivalently, $\Psi_j(\boldsymbol{\xi})$ (see Equation (3)), PCE belongs to the class of parametric metamodelling techniques [4].

The PCE multivariate terms $\Psi_j(\boldsymbol{\xi})$ satisfy an important property if they are defined as product of one-dimensional polynomials belonging to the so-called Askey scheme [3]. The Askey scheme defines sets of polynomials that are orthogonal on a specific domain (*support range*) if their variables ξ_{i_k} are modelled by specific continuous probability distributions. Due to the orthogonality of the polynomials belonging to the Askey scheme, a PCE where such polynomials are used as a basis can be interpreted as a Sobol decomposition [5], once the terms of the same dimensionality are clustered together [6]. Sobol decompositions are useful in variance-based sensitivity analysis as the variance of the output y can be decomposed as sum of the variances of the single terms, being their covariance equal to zero. Referring to the PCE in (3), the variance decomposition can then be simplified [6] as:

$$D = var(y) = \sum_{j=0}^{\infty} var\left(\alpha_j \Psi_j(\boldsymbol{\xi})\right) = \sum_{j=0}^{\infty} {\alpha_j}^2 \quad (4)$$

where the contribution of each multivariate term $\Psi_j(\boldsymbol{\xi})$ to the total output variance *D* is obtained analytically squaring its numerical coefficient α_j [6].

The *relative* contribution $S_{i_1,\ldots,i_t}$ of a given set of input variables $(i_1, \ldots, i_t)$ to the total output variance can then be obtained summing all the squared coefficients of the terms $\Psi_j(\boldsymbol{\xi})$ depending on the given set of input variables:

$$S_{i_1,\ldots,i_t} = \left(\sum_{j \in K} {\alpha_j}^2\right)/D \quad (5)$$

where K is used to indicate the set of subscripts j of the terms depending on the input variables $(i_1, \ldots, i_t)$.

The quantity $S_{i_1,\ldots,i_t}$ defined in Equation (5) is called *Sobol index* [5, 6, 7]: by definition Sobol index values belong to the range [0, 1] and their sum is unity. Sobol indices of the first order are defined as the relative variance contribution of the terms $\Psi_j(\xi_{i_1})$ depending on a single variable i_1:

$$S_{i_1} = \left(\sum_{j \in K} \alpha_j^2\right)/D \qquad (6)$$

whereas Sobol indices of n-th order provide information about the contribution of n variables and their interactions to the total output variance D [5, 6, 7]. A cumulative or total Sobol index, referred to a variable, is given by the sum of all the Sobol indices computed using the coefficients α_j of the terms $\Psi_j(\boldsymbol{\xi})$ depending at least on the specified variable.

Sobol indices allow to rank input variables according to their contribution to the total output variance. In this sense they ease metamodel dimensionality reduction through variable selection. Although Sobol indices can in general be obtained by other means (Monte Carlo experiments, for example [7, 8]), their extraction from PCE is one of the least expensive way to compute them, as no additional computer simulations or physical experiments are required other than the set used to build the PCE.

The reliability of the variable hierarchy based on Sobol indices depends critically on the accuracy of the numerical coefficients α_j in the polynomial chaos expansion (see Equation (3)). In the metamodelling framework here presented the *linear regression* or least squares approach [3] was used for tuning the coefficients of a PCE metamodel $\tilde{f}$, defined as a truncated PCE:

$$y = \sum_{j=0}^{P} \alpha_j \Psi_j(\boldsymbol{\xi}) \qquad P \in \mathbb{N} \qquad (6)$$

The accuracy of the metamodel can be improved at the cost of increasing the maximum order p of the truncated PCE. However, doing so also the total number of coefficients N to be tuned increases, as N can be computed by:

$$N = \frac{(n+p)!}{n!p!} \qquad (7)$$

where n is the number of input variables in the PCE [3]. So increasing the order of the PCE to improve accuracy requires the generation of a larger DoE to tune the extra coefficients. The N unknowns represented by the coefficients α_j in (3) were computed for the experiments shown in the following on a building data set made of a number of points larger than the number of unknowns, as recommended by Reference [3] and Reference [9]. A stochastic Latin Hypercube DoE was chosen to generate the input data for its space-filling properties [2, 8].

2.2 Metamodel constrained exploration

The main purpose a metamodel is built for is its inexpensive exploitation (see Figure 1): metamodel immediate evaluation allows to reduce the time required for engineering analysis [10, 11, 12, 13, 14]. An important part of the research activity was therefore dedicated to the development of a technique able to "explore" the metamodels generated by the methodology described in Section 2.1. The addressed problem can be defined as:

$$\mathit{find}\ \boldsymbol{x} \in S \mid \tilde{f}(\boldsymbol{x}) = k \tag{8}$$

under constraints:

$$l_i < g_i(\boldsymbol{x}) < u_i \quad \mathrm{i} = 1, \dots, \mathrm{p} \tag{9}$$

where $g_i(\boldsymbol{x})$ are p additional metamodels defined on the same design space as $\tilde{f}$ and l_i and u_i are the lower and upper bound of their feasible output. The problem defined in (8-9) will be referred to as *constrained search* or *constrained exploration* problem.

To solve it, the problem was reformulated as a constrained minimisation problem through the definition of the cost function F_c:

$$F_c = abs[f(\boldsymbol{x}) - k] + \sum_{i=1}^{p} w_i(l_i, u_i, g_i(\boldsymbol{x})) \tag{10}$$

where w_i are weight functions that assume a positive value if $\boldsymbol{x}$ is outside the feasible region defined on g_i's codomains (see expression (9)), otherwise they are zero. For the experiments described in this paper the weight functions w_i were defined using the exponential, to allow for a smooth increase of the cost function in the transition from feasible to unfeasible design space regions:

$$w_i(l_i, u_i, g_i(\boldsymbol{x})) = \begin{cases} 0 \ \ if \ \ l_i < g_i(\boldsymbol{x}) < u_i \\ e^{(g_i(x) - l_i)^2} - 1 \ \ if \ \ g_i(\boldsymbol{x}) < l_i \\ e^{(g_i(x) - u_i)^2} - 1 \ \ if \ \ g_i(\boldsymbol{x}) > u_i \end{cases} \tag{11}$$

The design points that produce a minimum of F_c defined by Equations (10-11) are in fact the solutions of the constrained exploration problem defined in (8-9).

2.2.1 Minimisation through Particle Swarm Optimisation algorithm

A stochastic technique was chosen to solve the minimisation problem defined by Equations (10-11). Although more efficient, deterministic (gradient-based) techniques are not able to perform a global exploration of the design space, the solutions they return generally depend on the initial guesses provided to the search algorithm and they are sensitive to noise.

Among the many metaheuristic techniques available (a brief survey can be found in Reference [15]) , the population-based Particle Swarm Optimisation algorithm (PSO) was selected for its simplicity [16]. PSO inspiring criterion is the concept of swarm. A swarm is defined as a set of particles, each characterised by a position and a velocity. At each algorithm iteration the position and the velocity of each point are computed taking into account the previous positions of all the particles as well as the position of the particle whose corresponding response is considered the best among all and up to the current iteration. As a result, iteration after iteration the swarm sweeps the entire design space and converges to the global optimum. The presence of a large swarm exploring the area around the solution found up to the latest iteration reduces the risk of returning local solutions.

In PSO different strategies can be used to ensure that the particles are kept within the design space boundaries during the search. Xu et al. [17] replaced the unfeasible coordinates of the particles generated outside the design space with the corresponding coordinates of a position randomly selected from the set of best positions recorded up to the current generation. A few preliminary tests showed that this strategy is not able to lead particles in areas close to the design space boundary. A new strategy was then introduced to increase the sampling frequency in the regions close to the design space boundary. The rules introduced to regenerate an unfeasible particle are:

$$\boldsymbol{X}_i^k(j) = \begin{cases} u_j & if \;\; \boldsymbol{X}_i^k(j) > u_i \\ l_j & if \;\; \boldsymbol{X}_i^k(j) < l_i \end{cases} \tag{12}$$

where $\boldsymbol{X}_i^k(j)$ is j-th coordinate of particle i at generation k and l_j and u_j are respectively the lower and the upper bound of variable j. Unfortunately, the strategy described by Equation (12) focuses the exploration on a narrow region along the design space boundary, so a combined approach was finally opted for: the unfeasible particles are regenerated selecting with equal probability either the strategy defined by Xu et al. or the strategy defined in Equation (12).

3 Aircraft dynamic landing analysis

The metamodelling framework introduced in Section 2.1 and the search algorithm described in Section 2.2 were validated on a real-life structural problem.

A PCE metamodel was generated to model the minimum shear force acting on a section of aircraft fuselage during landing. 10 input variables were identified and modelled using a uniform distribution. A Latin Hypercube DoE made of 3000 points was generated. To each point on the design space was associated the most critical value of the shear force recorded on the aircraft fuselage computer model during a period of approximately one second (the most critical value is the highest in absolute value, although it is actually negative according to the reference system used in the model). The maximum order of the PCE metamodel was set to 6.

For confidentiality reasons the actual physical parameters used as inputs cannot be disclosed, so the input variables will be named x_i with $i = 0, \dots, 9$. The variables' ranges and the cumulative Sobol indices extracted by the PCE metamodel are reported in Table 1. No significative differences in the magnitudes of the first order Sobol indices were observed, so total Sobol indices were considered for dimensionality reduction purposes. The three largest total Sobol indices are highlighted in bold in Table 1.

Variable	Range	Total Sobol index
x_0	[-2, 2]	**35.2892**
x_1	[0.5, 1.5]	4.1224
x_2	[0.5, 1.5]	**20.1297**
x_3	[-0.1, 0.1]	**49.7148**
x_4	[0.9, 1.1]	1.9040
x_5	[0.8, 1.2]	9.4272
x_6	[0.7, 1.3]	17.1599
x_7	[0.25, 1.75]	12.8212
x_8	[0.25, 1.75]	01.6608
x_9	[0.25, 1.75]	01.1519

Table 1: PCE metamodel input variables and their cumulative Sobol indices

The PSO search algorithm described in Section 2.2 was used to search for the set of input configurations resulting in specified target levels of the shear force on a specific fuselage section, which will be referred to as section "B". To make the search more challenging, constraints were introduced in the form of lower bounds on the minimum shear force (highest in absolute value) on neighbouring fuselage sections, which will be called section "A" (preceding section B) and "C" (following section B). Such constraints are a typical example of the functions g_i defined in Equation (9).

The search was performed on shear force metamodels of reduced dimensionality obtained from the ten-dimensional one returned by PCE. The results of the analysis on two-dimensional and a three-dimensional models are described in the following. All the PSO searches were performed on a desktop computer (Fujitsu Siemens Esprimo E5720, Intel Core 2 Duo E7200 2.53GHz with 1.99 GB Ram).

3.1 Search on two-dimensional metamodel

The ten-dimensional PCE metamodel was reduced to a function of the two most influential input variables, according to the cumulative Sobol indices provided in Table 1. x_3 and x_0 were assumed to vary in the ranges [-0.1, 0.1] and [-2.0, 2.0], respectively, whereas all the other variables were set to their average value (centre of the range, see Table 1).

The desired target level for the shear force on section B was set to 105% of the mean minimum shear force returned by the whole ten-dimensional PCE metamodel. The lower bound of the shear force on sections A and B was set to 105% of the mean minimum shear force returned by the respective ten-dimensional metamodels. The input parameters used for the PSO search algorithm are shown in Table 2.

Parameter	Value
Input variables	x_3 , x_0
Design space	[-0.1, 0.1] ×[-2.0, 2.0]
Shear force target level on section B	105% mean shear force on original metamodel
Shear force lower bound on section A and C	105% mean shear force on original metamodels
No. of independent PSO searches	10
Swarm initial centre coordinates	[0.0, 0.0]
Swarm initial radii	[0.1, 2.0]
No. of particles	100
No. of iterations	400

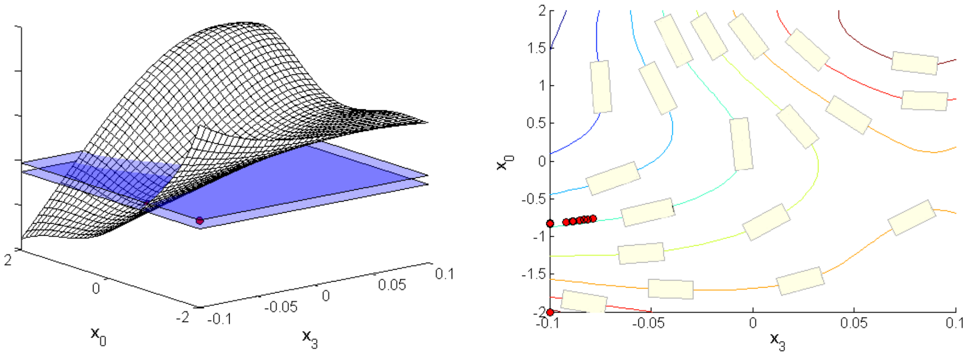
Table 2: input parameters for the PSO search on the two-dimensional metamodel

The points found by the PSO search algorithm are listed in Table 3. The first two columns of the table define the point found by the PSO algorithm. Column F_{Bnorm} reports the shear force on section B returned by the reduced metamodel in the point defined by the first two columns of the table and normalised by the desired target level. δ is the relative residual, computed as the difference between F_{Bnorm} and 1.0 multiplied by 100. F_{Anorm} and F_{Cnorm} are the corresponding normalised values of the shear force on section A and C, their lower bounds being used for the normalisation: values bigger than 1.0 violate the constraints. The points in the table are also plotted on the reduced metamodels of the shear force in sections A, B and C in Figure 2. The search was completed in almost 8 minutes (477.78 seconds).

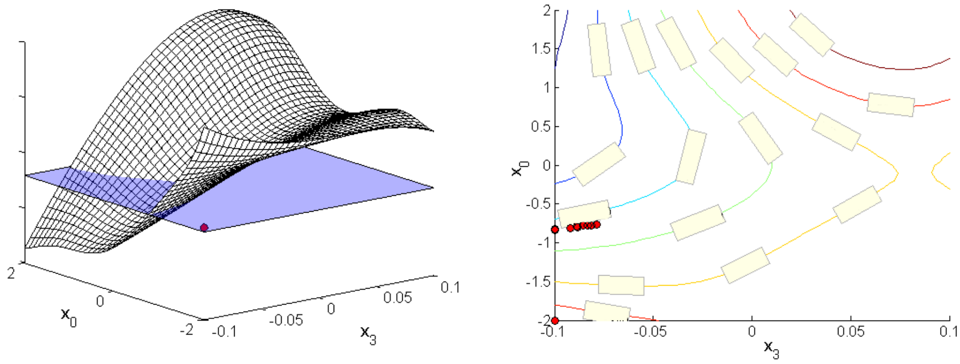
Design variables		Main objective		Constraints	
x_3	x_0	F_{Bnorm}	δ (%)	F_{Anorm}	F_{Cnorm}
-8.824941e-002	-8.027650e-001	0.990222	-0.977842	0.991317	0.998293
-9.184294e-002	-8.112971e-001	0.990764	-0.923577	0.991867	0.998360
-1.000000e-001	-2.000000e+000	0.991301	-0.869883	0.992341	0.999822
-8.310589e-002	-7.806096e-001	0.990533	-0.946736	0.991428	0.999519
-8.854922e-002	-7.933002e-001	0.991410	-0.858979	0.992488	0.999661
-1.000000e-001	-8.378425e-001	0.991212	-0.878796	0.991933	0.997679
-8.529922e-002	-7.834990e-001	0.991132	-0.886772	0.992122	0.999872
-8.130315e-002	-7.751179e-001	0.990377	-0.962304	0.991165	0.999634
-7.844540e-002	-7.653384e-001	0.990229	-0.977133	0.990803	0.999932
-1.000000e-001	-8.260076e-001	0.992639	-0.736083	0.993317	0.999322

Table 3: solutions found on the reduced two-dimensional metamodel

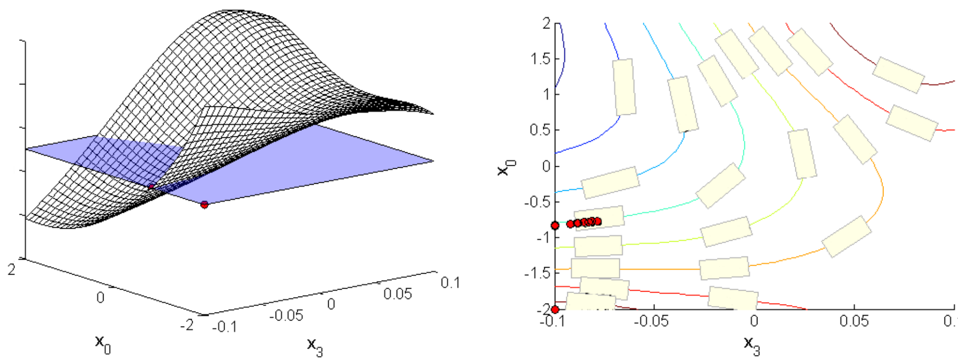
It can be noticed that the values of the minimum shear force on section B (column F_{Bnorm} and δ of Table 3) are distributed in a region centred in the target level. The amplitude of the region considered "acceptable", or tolerance on the solution, was set to 0.985927% of the target level. No solutions satisfying the given constraints were found using tolerances of lower orders of magnitude.



(a) Solutions (red dots) on metamodel describing shear force on fuselage section B: three-dimensional view (left) and contour view (right)



(a) Solutions (red dots) on metamodel describing shear force on fuselage section A: three-dimensional view (left) and contour view (right)



(a) Solutions (red dots) on metamodel describing shear force on fuselage section C: three-dimensional view (left) and contour view (right)

Figure 2: Solutions found by the PSO search on the two-dimensional metamodels of shear force in fuselage section B, A and C

A second PSO search was performed to assess whether the impossibility to find design points producing shear force values closer to the desired target level (section B) and satisfying the given constraints (section A, C) was due to errors in the PSO search algorithm or to the intrinsic behaviour of the aircraft dynamic model.

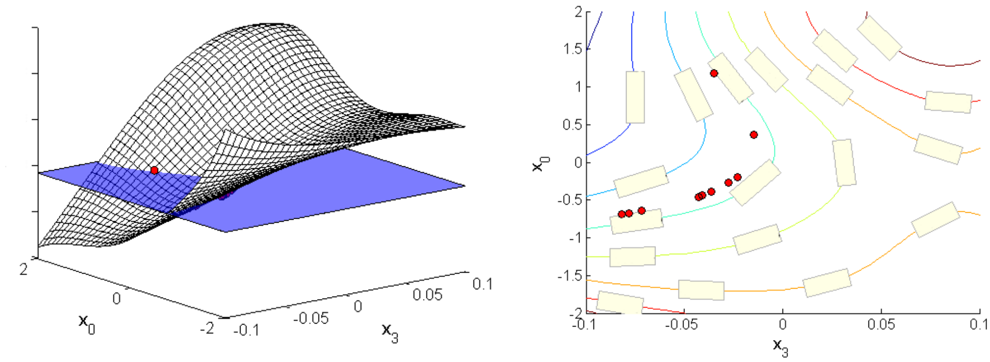
The lower bound on the shear force in sections A and C was decreased (increased in absolute value) from 105% to 110% of the corresponding shear force mean value. The amplitude of the interval defining the acceptable shear force on section B was reduced of three orders of magnitude (+/- 0.000986% target level), keeping the same target level as before (105% of the mean maximum shear force returned by the whole ten-dimensional PCE metamodel).

The solutions found by the PSO search are reported in Table 4 and plotted on the three response surfaces in Figure 3. The search was completed in almost 19 minutes (1135.52 seconds).

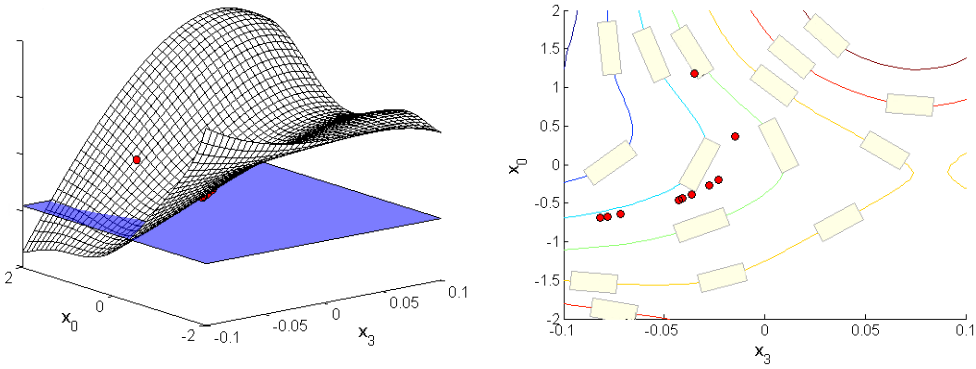
Design variables		Main objective		Constraints	
x_3	x_0	F_{Bnorm}	δ (%)	F_{Anorm}	F_{Cnorm}
-7.163566e-002	-6.407431e-001	1.000005	0.000503	0.954309	0.966694
-1.484286e-002	3.692041e-001	0.999993	-0.000730	0.942245	0.974594
-2.737174e-002	-2.739568e-001	1.000003	0.000286	0.947866	0.969763
-4.073198e-002	-4.359622e-001	1.000004	0.000365	0.949728	0.969704
-3.482956e-002	1.179369e+000	0.999993	-0.000720	0.930313	0.997217
-8.177373e-002	-6.871465e-001	1.000000	0.000020	0.955212	0.965053
-2.296140e-002	-1.950434e-001	0.999992	-0.000759	0.947286	0.969665
-7.817343e-002	-6.713121e-001	0.999991	-0.000927	0.954945	0.965645
-4.282618e-002	-4.552093e-001	1.000005	0.000503	0.950049	0.969626
-3.625386e-002	-3.903526e-001	0.999999	-0.000099	0.949063	0.969803

Table 4: solutions found on the reduced two-dimensional metamodel – relaxed constraints

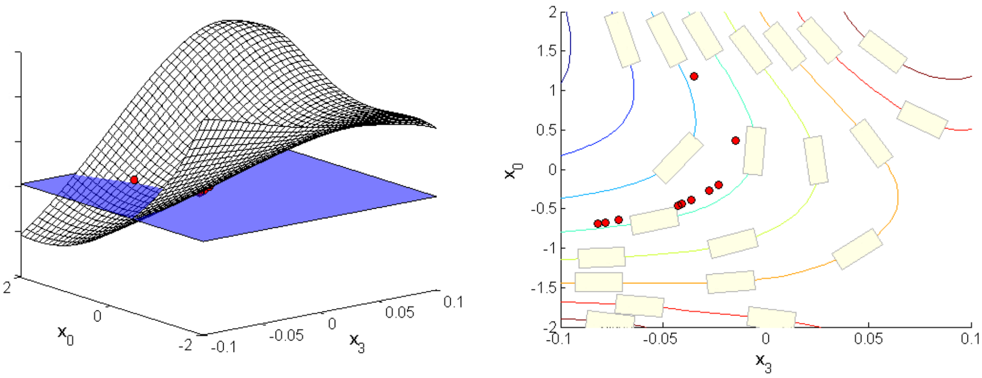
The ten design configurations found by the PSO algorithm (Table 4) are all within the reduced tolerance ($|\delta| < 0.000986$) and they all satisfy the specified constraints imposed on the shear force on sections A and C ($F_{Anorm} < 1, F_{Cnorm} < 1$). The lack of success of the first earch are then ascribable to the intrinsic behaviour of the aircraft dynamic model, and not to errors in the PSO search algorithm.



(a) Solutions (red dots) on metamodel describing shear force on fuselage section B: three-dimensional view (left) and contour view (right)



(a) Solutions (red dots) on metamodel describing shear force on fuselage section A: three-dimensional view (left) and contour view (right)



(a) Solutions (red dots) on metamodel describing shear force on fuselage section C: three-dimensional view (left) and contour view (right)

Figure 3: Solutions found by the PSO search on the two-dimensional metamodels of shear force in fuselage section B, A and C – relaxed constraints on sections A and C

3.2 Search on three-dimensional metamodel

The example described in the previous Section showed that the constrained search problem may not have any solution in case constraints on sections A and C are too tight. Expanding the feasible region defined on the additional responses g_i (see expression (9)) is a possible strategy to increase the likelihood of finding solutions.

The PSO search algorithm presented in Section 2.2.1 features the possibility to leave the constraints relaxation process to the algorithm itself. Once the original lower and upper bound on the output of metamodels g_i and their increment are specified, the tool is able to perform a set of independent searches, progressively reducing and increasing the initial lower and upper bounds of g_i feasible regions. The search stops as soon as at least one solution is found.

The automatic bound relaxation feature was validated on the same constrained search problem described in the previous example. The dimensionality of the problem was however increased to 3 dimensions, so the three most influential variables were considered. x_3, x_0 and x_2 were assumed to vary in the region [-0.1, 0.1]×[-2.0, 2.0]×[0.5, 1.5], whereas the remaining variables were set to their average values (see Table 1).

A set of input configurations producing a minimum shear force (maximum in absolute value) in section B equal to 105% of the mean shear force of the original ten-dimensional metamodel were sought, setting the same tolerance used in the second search described in Section 3.1 (+/- 0.000986% target level). The initial lower bounds on the minimum shear force on sections A and C were set to 105% of the mean of the corresponding ten-dimensional metamodels. At each relaxation step, the two upper bounds were reduced (relaxed) by 2% of the mean shear force value of the corresponding ten-dimensional metamodels. The input parameters for the PSO search algorithm are shown in Table 5.

Parameter	Value
	x_3, x_0, x_2
Design space	[-0.1, 0.1] ×[-2.0, 2.0] ×[0.5, 1.5]
Shear force target level on section B	105% mean shear force on original metamodel
Initial shear force upper bound on section A and C	105% mean shear force on original metamodel
No. of independent PSO searches	10
Swarm initial centre coordinates	[0.0, 0.0, 1.0]
Swarm initial radii	[0.1, 2.0, 0.5]
No. of particles	100
No. of iterations	400

Table 5: input parameters for the PSO search on the three-dimensional metamodel

The first set of searches, with the initial lower bounds on the shear force on section A and C, was not able to find any solutions. After relaxing the lower bounds by a

2% decrement, a few solutions were instead found. The points found and the corresponding shear forces on section B are reported in Table 6. In Table 7 the corresponding values of the shear forces on sections A and C are given. This successful second set of searches was completed in about 52 minutes (3113.5 seconds).

Design variables			Main objective	
x_3	x_0	x_2	F_{Bnorm}	δ (%)
-6.023619e-02	4.535519e-02	7.098932e-01	1.000003	0.000266
-3.485146e-02	-2.799104e-01	9.576544e-01	1.000004	0.000414
-6.470957e-02	-5.681424e-01	9.776783e-01	0.999997	-0.000306
1.527897e-02	-2.923179e-03	1.285384e+00	1.000009	0.000927
-3.309159e-02	-1.286333e-01	9.115471e-01	0.999996	-0.000404
-1.414329e-02	1.592475e-01	1.000198e+00	1.000007	0.000739
-3.520328e-02	-6.931131e-02	8.806484e-01	1.000003	0.000276
-4.864929e-02	-6.730552e-01	1.098390e+00	0.999995	-0.000513
-5.243581e-02	-5.328829e-01	1.001227e+00	1.000001	0.000108
-4.710283e-02	-3.158177e-01	9.101925e-01	0.999994	-0.000582

Table 6: solutions found on three-dimensional metamodel

Design variables			Constraints	
x_3	x_0	x_2	F_{Anorm}	F_{Cnorm}
-6.023619e-02	4.535519e-02	7.098932e-01	0.987519	0.984127
-3.485146e-02	-2.799104e-01	9.576544e-01	0.975878	0.995905
-6.470957e-02	-5.681424e-01	9.776783e-01	0.980605	0.994748
1.527897e-02	-2.923179e-03	1.285384e+00	0.969592	0.995801
-3.309159e-02	-1.286333e-01	9.115471e-01	0.975944	0.994176
-1.414329e-02	1.592475e-01	1.000198e+00	0.971255	0.998452
-3.520328e-02	-6.931131e-02	8.806484e-01	0.976575	0.992986
-4.864929e-02	-6.730552e-01	1.098390e+00	0.976323	0.995767
-5.243581e-02	-5.328829e-01	1.001227e+00	0.978222	0.996175
-4.710283e-02	-3.158177e-01	9.101925e-01	0.978626	0.994393

Table 7: values of the normalised shear forces on section A and C at the design points presented in Table 6.

4 Conclusion

This paper has presented a framework that can be used to both generate and exploit metamodels. Polynomial chaos expansion has been selected for its ability to build explicit metamodels and to provide sensitivity information at no extra sampling cost, easing the dimensionality reduction process. A search algorithm based on particle swarm optimisation (PSO) has also been presented and used to explore metamodels generated by polynomial chaos expansion.

The experiments showed in the paper prove that the two methods can be effectively used together to optimise aircraft non-specific design, although the framework here presented is general enough to be applied to any engineering problem where the identification of input configurations producing a specified target level of the response under analysis is required.

Future work will mainly focus on the improvement of the metamodel generation stage. The increase in the maximum PCE order to model high-dimensional and highly non-linear responses is likely to make data gathering excessively expensive. As a result, alternative methods to PCE are currently under study. Non parametric metamodelling techniques like genetic programming (GP) or moving least squares (MLSM) have produced promising results. Furthermore, a better integration of the metamodel building and exploitation stages is required to encourage the use of the framework at industrial scale.

References

[1] E. Vladislavleva, “Model-based Problem Solving through Symbolic Regression via Pareto Genetic Programming", PhD dissertation Tilburg University, the Netherlands, 2008.

[2] T. W. Simpson, J. D. Peplinski, P. N. Koch and J. K. Allen, “Metamodels for computer-based engineering design: survey and recommendations”, Engineering with Computers, 17, 129—150, Springer-Verlag London Limited, 2001.

[3] M. S. Eldred, C. G. Webster, P. G. Constantine, "Evaluation of non-intrusive approaches for Wiener-Askey generalized polynomial chaos", Proceedings of the 49th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference, AIAA, Schaumburg, IL, USA, 2008.

[4] J. H. Friedman, "Multivariate adaptive regression splines", The Annals of Statistics, 19(1):1—141, 1991.

[5] I. M. Sobol, “Sensitivity estimates for nonlinear mathematical models”, Mathematical Modelling and Computational Experiment, 1(4): 407—414, John Wiley & Sons, 1993.

[6] B. Sudret, "Global sensitivity analysis using polynomial chaos expansions", Reliability Engineering & System Safety, 93(7), 2008.

[7] S. R. Arwade, M. Moradi, A. Louhghalam, “Variance decomposition and global sensitivity for structural systems”, Engineering Structures, 32:1—10, 2010.

[8] J. C. Helton and F. J. Davis, “Latin hypercube sampling and the propagation of uncertainty in analyses of complex systems”, Reliability engineering and system safety, 81:23—69, Elsevier, 2003.

[9] G. Georgiou, J. E. Cooper, “Virtual Prototyping and MDO of Aircraft Structures”, Proceedings of the DiPaRTLoads and Aeroelastics Workshop, Bristol, December 2011.

[10] M. H. B. Bonte, A. H. van den Boogaard, J. Huétink, “Solving optimisation problems in metal forming using finite element simulation and metamodelling techniques”, Proceedings of the Automatic Process Optimization in Materials Technology (APOMAT) Conference, Morschach, Switzerland, 2005.
[11] V. V. Toropov, U. Schramm, A. Sahai, R. D. Jones, T. Zeguer, “Design optimization and stochastic analysis based on the moving least squares method”, Proceedings of the Sixth World Congress of Structural and Multidisciplinary Optimization, Rio de Janeiro, Brasil, 2005.
[12] F. Harewood, R. Thornton, P. Sharp, “Step Change in Design: Exploring Sixty Stent Design Variations Overnight”, Proceedings of the biennial Altair CAE Technology Conference, UK, 2007.
[13] S. Shahpar, A. Polyinkin, V. V. Toropov, “Large scale optimization of transonic axial compressor rotor blades”, Proceedings of the 49th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conference, Schaumburg, Illinois, USA, 2008.
[14] M. Ramu, V. Prabhu Raja, P. R. Thyla, M. Gunaseelan, “Design optimization of complex structures using metamodels”, Jordan Journal of Mechanical and Industrial Engineering, 4(5), 2010.
[15] L. Lamberti, C. Pappalettere, "A Fast Big Bang-Big Crunch Optimization Algorithm for Weight Minimization of Truss Structures", in Y. Tsompanakis, B.H.V. Topping, (Editors), "Proceedings of the Second International Conference on Soft Computing Technology in Civil, Structural and Environmental Engineering", Civil-Comp Press, Stirlingshire, UK, Paper 11, 2011. doi:10.4203/ccp.97.11
[16] J. Kennedy, R. Eberhart, “Particle swarm optimization”, Proceedings of the IEEE International Conference on Neural Networks, 4:1942—1948, IEEE, 1995.
[17] Y.J. Xu, W.H. Zhang, D. Chamoret, M. Domaszewski, "Minimizing Thermal Residual Stresses in Ceramic Matrix Composites by using Particle Swarm Optimization Algorithm", in Y. Tsompanakis, B.H.V. Topping, (Editors), "Proceedings of the Second International Conference on Soft Computing Technology in Civil, Structural and Environmental Engineering", Civil-Comp Press, Stirlingshire, UK, Paper 37, 2011. doi:10.4203/ccp.97.37