GP Basics

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Focus: Selection of Run Parameters

- Population vs. generation size
- Mutation vs. crossover
- Choosing fitness function
- Training vs. forecast fitness
- Predictability of dependent variable
- Financial market Applications
  - Stock returns
  - Stock prices
  - Exchange rate

Population vs. Generation Size

- Basically there is no agreement in the literature.
- This is a research opportunity that will gain immediate attention.
- Suggested Strategy:
  - Select 5-10 series with known or measurable levels of complexity.
  - Run 100 searches twice to obtain best equation for each. Use 1000/100 for population/generation sizes once and 100/1000 for population/generation sizes another.
  - Select the fittest 10 from each run and compute the mean fitness for all selected outcomes.
  - Test the hypothesis that mean differences between those means is zero.

Population vs. Generation Size (Cont'd)

- Select i series, for i = 1, …, n.
- Obtain lowest j MSEs, for j = 1, …, k.
- Compute mean MSE for the ith series:

$$\overline{MSE_i} = \frac{1}{k}\left(\sum_{j=1}^{k} MSE_j\right)$$

- Test H_o:

$$\overline{MSE_d} = \frac{1}{n}\sum_{i=1}^{n}\left(\overline{MSE_{100/1000}} - \overline{MSE_{100/1000}}\right)_i = 0$$

- The test statistic is:

$$t = \frac{\overline{MSE_d}}{s_d / \sqrt{n}}$$

Mutation vs. Crossover

- GP researchers favor crossover while EP researchers favor mutation.
- Strategy to test the efficacy of either may depend on the relative complexity of a series. I would guess that more complex series may demand higher % of mutation.
- This is another area of research that demands immediate attention and can be conducted in a manner similar to the one described for determining population and generation sizes.

Choosing Fitness Function

- There are several fitness functions to choose from.
  - MSE
  - MAPE
  - Conditional MSE

$$P\left(\sum_{t=(T+1)}^{(T+K+L)}(x_t-\hat{x}_t)^2 \,|\, \min\sum_{t=1}^{T}(x_t-\hat{x}_t)^2\right) \geq$$
$$P\left(\sum_{t=(T+1)}^{(T+K+L)}(x_t-\hat{x}_t)^2 \,|\, \min\sum_{t=T+1}^{(T+K)}(x_t-\hat{x}_t)^2 \,\&\, \text{reasonable}\sum_{t=1}^{T}(x_t-\hat{x}_t)^2\right)$$

  - Conditional MAPE

$$P\left(\sum_{t=(T+1)}^{(T+K+L)}(x_t-\hat{x}_t)^2 \,|\, \min\sum_{t=1}^{T}(x_t-\hat{x}_t)^2\right) \geq$$
$$P\left(\sum_{t=(T+1)}^{(T+K+L)}(x_t-\hat{x}_t)^2 \,|\, \min\sum_{t=T+1}^{(T+K)}|x_t-\hat{x}_t| \,\&\, \text{reasonable}\sum_{t=1}^{T}(x_t-\hat{x}_t)^2\right)$$

Training vs. Forecast Fitness

- Selection of fitness function depends on the nature of the values of the dependent variable and the objective.
    - Selecting a function for cyclical data with large spreads between highs and lows can be a challenge. MSE will provide accurate predictions of larger values. MAPE will provide more accurate predictions of smaller values.
    - Conditional functions are useful when recent dynamics are believed to be more dominant than earlier dynamics.

Conditional			Unconditional		
	MAPE	MSE		MAPE	MSE
R 38 U	19.5	583.7	R 1 U	19.7	530.5
R 22 U	21.6	541.2	R 35 U	24.6	948.9
R 37 U	21.8	608.4	R 91 U	26.4	1082.8
R 42 U	23.8	565.6	R 79 U	28.2	1324.0
R 62 U	23.8	628.5	R 53 U	28.3	1890.7
R 8 U	24.0	605.5	R 26 U	29.5	1207.4
R 1 U	24.1	567.7	R 22 U	32.1	1003.1
R 44 U	24.6	701.9	R 54 U	33.7	1375.7
R 60U	25.5	653.1	R 83 U	34.9	1152.8
R 31 U	35.0	1987.7	R 13 U	36.8	1218.2

R 47 C	22.6	623.8	R 8 U	22.7	515.8
R 55 C	29.1	1553.2	R 88 U	25.4	608.5
R 70 C	29.9	932.7	R 15 U	28.3	784.8
R 41 C	31.1	985.1	R 2 U	32.0	922.2
R 15 C	31.2	1070.7	R 76 U	32.3	863.7
R 96 C	31.7	1111.2	R 24 U	34.7	979.4
R 50 C	33.1	1848.7	R 16 U	35.2	1098.6
R 66 C	35.0	1793.2	R 1 U	41.1	1428.6
R 63 C	35.7	1104.5	R 58 U	41.2	1741.1
R 44 C	36.6	1244.2	R 78 U	46.9	1872.0

R 94 C	21.2	614.0	R 50 U	20.9	549.0
R 33 C	21.6	652.3	R 52 U	21.0	565.3
R 7 C	22.0	1289.9	R 34 U	21.1	605.0
R 20 C	22.2	675.3	R 68 U	21.2	616.0
R 45 C	23.3	1003.4	R 47 U	23.9	816.2
R 74 C	24.2	1090.9	R 53 U	24.1	735.3
R 8 C	26.4	1667.0	R 60 U	24.9	1187.4
R 22 C	26.9	977.3	R 20 U	28.7	989.0
R 56 C	28.8	1177.2	R 90 U	29.3	1017.9
R 80 C	33.9	2169.9	R 10 U	36.2	2049.2

R 12 C	20.9	891.0	R 96 U	24.5	814.9
R 52 C	21.1	749.7	R 10 U	24.5	909.5
R 77 C	21.5	644.7	R 31 U	27.1	873.5
R 51 C	23.2	676.1	R 39 U	27.3	1122.7
R 8 C	23.9	741.6	R 17 U	28.6	992.2
R 85 C	25.6	1058.3	R 6 U	28.9	1172.4
R 23 C	29.7	1344.1	R 78 U	29.5	1235.9
R 11 C	30.5	1322.9	R 48 U	30.5	1143.8
R 87 C	30.8	1616.6	R 74 U	30.8	1525.3
R 33 C	33.0	1661.8	R 71 U	32.2	1365.9

R 41 C	20.2	631.4	R 34 U	22.7	539.4
R 22 C	20.3	637.1	R 71 U	22.9	653.6
R 85 C	21.9	610.9	R 66 U	22.9	695.4
R 14 C	24.5	608.1	R 30 U	23.2	653.8
R 82 C	25.0	708.9	R 84 U	24.5	649.4
R 13 C	26.3	878.4	R 69 U	25.8	804.8
R 86 C	28.4	944.2	R 53 U	28.7	1207.2
R 26 C	28.8	1024.6	R 39 U	31.0	933.9
R 42 C	29.4	1025.0	R 16 U	34.9	1152.8
R 94 C	30.1	1180.9	R 57 U	41.0	1414.1

Comparative Analysis

	Conditional			Unconditional	
	MAPE	MSE		MAPE	MSE
Average	26.51	999.68	Average	28.94	1030.79
Var	22.47	177955.3	Var	36.63	138191.8

	MAPE	MSE
$s_p^2 =$	5.4274	29034

Ho: Ave. MAPE for Conditional = Ave. MAPE for Unconditional

MAPE $t =$ -2.33 (72%)

Ho: Ave. MSE for Conditional = Ave. MSE for Unconditional

MSE $t =$ -0.408 (44%)

Recommendation

- Use the conditional MSE fitness.
- Select the fittest 10.
- Plot actual, fitted, *ex post* forecast, and *ex ante* forecast.
- Discard of any illogical forecast.
- Select that equation that seems to produce the best *ex post* forecast with the most logical *ex ante* forecast.

Predictability of Dependent Variable

- The predictability η-metric measures percentage predictable information in a variable Y_t. The η-metric applies to time series in general.

$$\eta = \max\left\{0,\ \frac{1}{n}\sum_{i=1}^{n}\left(1-\frac{SSE_Y}{SSE_S}\right)_n\right\}$$

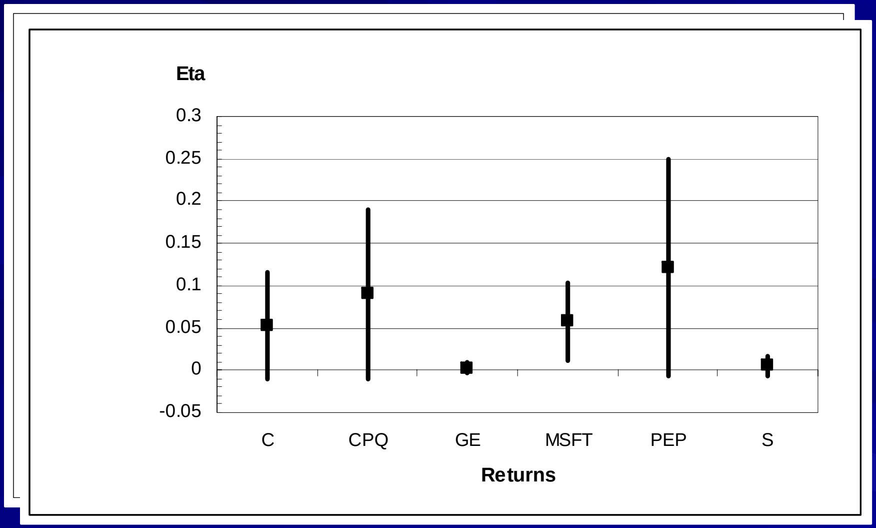
(S = Y_t with order of its observations randomly shuffled.)

- The metric can be used to conduct one of two hypothesis tests:
  - Ho: Y_t is GP-predictable,
    Ha: Y_t is not GP-predictable.
  - Ho: $Y1_t$ and $Y2_t$ are equally GP-predictable,
    Ha: $Y1_t$ and $Y2_t$ are not equally GP-predictable.

Application: Stock Returns

Predictability of Stock Returns						
	30-minute:		1-minute:		PCRs:	
Stocks	R^2	η (%)	R^2	η (%)	R^2	η (%)
BA	0.07	0.00	0.12	0.00	0.32	38.59
GE	0.03	0.00	0.10	1.07	0.59	72.44
GM	0.49	0.00	0.13	10.76	0.54	58.32
IBM	0.46	0.00	0.21	0.00	0.28	30.75
S	0.01	2.15	0.11	6.32	0.42	70.70
T	0.08	0.00	0.45	50.10	0.59	41.48
WMT	0.07	13.74	0.30	0.00	0.83	91.46
XON	0.02	7.01	0.11	0.00	0.56	47.67

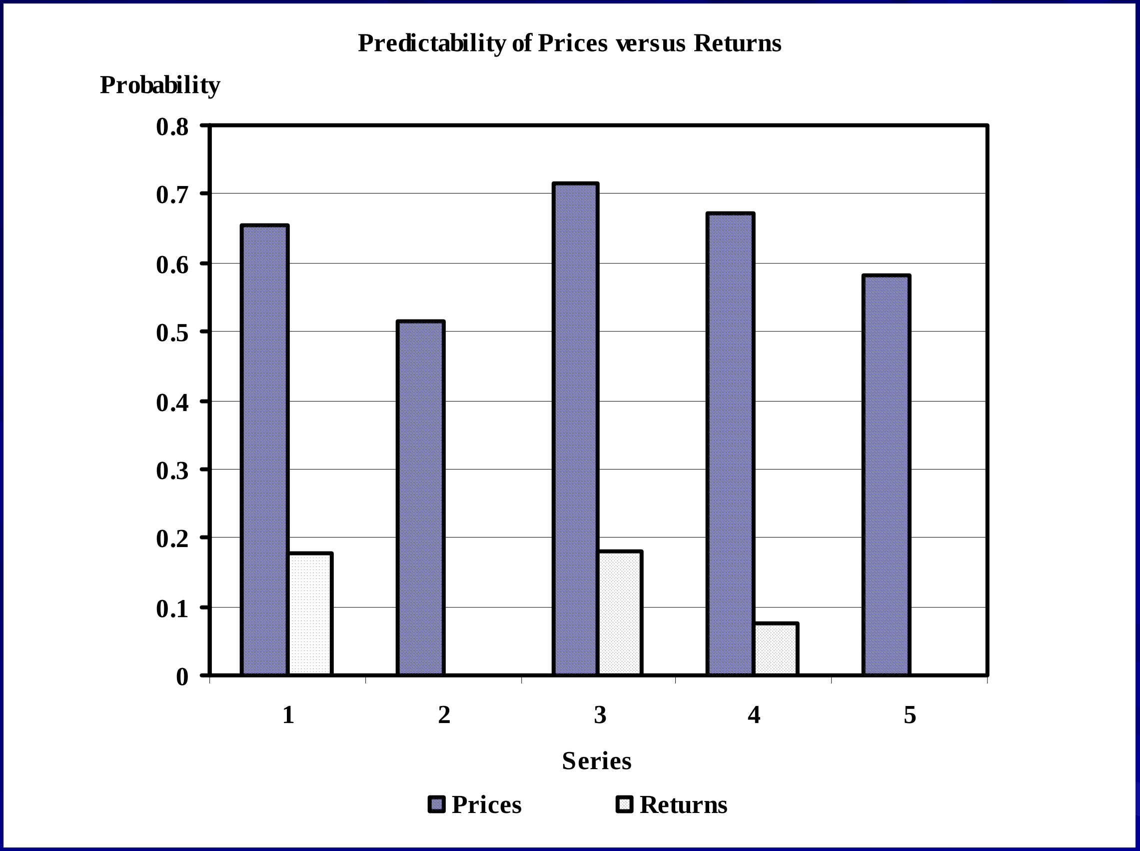
Price Levels vs. Returns

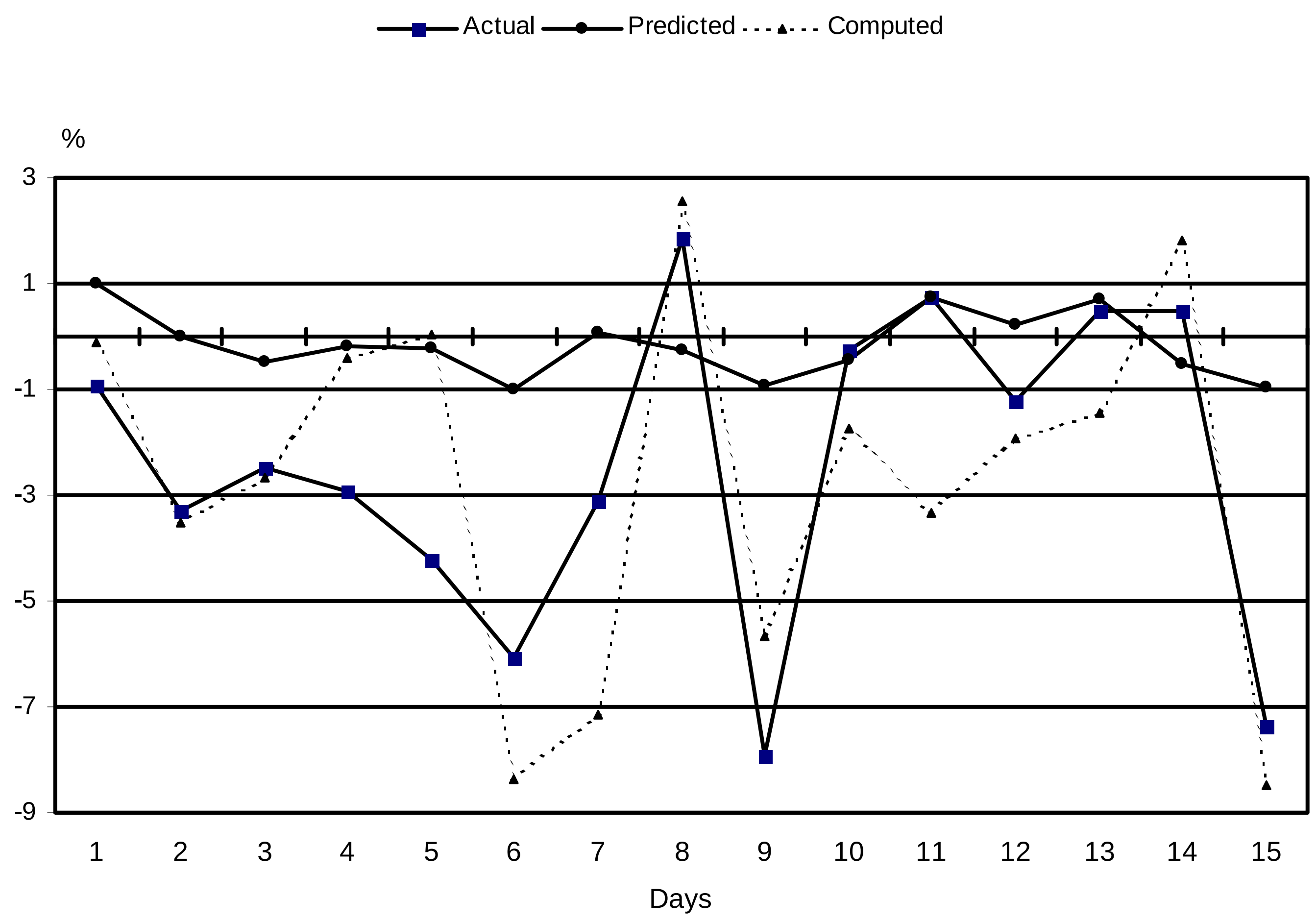


Application: Stock Prices

Testing mean difference between predicted and computed predicted prices

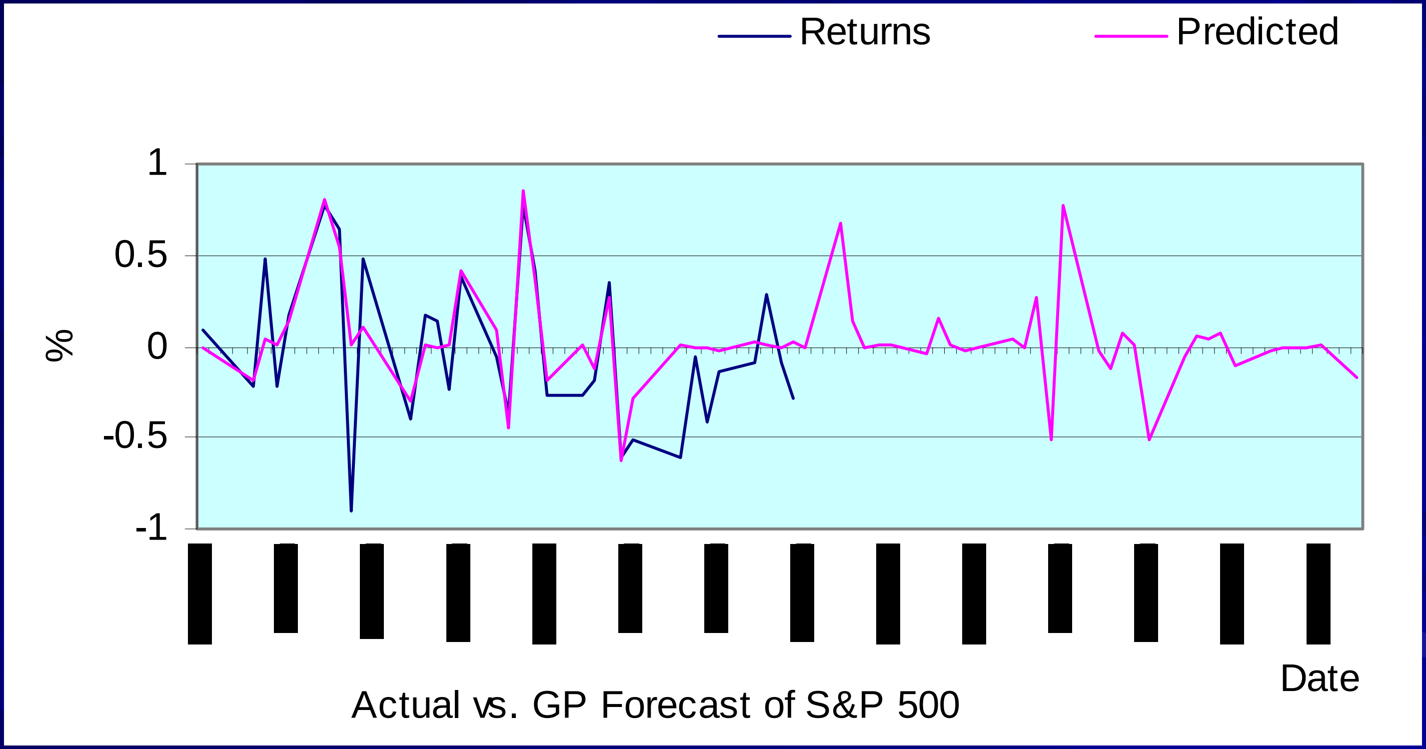
			MSE	
	n	**t**	**Predicted**	**Computed**
C	100	-2.400	2.327	2.873
CPQ	100	-2.895	0.831	1.405
GE	100	-4.341	3.162	4.745
MSFT	100	-4.097	1.636	2.158
PEP	100	-2.611	0.987	1.788
S	100	-2.695	1.910	2.707
All	600	-6.374	1.809	2.613


Predictability of Prices versus Returns
Probability
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1
2
3
4
5
Series
Prices
Returns

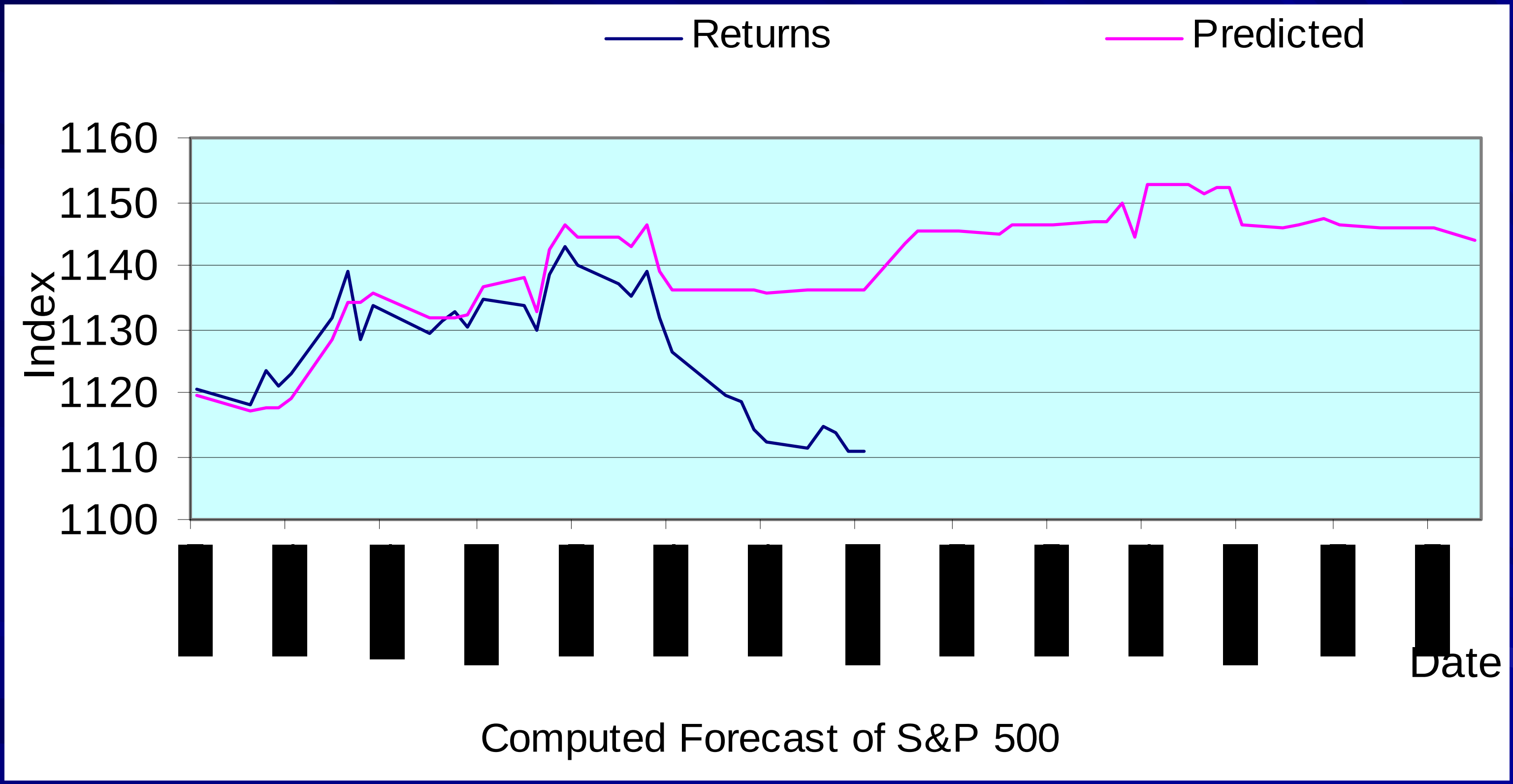


Actual, Predicted, and Computed Returns

S&P 500

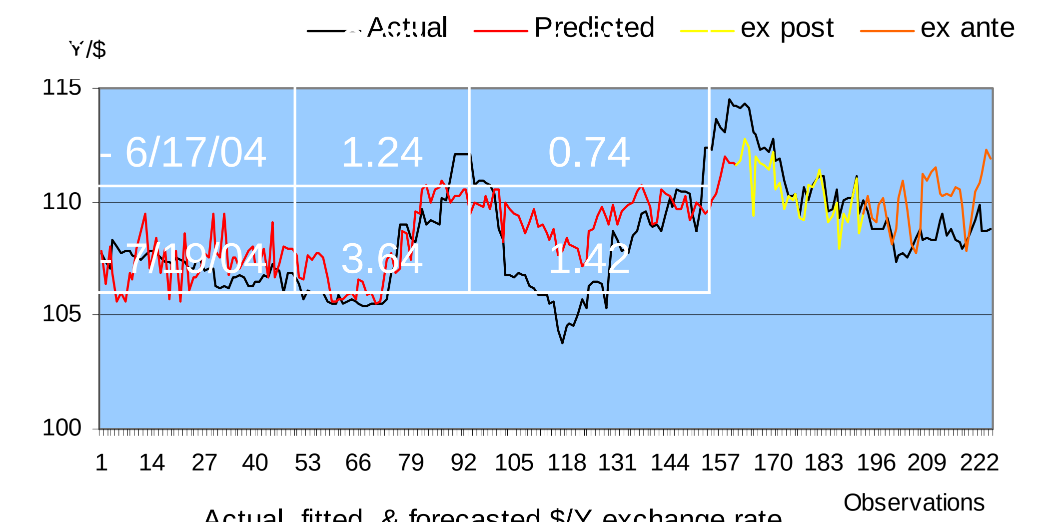

Returns
Predicted
1
0.5
0
-0.5
-1
%
Date
Actual vs. GP Forecast of S&P 500

S&P 500 Index Forecast



Application: Yen/$ Exchange Rate Forecast

	Period	MSE	MAPE
Training	12/	[illegible]	[illegible]
ex post	5/1 - 6/17/04	1.24	0.74
ex ante	6/1 - 7/19/04	3.64	1.42



Actual, fitted, & forecasted $/Y exchange rate

Summary & Conclusion

- Selecting between larger population and larger number of generations is unclear. It may depend on software written as well as complexity of series investigated.
- Favoring mutation over crossover seems logical although more research is needed.
- MSE seems to be the most logical fitness function to use. Using conditional MSE may help obtain better forecasts, but this also remains unclear and more research is needed to confirm its superiority.
- Using minimum lag length (MLL) models provides more useful for decision making forecasts. They may improve accuracy of forecasting of nonlinear systems.
- It may be easier to predict natural nonlinear phenomena than it is to predict outcomes of multi-agent decision where the agents are humans.

Next Topic

Wavelets