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MODIFICATION OF THE CRITIC METHOD USING FUZZY ROUGH NUMBERS

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Abstract:

This paper presents a new approach in the modification of the CRiteria Importance Through Intercriteria Correlation (CRITIC) method using fuzzy rough numbers. In the modified CRITIC method (CRITIC-M), the normalization procedure of the home matrix elements was improved as well as the aggregation function for information processing in the normalized home matrix. By introducing a new way of normalization, smaller deviations between normalized elements are obtained, which affect smaller values of standard deviation. Thus, the relationships between the data in the initial decision matrix are presented in a more objective way. The introduction of a new way of aggregating the values of weights in the CRITIC-M method enables a more comprehensive view of information in the initial decision matrix, which leads to obtaining more objective values of weights. A new concept of fuzzy rough numbers was used to address uncertainties in the CRITIC-M methodology.

Keywords: MCDM, fuzzy sets, rough sets, fuzzy rough numbers, CRITIC-M, DIBR

1. Introduction

Determining criterion weights is one of the key problems that arises in multi-criteria optimization models. In order to develop effective methods for determining the weight of the criteria, researchers around the world in recent years in the literature pay considerable attention to this problem. Most authors suggest dividing the model for determining the weights of criteria into subjective and objective [1].

Subjective approaches reflect the subjective opinion and intuition of the decision maker. In this approach, the weight of the criteria are determined based on the preferences of the decision maker. Traditional methods of determining weights of criteria include tradeoff method [2], proportional (ratio) method, Swing method [3] and Conjoint method [4], AHP model [5], SMART method [6], MACBETH method [7], Direct point allocation method [8], RDSW method [9], RC method [10], WLS method [11] and FPP method [12]. Recent subjective methods include multipurpose linear programming [13], linear programming [14], SWARA method [15], BWM [16] and FUCOM [17].

Among the most known objective methods are the following: Entropy method [18], CRITIC method [19] and FANMA method whose name was derived from the names of the authors of the method [20].

The CRITIC method is one of the most well-known and most frequently used objective methods, which uses standard deviations of the standardized criterion values of variants to determine the contrast of criteria, as well as the correlation coefficients of all pairs of columns. In this study, a modification of the CRITIC method in a fuzzy rough environment was proposed.

The rest of the work is organized as follows. The following section shows the preliminary settings for fuzzy rough numbers. Section 3 shows a modification of the CTIRIC method in a fuzzy rough environment. The fourth section of the paper presents the application of the fuzzy rough CRITIC-M method through an example from the literature. Concluding remarks and directions for future research are given in Section 5.

2. Preliminaries on fuzzy rough numbers

In the fuzzy rough concept, fuzzy theory was used to represent uncertainty in information, while rough theory was used to create flexible boundary intervals of fuzzy numbers. The use of hybrid fuzzy rough numbers eliminates the limitation of classic fuzzy type 2 numbers that have a predefined imprint of uncertainty. Hybrid fuzzy rough numbers (FRNs) are based on the basic concept of conventional rough numbers. Assuming that $\mathbb{N}$ denotes the universe containing all decision maker (DM) preferences involved in decision making and that triangular fuzzy numbers represent these preferences $\tilde{\tau}_i = \left(\tau_i^{(l)}, \tau_i^{(m)}, \tau_i^{(u)}\right)$. Then DMs preferences can be divided into x classes that satisfy the condition that $\tilde{\tau}_1 \le \tilde{\tau}_2 \le, ..., \le \tilde{\tau}_x$. If we assume that $\widetilde{\Lambda}$ is a collection of $\left(\tilde{\tau}_1, \tilde{\tau}_2, ..., \tilde{\tau}_x\right)$ and ϑ is an arbitrary element of $\mathbb{N}$, then the lower approximation of class $\tilde{\tau}_i$ can be defined as follows:

$$\underline{Apr}\left(\tau_i^{(l)}\right) = \bigcup_{1\le i\le x}\left\{\vartheta \in \mathbb{N} / \widetilde{\Lambda}(\vartheta) \le \tau_i^{(l)}\right\} \tag{1}$$

$$\underline{Apr}\left(\tau_i^{(m)}\right) = \bigcup_{1\le i\le x}\left\{\vartheta \in \mathbb{N} / \widetilde{\Lambda}(\vartheta) \le \tau_i^{(m)}\right\} \tag{2}$$

$$\underline{Apr}\left(\tau_i^{(u)}\right) = \bigcup_{1\le i\le x}\left\{\vartheta \in \mathbb{N} / \widetilde{\Lambda}(\vartheta) \le \tau_i^{(u)}\right\} \tag{3}$$

Also we can define the upper approximation:

$$\overline{Apr}\left(\tau_i^{(l)}\right) = \bigcup_{1\le i\le x}\left\{\vartheta \in \mathbb{N} / \widetilde{\Lambda}(\vartheta) \ge \tau_i^{(l)}\right\} \tag{4}$$

$$\overline{Apr}\left(\tau_i^{(m)}\right) = \bigcup_{1\le i\le x}\left\{\vartheta \in \mathbb{N} / \widetilde{\Lambda}(\vartheta) \ge \tau_i^{(m)}\right\} \tag{5}$$

$$\overline{Apr}\left(\tau_i^{(u)}\right) = \bigcup_{1\le i\le x}\left\{\vartheta \in \mathbb{N} / \widetilde{\Lambda}(\vartheta) \ge \tau_i^{(u)}\right\} \tag{6}$$

Then we can present the lower limit of $\tilde{\tau}_i$ as follows:

$$\underline{Lim}\left(\tau_i^{(l)}\right) = \left(\frac{1}{N_{Ll}}\sum_{i,j=1}^{N_{Ll}} \tau_i^{(l)\delta_1}\left(\prod_{j=1}^{N_{Ll}} \tau_j^{(l)\delta_2}\right)^{\frac{1}{N_{Ll}-1}}\right)^{\frac{1}{\delta_1+\delta_2}} \Bigg| \tau_i^{(l)\delta_1}, \tau_j^{(l)\delta_2} \in \underline{Apr}(\tau_i^{(l)}) \tag{7}$$

$$\underline{Lim}\left(\tau_i^{(m)}\right) = \left(\frac{1}{N_{Lm}}\sum_{i,j=1}^{N_{Lm}} \tau_i^{(m)\delta_1}\left(\prod_{j=1}^{N_{Lm}} \tau_j^{(m)\delta_2}\right)^{\frac{1}{N_{Lm}-1}}\right)^{\frac{1}{\delta_1+\delta_2}} \Bigg| \tau_i^{(m)\delta_1}, \tau_i^{(m)\delta_2} \in \underline{Apr}(\tau_i^{(m)}) \tag{8}$$

$$\underline{Lim}\left(\tau_i^{(u)}\right) = \left(\frac{1}{N_{Lu}}\sum_{i,j=1}^{N_{Lu}} \tau_i^{(u)\delta_1}\left(\prod_{j=1}^{N_{Lu}} \tau_j^{(u)\delta_2}\right)^{\frac{1}{N_{Lu}-1}}\right)^{\frac{1}{\delta_1+\delta_2}} \Bigg| \tau_i^{(u)\delta_1}, \tau_i^{(u)\delta_2} \in \underline{Apr}(\tau_i^{(u)}) \tag{9}$$

Also we can define the upper limits of $\tilde{\tau}_i$ as follows:

$$\overline{Lim}\left(\tau_i^{(l)}\right)=\left(\frac{1}{N_{Ul}}\sum_{i,j=1}^{N_{Ul}}\tau_i^{(l)\delta_1}\left(\prod_{j=1}^{N_{Ul}}\tau_j^{(l)\delta_2}\right)^{\frac{1}{N_{Ul}-1}}\right)^{\frac{1}{\delta_1+\delta_2}}\left|\tau_i^{(l)\delta_1},\tau_j^{(l)\delta_2}\in\overline{Apr}(\tau_i^{(l)})\right. \tag{10}$$

$$\overline{Lim}\left(\tau_i^{(m)}\right)=\left(\frac{1}{N_{Um}}\sum_{i,j=1}^{N_{Um}}\tau_i^{(m)\delta_1}\left(\prod_{j=1}^{N_{Um}}\tau_j^{(m)\delta_2}\right)^{\frac{1}{N_{Um}-1}}\right)^{\frac{1}{\delta_1+\delta_2}}\left|\tau_i^{(m)\delta_1},\tau_j^{(m)\delta_2}\in\overline{Apr}(\tau_i^{(m)})\right. \tag{11}$$

$$\overline{Lim}\left(\tau_i^{(u)}\right)=\left(\frac{1}{N_{Uu}}\sum_{i,j=1}^{N_{Uu}}\tau_i^{(u)\delta_1}\left(\prod_{j=1}^{N_{Uu}}\tau_j^{(u)\delta_2}\right)^{\frac{1}{N_{Uu}-1}}\right)^{\frac{1}{\delta_1+\delta_2}}\left|\tau_i^{(u)\delta_1},\tau_j^{(u)\delta_2}\in\overline{Apr}(\tau_i^{(u)})\right. \tag{12}$$

Then, we can represent FRN $\tilde{\tau}_i$ as follows:

$$FRN\left(\tilde{\tau}_i\right)=\left(\left[\tau_i^{(l)-},\tau_i^{(l)+}\right],\left[\tau_i^{(m)-},\tau_i^{(m)+}\right],\left[\tau_i^{(u)-},\tau_i^{(u)+}\right]\right) \tag{13}$$

3. Fuzzy rough CRITIC method

In the following part, the modified fuzzy rough CRITIC method algorithm is presented and testing is performed on an example from the literature.

Step 1. Construct the basic fuzzy rough decision matrix ($\Im$). We will assume that the evaluation of alternatives was performed by e experts using the fuzzy scale. Also, we will assume that expert preferences are presented in the home matrix $\Im^b=\left[\tilde{\vartheta}_{ij}^b\right]_{m\times n}$ where $1\le b\le e$; *i=1,...,m; j=1,...,n*; and $\tilde{\vartheta}_{ij}^b=\left(\vartheta_{ij}^{b(l)},\vartheta_{ij}^{b(m)},\vartheta_{ij}^{b(u)}\right)$ represent linguistic variables from the fuzzy scale used by expert *e*. For each element $\theta_{ij}^{e(l)}$, $\theta_{ij}^{e(m)}$ and $\theta_{ij}^{e(u)}$ from $\Im^b=\left[\tilde{\vartheta}_{ij}^b\right]_{m\times n}$ we form matrices of the aggregated sequences of experts $\Im^{b(l)}=\left[\tilde{\vartheta}_{ij}^{b(l)}\right]_{m\times n}$, $\Im^{b(l)}=\left[\tilde{\vartheta}_{ij}^{b(m)}\right]_{m\times n}$ and $\Im^{b(l)}=\left[\tilde{\vartheta}_{ij}^{b(u)}\right]_{m\times n}$. Using expressions (1)-(12) sequence $\vartheta_{ij}^{e(l)}$, $\vartheta_{ij}^{e(m)}$ and $\vartheta_{ij}^{e(u)}$ are transformed into fuzzy rough number $\overline{\vartheta}_{ij}^b=\left(\left[\overline{\vartheta}_{ij}^{b(l)-},\overline{\vartheta}_{ij}^{b(l)+}\right],\left[\overline{\vartheta}_{ij}^{b(m)-},\overline{\vartheta}_{ij}^{b(m)+}\right],\left[\overline{\vartheta}_{ij}^{b(u)-},\overline{\vartheta}_{ij}^{b(u)+}\right]\right)$; $1\le b\le e$. For fusion fuzzy rough values $\overline{\vartheta}_{ij}^b$ ($1\le b\le e$)the fuzzy rough weighted geometric Bonferroni function was used. This is how the aggregated fuzzy rough matrix $\Im=\left[\overline{\vartheta}_{ij}\right]_{m\times n}$ is defined.

Step 2. The elements of the matrix $\Im=\left[\overline{\vartheta}_{ij}\right]_{m\times n}$ are normalized as follows:

$$\overline{\xi}_{ij}=\begin{cases}\left[\left(\dfrac{\overline{\vartheta}_{ij}^{(l)-}}{\vartheta_{j+}^U},\dfrac{\overline{\vartheta}_{ij}^{(l)+}}{\vartheta_{j+}^U}\right),\left(\dfrac{\overline{\vartheta}_{ij}^{(m)-}}{\vartheta_{j+}^U},\dfrac{\overline{\vartheta}_{ij}^{(m)+}}{\vartheta_{j+}^U}\right),\left(\dfrac{\overline{\vartheta}_{ij}^{(u)-}}{\vartheta_{j+}^U},\dfrac{\overline{\vartheta}_{ij}^{(u)+}}{\vartheta_{j+}^U}\right)\right]; & if\quad j\in B,\\ \left[\left(\dfrac{\vartheta_{j-}^U}{\overline{\vartheta}_{ij}^{(u)+}},\dfrac{\vartheta_{j-}^U}{\overline{\vartheta}_{ij}^{(u)-}}\right),\left(\dfrac{\vartheta_{j-}^U}{\overline{\vartheta}_{ij}^{(m)+}},\dfrac{\vartheta_{j-}^U}{\overline{\vartheta}_{ij}^{(m)-}}\right),\left(\dfrac{\vartheta_{j-}^U}{\overline{\vartheta}_{ij}^{(l)+}},\dfrac{\vartheta_{j-}^U}{\overline{\vartheta}_{ij}^{(l)-}}\right)\right]; & if\quad j\in C\end{cases} \tag{14}$$

where $\vartheta_{i+}^{U} = \max_{1\le i\le m}(\vartheta_{ij}^{(u)+})$ and $\vartheta_{i-}^{L} = \min_{1\le i\le m}(\vartheta_{ij}^{(l)-})$.

Step 3: Construct a matrix of linear correlations. The amount of information W_j contained in criterion *j* is determined by applying expression (15):

$$W_j = \sigma_j \sum_{k=1}^{n}(1-l_{kj}) \tag{15}$$

Step 4: Calculations of weight coefficients of criteria:

$$w_j = \frac{\frac{\overline{\xi}_j}{1-\overline{\xi}_j}\cdot W_j}{\sum_{j=1}^{n}\left(\frac{\overline{\xi}_j}{1-\overline{\xi}_j}\cdot W_j\right)} \tag{16}$$

Example:

We will assume that the multi-criteria model considers the evaluation of three alternatives under five criteria. We will also assume five experts evaluated the alternatives using the fuzzy scale presented in Table 1.

Linguistic terms	Membership function
Absolutely low (AL)	(1, 1.5, 2.5)
Very low (VL)	(1.5, 2.5, 3.5)
Low (L)	(2.5, 3.5, 4.5)
Medium low (ML)	(3.5, 4.5, 5.5)
Equal (E)	(4.5, 5.5, 6.5)
Medium high (MH)	(5.5, 6.5, 7.5)
High (H)	(6.5, 7.5, 8.5)
Extremly high (EH)	(7.5, 8.5, 9.5)
Absolutely high (AH)	(8.5, 9, 10)

Table 1. Fuzzy scale

Experts' assessments of alternatives are presented in Table 2.

	A1	A2	A3
C1	EH,EH,EH,AH,AH	H,EH,H,MH,H	VL,L,L,L,L
C2	AH,AH,AH,AH,EH	E,ML,ML,ML,E	AH,AH,AH,H,AH
C3	EH,AH,AH,EH,AH	H,EH,H,EH,H	EH,H,H,EH,AH
C4	EH,AH,AH,H,AH	H,H,H,H,EH	MH,MH,E,MH,E
C5	VL,VL,AL,VL,VL	E,E,ML,ML,ML	AL,AL,VL,VL,AL

Table 2. Expert evaluation of alternatives

By applying expressions (1) - (12) the expert estimates were transformed into fuzzy rough values, Table 3.

Crit.	A1	A2	A3
C1	([7.56,8.28],[8.53,8.89],[9.53,9.89])	([5.96,6.75],[6.97,7.76],[7.97,8.76])	([1.93,2.44],[2.95,3.44],[3.96,4.45])
C2	([7.97,8.50],[8.74,9.00],[9.74,10.0])	([3.56,4.13],[4.56,5.13],[5.56,6.14])	([7.41,8.39],[8.19,8.92],[9.20,9.92])
C3	([7.70,8.43],[8.60,8.97],[9.60,9.97])	([6.56,7.14],[7.56,8.14],[8.56,9.14])	([6.70,7.64],[7.70,8.49],[8.70,9.49])
C4	([7.16,8.43],[8.07,8.97],[9.07,9.97])	([6.50,6.81],[7.5,7.81],[8.50,8.810])	([4.70,5.33],[5.70,6.33],[6.70,7.33])
C5	([1.22,1.50],[1.93,2.50],[2.95,3.50])	([3.56,4.13],[4.56,5.13],[5.56,6.14])	([1.03,1.31],[1.55,2.11],[2.56,3.12])

Table 3. Fuzzy rough home matrix

Using the expression (14), the elements from Table 3 were normalized. Then, using the expressions (15) and (16), the matrices of linear correlations of fuzzy rough elements were defined and the final values of the weighting coefficients were determined as follows:

$w_1 = 0.153;$

$w_2 = 0.380;$

$w_3 = 0.189;$

$w_4 = 0.118;$

$w_5 = 0.160.$

5. Conclusion

This research presents a modification of the CRITIC method using fuzzy rough numbers. Fuzzy rough numbers are applied because part of the uncertainty and subjectivity are neglected in the classic fuzzy and rough models. Given the well-known performance of fuzzy sets in representing uncertainties and confirmed advantages of rough numbers in subjectivity manipulation, a modification of the CRITIC method based on information processing using hybrid fuzzy rough numbers is proposed. Also, the application of the fuzzy rough CRITIC method is shown in an example that considers the evaluation of three alternatives under five criteria.

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