A New Methodology for Optimizing Evasive Maneuvers Under Uncertainty in the Extended Two-Dimensional Pursuer/Evader Problem

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ABSTRACT: Traditional analytic or control-theoretic solutions to the problem of optimizing evasive maneuvers in the extended two-dimensional pursuer/evader problem require the evader to execute specific sequences of maneuvers at precise pursuer/evader distances. These solutions depend upon several pursuer-specific characteristics, and fail to effectively account for uncertainty about the state of the pursuer. This paper describes the implementation of a genetic programming system that evolves optimized solutions to the extended two-dimensional pursuer/evader problem that do *not* depend upon knowledge of pursuer's *current state*. Best-of-run programs execute strategies by which an evader may maneuver to successfully evade a pursuer starting from a wide range of relative initial positions, under conditions where the state of the pursuer is unknown or uncertain.

I. INTRODUCTION

The two-dimensional pursuer/evader problem (Hamalainen and Ehtamo, 1990) is a competitive zero-sum game in which a faster, more agile pursuer is given a limited amount of time to capture an evader as both are traveling across a plane. The game ends favorably for the evader if it manages to stay outside the *capture radius* of the pursuer (the maximum distance at which the pursuer is considered to have captured the evader) for the duration of the encounter. A solution to the two-dimensional pursuer/evader problem must incorporate an *optimized strategy* for maneuvering the evader in a manner that successfully escapes the pursuer, regardless of the initial conditions of the system.

(Moore and Garcia, 1997a) described a genetic programming (GP) system that identifies optimized solutions to the *extended two-dimensional pursuer/evader problem* (E2DPE). Formal evaluation of this system incorporated physical data (including mass) and performance characteristics (such as maximum thrust, maximum turning rate, fuel consumption rate, and drag coefficients) of an F-16C aircraft evader (Lambert and Munson, 1994) and various types of Soviet surface-to-air missile (SAM) pursuers (Cullen and Foss, 1995). Each best-of-run program was evolved against a training population of pursuers of a single SAM type, starting from a variety of positions. Subsequent testing against pursuers of the same type demonstrated that the resulting best-of-run programs were capable of evading a high percentage of pursuers from a large, representative test population. Unfortunately, best-of-run programs optimized against one type of pursuer generally did not perform optimally when tested against other types of pursuers. Subsequent research (Moore and Garcia, 1997b) investigated the impact of uncertainty about the *type* of pursuer in the E2DPE problem, and identified a methodology for evolving programs that exhibit optimized performance against multiple pursuer types, by using training populations that reflect specific probability distributions over those types.

These GP solutions to the E2DPE problem assumed that the evader had total knowledge of the *state* of the pursuer (its relative position, velocity, and acceleration vectors) for the duration of each encounter. The resulting best-of-run programs used selection statements based upon the current relative pursuer/evader distance to execute specific sequences of thrusting and turning commands. In contrast, the research described in this paper investigates the problem of identifying optimized evasion strategies in the E2DPE problem that do *not* depend upon knowledge of the *current state* of the pursuer. The resulting methodology represents an

alternative to current analytic and control-theoretic solutions to the E2DPE problem.

II. AN OVERVIEW OF THE EXTENDED TWO-DIMENSIONAL PURSUER/EVADER PROBLEM

As shown in Figure 1, the E2DPE problem models pursuer P and evader E as point masses whose motions across a plane are controlled by thrusting forces (applied in the direction of the velocity vector) and turning forces (applied in a direction that is perpendicular to the velocity vector). Both P and E are affected by drag forces and momentum; instantaneous changes in direction are not possible. Since acceleration (a), force (F), and mass (m) are related by the equation a = F/m, the acceleration of P or E depends on its current mass, as well as the magnitude of the applied thrusting and turning forces. The effects of these forces depend on the current state of P or E. The maximum distance over which P pursues E depends on the type of pursuer. P captures E as soon as the distance between them becomes less than a pursuer-specific capture radius.

Prior to the start of the encounter, the pursuer uses the initial state of the evader to predict an *intercept point.* The pursuer then accelerates at maximum thrust towards the intercept point. If the evader fails to maneuver, the pursuer captures the evader at (or very close to) the intercept point. If the evader maneuvers, the pursuer relies upon the highly effective *proportional navigation* technique (Ball, 1985) to compensate for those maneuvers. Proportional navigation causes the pursuer to accelerate in the direction perpendicular the line-of-sight from the pursuer to the evader; the magnitude of this acceleration is calculated by the equation

$$n_c = N' V_c (d\lambda/dt)$$

where N' is a unitless designer-chosen gain known as the *effective navigation ratio*, and V_c is the pursuer-evader closing velocity vector (the negative rate of change of the distance from the pursuer to the evader). The time derivative of the line-of-sight angle λ is known as the line-of-sight rate. For practical guidance systems, optimal values for N' range between 3 and 5 (Ramo and Pucket, 1959).

The evader maneuvers by executing specific combinations of thrusting and turning forces in specific sequences. The *optimal strategy* for the evader (Zarchan, 1990) is to maneuver in a manner that maximizes the likelihood of evading the pursuer, regardless of the initial state of the evader and the relative starting position of the pursuer.

Note that by rotating the reference coordinate system at the starting location of the pursuer, the initial pursuer/evader line-of-sight angle λ_0 may be considered constant for all pursuer/evader pairs. For this reason, the only variables necessary to describe the initial

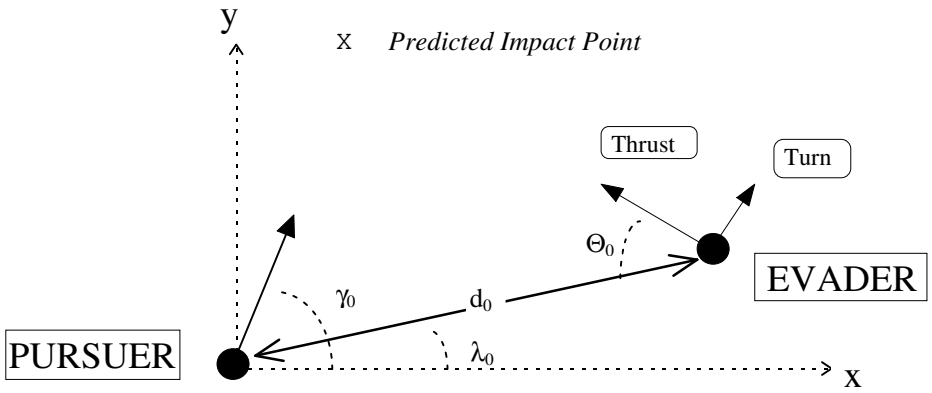

Figure 1. The Extended Two-Dimensional Pursuer/Evader Problem (Initial Conditions).

configuration of each confrontation are the *line-of-sight distance* between the evader and the pursuer, and the *velocity vector* of the evader at the beginning of the encounter.

For each pursuer type, the minimum and maximum effective range of the pursuer defines the range of possible initial line-of-sight distances. The aggregate fitness of a specific

program reflects its fitness when evaluated against each of the initial pursuer positions in a specified training population. Optimal evasion programs output commands that assert *thrusting and turning forces* at appropriate moments in order to accomplish maneuvers that optimize evader survivability.

III. PRIOR RESEARCH

A GP solution to the E2DPE problem was successfully implemented (Moore and Garcia, 1997a). During each generation of the GP approach, each member of a population of programs for maneuvering the evader was evaluated against each member of a training population of pursuers. Each GP run optimized maneuvers against a single type of pursuer. Proportional navigation was used by all pursuers. The pursuer and evader had complete knowledge of each other's current state (relative position, velocity, and acceleration). A fitness function was used to qualitatively evaluate the survivability of each evasion program during a simulated encounter; the aggregate fitness of a particular program reflected its overall survivability when independently evaluated against all of the pursuers in the training population. Fitness-proportionate reproduction, together with crossover, was used to create each new generation. Each run was terminated after a fixed number of generations.

A tableau for our GP solution to the original E2DPE problem (Moore and Garcia, 1997a) is shown Figure 2. Cartesian coordinates were used to represent components of position, velocity, and acceleration for both the evader and the pursuer. This simple set of terminals proved to be sufficient to allow the GP system to converge to a solution of this problem. The E2DPE problem fixed the origin of the coordinate system (x = 0, y = 0) at the position of the pursuer. PX and PY thus continually designated the relative displacement from the pursuer to the evader. For each of the functions used by this GP system, an "argument" consists of any syntactically valid composition of functions, variables, and constants that returns a floating-point value in the range [-1.0 … +1.0].

Function ifDistance is a three-argument selection function: if the current distance between the pursuer and evader is less than the percentage of the pursuer's maximum pursuit distance specified by the absolute value of the first argument, then the second argument is evaluated; otherwise the third argument is evaluated. Functions ifPX and ifPY are two-argument selection functions: ifPX evaluates its first argument if the current x-position of the evader relative to the pursuer is negative; otherwise ifPX evaluates its second argument. Function ifPY is defined similarly for the relative y-position of the evader. Functions ifDistance, ifPX, and ifPY return the value of the evaluated argument.

Functions setThrust and hardTurn are single-argument functions. Function setThrust causes the thrust output of the evader to be set to the percentage of its maximum thrust specified by its argument; for example, setThrust (0.9) will set evader thrust to 90% of its maximum possible value. setThrust ignores the sign of its argument: thrust is assumed to act in the direction of the current evader velocity vector. Function hardTurn causes the evader to execute a turn whose g-force equals the percentage of the maximum allowable turning force of the evader/pilot system specified by its argument; for example, if the maximum turning force is 4 gravities (g's), then hardTurn (0.5) will cause the evader to execute a 2g turn in a direction which is perpendicular and to the right of the current evader velocity vector; the function call hardTurn (-0.5) would result in a 2g evader turn to the left. Both setThrust and hardTurn are assumed to act instantaneously, and both return the value of their input argument.

Fitness cases were identified by two floating-point values ranging from 0.0 to 1.0. The first value, denoted J, identifies the initial line-of-sight distance from the pursuer to the evader. If D_{min} and D_{max} denote the minimum and maximum effective initial distances for the pursuer, then the initial line-of-sight distance d_0 may be calculated as follows:

$$d_0 = D_{min} + (J * (D_{max} - D_{min}))$$

Objective:	Determine an optimized evasion strategy for the Extended Two-Dimensional Pursuer/Evader (E2DPE) problem.
Terminal Set:	***R***, the ephemeral random floating-point constant ranging from -1.0 to 1.0.
Function Set:	ifDistance, ifPX, ifPY setThrust

	hardTurn
Fitness Cases:	Numerous fitness cases which differ according to the distance from the pursuer to the evader at the start of the encounter, as well as the acute angle between the initial velocity vector of the evader and the line-of-sight vector from evader to pursuer.
Raw Fitness:	The number of encounters during which the distance from pursuer to evader is less than or equal to the capture radius of the pursuer.
Standardized Fitness:	Same as Raw Fitness for this problem.
Hits:	The number of fitness cases that result in capture of the evader prior to the maximum pursuit time of the pursuer. (The encounter is also terminated if the distance between the evader and the pursuer exceeds a pursuer-specific value, at which time the pursuer is considered to have missed the evader.)
Wrapper:	N/A
Parameters:	Population Size M = 100, Maximum Number of Generations G = 21.
Success Predicate:	None.

Figure 2. A Tableau for the State-Dependent Solution to Extended Two-Dimensional Pursuer/Evader Problem

D_{min} and D_{max} depend upon the type of pursuer. The second value, denoted K, identifies the angle that the initial velocity vector of the incoming evader makes with the line-of-sight from the evader to the pursuer. Let Θ_0 denote this angle. If Θ_{min} and Θ_{max} denote the minimum and maximum initial value of Θ, then Θ_0 may be calculated in the following manner:

$$\Theta_0 = \Theta_{min} + (K * (\Theta_{max} - \Theta_{min}))$$

To maintain the relative geometry illustrated in Figure 1 for the pursuer/evader problems addressed by this research, Θ_{min} and Θ_{max} described a range of values between 10 and 80 degrees. For this study, the magnitude of the evader's initial velocity vector (its "speed") was assumed to be the same for each encounter. Thus, each fitness case corresponded to a specific combination of values from sets J and K.

The research described in (Moore and Garcia, 1997a) identified a methodology for evolving optimized evasion strategies (in the form of programs) for a variety of pursuer populations. We began with the two-dimensional problem to reduce the required amount of computation as much as possible. Implicit in the model described above is the assumption that the magnitude of the turning force is independent of the velocity of the evader. Additionally, limitations on the maximum sustainable g-force of the evader were imposed by restricting the magnitude of the evader's turning force; the contribution of thrust to current evader g-force was ignored. We also assumed that the evader stalled if its speed fell below a specified minimum value, resulting in a "capture" for the pursuer. For the purposes of this study, all evaders were assumed to be traveling *inbound* (towards the pursuer) at the start of each confrontation, with lead angle $\gamma_0 > \lambda_0$, as shown in Figure 1. Each best-of-run program represented an optimized evasion technique for a specific type of evader against a specific type of pursuer.

The evader used in (Moore and Garcia, 1997a) was modeled using physical data and performance characteristics of an F-16C aircraft evader, as described in (Stevens and Lewis, 1992). Each pursuer was modeled using physical data and performance characteristics of an SA-6, SA-13, or SA-15 SAM pursuer, in a manner similar to the approach described in (Klos et al, 1994). Programs were evolved using a training population consisting of a single type of pursuer, starting from a variety of initial positions. Best-of-run programs were evolved for each type of pursuer using a fixed training population of 32 pursuers described by the cross-product of sets J and K:

J = {0.2, 0.3, 0.4, 0.45, 0.5, 0.6, 0.7, 0.8}
K = {2/9, 3/9, 4/9, 5/9}

This combination of values includes several of the most difficult pursuer/evader encounters, which occur when the values for both J_i and K_i are small (denoting small initial pursuer/evader distances and lead angles).

Five separate training runs were performed for each type of pursuer, with each run using a

different random number seed during the creation and subsequent evolution of the program population. The particular set of random number seeds used for this study was:

Seed ∈ {0.2, 0.4, 0.6, 0.8, 1.0}

After evolving five best-of-run programs for each type of pursuer, these programs were subsequently tested against a population of 128 pursuers, described by the following set values for J and K:

J = {0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.45, 0.5, 0.55, 0.6, 0.65, 0.7, 0.75, 0.8, 0.85}

K = {1/9, 2/9, 3/9, 4/9, 5/9, 6/9, 7/9, 8/9}

The results of these tests, showing the number and percentage of successful evasions for each best-of-run program evolved against each type of pursuer, are shown in Figure 3. Note that each entry represents the result of testing a program optimized against a specific type of pursuer, using a test population consisting of the same type of pursuers.

IV. INTRODUCING UNCERTAINTY ABOUT THE STATE OF THE PURSUER

Uncertainty introduces a degree of complexity into the E2DPE problem that is difficult to model using traditional analytical and control-theoretic approaches (Krasovskii and Subbotin, 1988). Techniques for evolving optimized solutions to the E2DPE problem under uncertainty about the *type* of pursuer were examined in (Moore and Garcia, 1997b). In contrast, this study is concerned with determining a methodology for using GP to evolve programs that exhibit optimized performance when the *state* of the pursuer is either unknown or uncertain. "Uncertainty about pursuer state" occurs when the evader is deprived of important information describing the current state of the pursuer. The critical question addressed by this paper is:

Can genetic programming be used to evolve programs that perform near-optimally in situations where the state of the pursuer is either unknown or uncertain?

If so, these programs would represent an alternative to classic analytic and control-theoretic solutions to pursuer/evader problems (Shinar and Steinberg, 1977; Grimm and Well, 1990; Zarchan, 1990; Barron, 1995), which require precise knowledge of the state of the pursuer to identify optimal evasion strategies.

To begin to answer this question, we replaced the selection functions ifDistance, ifPX, and ifPY (as defined in Figure 2) with an ifTime function. In this manner, dependency upon knowledge of the state of the pursuer was eliminated from the programs evolved by the GP

PROGRAM	SA-6	SA-13	SA-15
1 (seed = 1.0)	125 (97.7%)	116 (90.6%)	111 (86.7%)
2 (seed = 0.8)	128 (100%)	115 (89.8%)	113 (88.3%)
3 (seed = 0.6)	128 (100%)	120 (93.8%)	112 (87.5%)
4 (seed = 0.4)	127 (99.2%)	115 (89.8%)	113 (88.3%)
5 (seed = 0.2)	128 (100%)	120 (93.8%)	114 (89.1%)
Average:	127.2 (99.4%)	117.2 (91.6%)	112.6 (88.0%)
Variance:	1.36 (1.06%)	5.36 (4.18%)	1.04 (0.81%)

Figure 3. Test Results for the State-Dependent Solution to the Extended Two-Dimensional Pursuer/Evader Problem

system. ifTime is a three-argument function; if the time elapsed since the beginning of the pursuer/evader encounter is less than the percentage of the maximum duration of the encounter specified by the absolute value of the first argument, then the second argument is evaluated; otherwise, the third argument is evaluated. ifTime returns the value of the evaluated argument. For example, the program fragment

(ifTime 0.25 (setThrust 0.7) (hardTurn -0.6))

is interpreted as follows: If the time elapsed since the start of the encounter is less than 25% of the maximum encounter duration, set the thrust equal to 70% of the maximum thrust value; otherwise turn to the left at 60% of the maximum turning rate. Note that ifTime requires the evader to know the starting time of

the encounter, but does not otherwise depend upon knowledge about the pursuer's current state. The maximum duration of each encounter depends upon the *type* of pursuer, and was assumed during this study to be equal to the time required by the pursuer to traverse from its initial launch position to a distance equal to 125% of its maximum effective range.

Using this new function set, training runs 1-5 were repeated for each type of pursuer, and a new set of programs were optimized for solving the E2DPE problem under conditions in which the current state of the pursuer was unknown. Note that the *type* of pursuer (SA-6, SA-13, or SA-15) was assumed to be known; each pursuer in the training population was assumed to use proportional navigation with an effective navigation ratio N' = 4.

Each best-of-run program was subsequently tested against a large, representative test population of 128 pursuers described by the cross-product of sets J and K, as defined above. Each pursuer in each test population was of the same type as the pursuers in the corresponding training population. The results of these tests are summarized in Figures 4, 5, and 6.

PROGRAM	RESULTS
SA-6 Test 1 (seed = 1.0)	124 (96.9%)
SA-6 Test 2 (seed = 0.8)	128 (100%)
SA-6 Test 3 (seed = 0.6)	126 (98.4%)
SA-6 Test 4 (seed = 0.4)	128 (100%)
SA-6 Test 5 (seed = 0.2)	128 (100%)
Mean = 126.8 (99.1%)	Variance = 2.56 (2.00%)

Figure 4. State-Independent Solutions to the E2DPE Problem: SA-6 Test Results

PROGRAM	RESULTS
SA-13 Test 1 (seed = 1.0)	118 (92.2%)
SA-13 Test 2 (seed = 0.8)	120 (93.8%)
SA-13 Test 3 (seed = 0.6)	120 (93.8%)
SA-13 Test 4 (seed = 0.4)	120 (93.8%)
SA-13 Test 5 (seed = 0.2)	119 (93.0%)
Mean = 119.4 (93.3%)	Variance = 0.64 (0.50%)

Figure 5. State-Independent Solutions to the E2DPE Problem: SA-13 Test Results

PROGRAM	RESULTS
SA-15 Test 1 (seed = 1.0)	113 (88.3%)
SA-15 Test 2 (seed = 0.8)	114 (89.1%)
SA-15 Test 3 (seed = 0.6)	114 (89.1%)
SA-15 Test 4 (seed = 0.4)	114 (89.1%)
SA-15 Test 5 (seed = 0.2)	112 (87.5%)
Mean = 113.4 (88.6%)	Variance = 0.64 (0.50%)

Figure 6. State-Independent Solutions to the E2DPE Problem: SA-15 Test Results

V. ANALYSIS OF TEST RESULTS

Test results clearly demonstrate that the GP system developed during this study was capable of evolving programs that exhibited optimized survivability *without* depending upon knowledge of the pursuer's current state. For SA-6 pursuers, the average fitness of best-of-run programs evolved under conditions of uncertainty about pursuer state differed from that of programs evolved under conditions of certainty by only 0.3%; for SA-13s, this difference was only 1.7%; and for SA-15s, this difference was only 0.6%. In light of the variances indicated in Figure 3, *none* of these differences are considered to be statistically significant. This study demonstrated that certainty about the starting time of the encounter and the type of pursuer provides enough information to the GP system to allow it to produce effective solutions to the E2DPE problems illustrated by Figure 1.

VI. CONCLUSIONS

The GP system described in this paper defines a new methodology for identifying optimized solutions to the E2DPE problem. The advantage of the new methodology is that the resulting evasion programs exhibit optimized performance, *without* requiring information describing the state of the pursuer. The GP methodology used in this project thus provides a medium for automatically creating optimized solutions to a class of difficult strategy optimization problems that may be superior to classical analytic and game-theoretic solutions. As part of on-going dissertation research (Moore, 1997), the methodology established by this study was subsequently extended to integrate the use of countermeasures (such as chaff, flares, and jamming) with maneuvers to further enhance the survivability of the evader under conditions of uncertainty. The dissertation research ultimately defined a genetic programming solution to the three-dimensional missile countermeasures optimization problem.

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