MODELING 2D STEADY FLOW THROUGH A POROUS MEDIUM WITH FREE SURFACE USING PHYSICS-INFORMED NEURAL NETWORKS

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Abstract:

The problems of flow through a porous medium appear in a variety of engineering branches. In the context of various numerical methods, such as the finite element method (FEM), a few procedures have been developed for determining the potential field in the cases in which a free surface exists [1]. Although these procedures provide good results, they are time-consuming. Widely used machine learning techniques can bridge this problem. This research aims to show the application of the machine learning method for the simulation of a steady free surface flows within a porous structure using a novel method called physics-informed neural networks (PINNs). Physical laws were incorporated in the neural network in the form of partial differential equations (PDE) with corresponding initial and boundary conditions, utilizing Darcy's law and Laplace's equation of continuity. The steady free surface flow, which is approximated using the PINN, was compared with the results obtained by analytical solution.

Keywords: porous media, steady flow, free surface, physics-informed neural networks

1. Introduction

Most of the two- and three-dimensional flow problems can be described by a system of non-linear partial differential equations for which analytical solutions are not available unless a great number of assumptions are made to simplify the problem and make the equations applicable only to idealized cases [2]. Also, various numerical methods can be applied in complicated flow problems and one of them is the finite element method. In the context of the finite element method, several procedures have been developed to handle the important class of the steady free surface flow problem [1]. Due to the FEM requirement of modifying the mesh, it becomes necessary to evaluate element properties for the whole discretized mesh at each cycle of iteration for steady flow. The physics-informed neural networks approach can provide good results with less computational effort, time savings and better flexibility in solving inverse problems such as parameter identification and data assimilation. Physics-informed neural networks are trained to solve supervised learning tasks while respecting any given law of physics described by general nonlinear partial differential equations [3]. The major innovation with PINNs is the introduction of a residual network that encodes the governing physics equations, takes the output of a deep-learning network, called the approximator, and calculates a residual value. The residual of the differential equation is minimized by training the neural network. The PINN calculates differential operators on graphs using automatic differentiation. This work presents the potential of using PINNs for modeling two-dimensional steady flow through a porous medium with a free surface by joining the physical laws with the initial and boundary conditions in the loss function.

2. Materials and Methods

2.1 2D steady flow through a porous medium with a free surface – basic physical laws

Two-dimensional steady flow through a porous medium is governed by a difference in potential on two surfaces. The flow velocity q of the fluid, also known as Darsy's velocity, represents the volume of the fluid flowing per unit time, per unit area of the porous medium. Darcy's velocity can be described using potential ϕ through the relation called Darcy's law:

$$q = -K\nabla\phi, \tag{1}$$

where K represents the material properties of an orthotropic material, and $\nabla\phi$ is the potential gradient. Here, K is also known as the permeability matrix with the following form:

$$K = \begin{bmatrix} k_x & 0 \\ 0 & k_y \end{bmatrix}, \tag{2}$$

where k_x and k_y are coefficients of permeability in the x and y axes respectively. The operator ∇ is defined by:

$$\nabla = \begin{bmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \end{bmatrix}. \tag{3}$$

The component form of equation (1) is:

$$q_i = -k_i \frac{\partial \phi}{\partial x}, \; i = x, y, \tag{4}$$

where x and y are principal axes for orthotropic materials. Water is assumed to be incompressible, therefore, the mass conservation equation is reduced to:

$$\nabla q = 0. \tag{5}$$

Using Darcy's law, the mass conservation equation can be redefined as:

$$k_x \frac{\partial^2 \phi}{\partial x^2} + k_y \frac{\partial^2 \phi}{\partial y^2} = 0. \tag{6}$$

2.2 Test case

The domain $D \subset \mathbb{R}^2$ occupied by the porous medium is assumed to be either wet or dry. The problem we examined is the flow with the free surface, defined as the boundary line between the wet and dry soils, shown in Figure 1. The assumptions are that the flow is a two-dimensional steady flow that obeys Darcy's law, and that the soil has homogeneous and isotropic characteristics with coefficients of permeability $k_x = k_y = 1$. Because of these assumptions, the flow can be determined by the potential ϕ, which can be obtained from (6) with conditions shown in Figure 1.

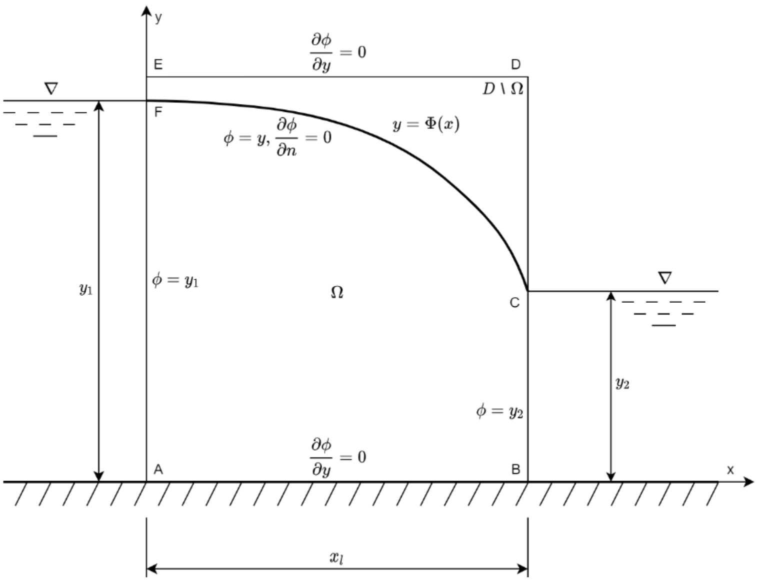

Figure 1 – Initial and boundary conditions for steady flow through a porous medium with a free surface

The flow region $\Omega \subset D$ is indicated in Figure 1 as $ABCF$ region. The flow occurs between reservoirs of height y_1 and y_2, $y_1 > y_2$. The problem for the case was solved when the heights of these reservoirs are $y_1 = 2\,m$ and $y_2 = 1m$. Furthermore, initial conditions apply at the beginning and the end of the dam of length $x_l = 2m$, where $\phi(0m, y) = 2m$ and $\phi(2m, y) = 1m$. Additionally, at the crest and the bottom of the dam we apply condition $\frac{\partial \phi}{\partial y} = 0$. The location of the curve $y = \Phi(x)$ that represents the free surface is unknown, as well as the potential field ϕ. The only fact that allows us to find the free surface is that every point (x, y) on the free surface has the potential $\phi = y$ with a condition $\frac{\partial \phi}{\partial n} = 0$, with n being a unit vector normal to the free surface.

2.3 Embedding physical laws into neural network

We propose a solution using PINNs to solve the PDE (6) with indispensable boundary conditions presented in Figure 1. For the flow modeling, we made a neural network consisting of two subnets: an approximator network and a residual network. The approximator network takes x and y as input, and provides a solution $\hat{\phi}(x, y)$ of the PDE (6) at a given input point (x, y). The approximator network is trained on a set of so-called *collocation points*. Each collocation point (x, y) belongs to the domain occupied by the porous medium. The residual network takes the output of the approximator network as its input. This network is not trained at all but its role is crucial for embedding physical laws through PDE and boundary conditions into the neural network. The final outputs of the PINN model are outputs of the residual network obtained as:

$$r = \hat{\phi}(x, y) - \phi(x, y), \tag{7}$$

$$i_0 = \hat{\phi}(0, y) - \phi(0, y), \tag{8}$$

$$i_1 = \hat{\phi}(2, y) - \phi(2, y), \tag{9}$$

$$b_0 = \frac{\partial \hat{\phi}(x,0)}{\partial y} - \frac{\partial \phi(x,0)}{\partial y}, \tag{10}$$

$$b_1 = \frac{\partial \hat{\phi}(x,2.1)}{\partial y} - \frac{\partial \phi(x,2.1)}{\partial y}, \tag{11}$$

$$b_2 = \begin{cases} \frac{\partial \hat{\phi}(x,y)}{\partial n} - \frac{\partial \phi(x,y)}{\partial n}, & \phi = y \\ 0, & \phi \neq y \end{cases}, \tag{12}$$

where $\hat{\phi}(x, y)$ is output of the approximator network and $\phi(x, y)$ is determined by the PDE (6) and boundary and initial conditions from Figure 1. The solution is reached by finding the weights and biases of the approximator network that minimize a loss function which is composed of the residuals (7)-(12) over a set of collocation points. The goal is to minimize the Mean Squared Error (MSE) loss function with the following form:

$$MSE = \frac{1}{N_{x_r,y_r}} \sum |r|^2 + \sum_{i=0}^{1} \left(\frac{1}{N_{x_i,y_i}} \sum |i_i|^2 \right) + \sum_{j=0}^{2} \left(\frac{1}{N_{x_j,y_j}} \sum |b_j|^2 \right), \tag{13}$$

where N_{x_r,y_r} represents the total number of collocation points on $\phi(x, y)$, and N_{x_i,y_i} and N_{x_j,y_j} represent numbers of collocation points of the corresponding initial and boundary conditions.

2.4 Creating the PINN model

For creating the PINN model we employed SciANN [4]. SciANN is a Python package for scientific computing and physics-informed deep learning using artificial neural networks, written in Python using Keras and TensorFlow backends. In order to solve the problem we required a model that receives x and y coordinates as input. The resulting model was trained on collocation points, i.e., a grid of points (x_i, y_i) with x coordinates in range $[0m, 2m]$, and y coordinates in range $[0m, 2.1m]$ in accordance with the test case. In order to obtain a precise form of the free surface, the grid of points had to be denser and above the line $y = 1$, where the free surface was expected. The points were arranged with spatial steps $\Delta x_1 = \Delta y_1 = 1 \cdot 10^{-2} m$ below the line $y = 1$, and with spatial steps $\Delta x_2 = \Delta y_2 = 2.5 \cdot 10^{-3} m$ above the line $y = 1$. We constructed a physics-informed neural network that contains 8 layers with 20 neurons per layer. All neurons located in hidden layers perform a sigmoid activation function. The neural network was trained for 600 epochs with learning rate $2 \cdot 10^{-3}$ and batch size of 1024 to minimize MSE (13).

3 Results and discussion

We compared the free surface $\widehat{\Phi}(x)$ given by the trained model with a known analytic form of the free surface $\Phi(x) = \sqrt{4 - 1.5x}$. The comparison was made at points different than those used for the training process. According to Figure 1, the free surface consists of points (x, y) in which the condition $\phi = y$ is fulfilled. The obtained results are given in Figure 2.

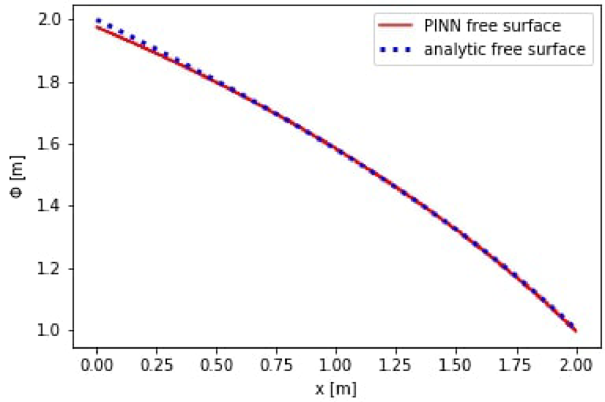

Figure 2 – Free surface calculated using analytic equation and PINN

As can be seen from Figure 2, free surfaces provided by the PINN model and calculated by the analytic method are very similar. The value of the Root Mean Squared Error (RMSE) calculated between the analytic $\Phi(x)$ and the predicted $\widehat{\Phi}(x)$ free surface is $RMSE = 0.0072m$.

4. Conclusions

Based on the presented results, we may conclude that the created PINN model can solve the steady flow problem with the free surface, predicting not only the potential field as a solution of the PDE, but also the position of the free surface. With respect to the obtained results, future research will be directed toward solving transient flow through the soil with inhomogeneous and anisotropic characteristics. Hyperparameter tuning using evolutionary algorithms will also be considered.

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