Sequential Dynamical Systems and Simulation

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Sequential Dynamical Systems

Sequential dynamical systems (SDS) are a new class of dynamical systems that are motivated by the generic structure of computer simulations.

An SDS consists of three elements:

- A graph where each vertex has associated a binary state (the support structure in a simulation).
- A set of local functions that each can update the state of a vertex (the agents).
- An update schedule that decides the order in which the states will be updated.

Thus, an SDS is a triple consisting of a graph Y, local functions (f_i) and an update ordering π.

Sequential cellular automata (sCA)

The states of a classical cellular automaton are updated in parallel. However, most natural phenomena are inherently sequential. That should motivate one to consider sequentially updated cellular automata.

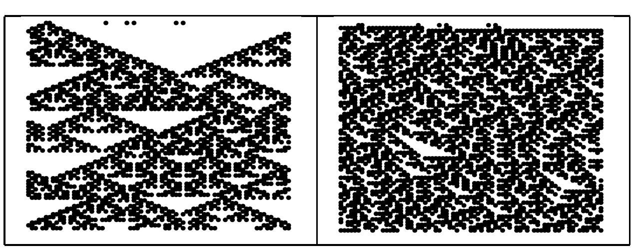

Figure 1: Order matters. Here $Y = \mathrm{Circ}_{75}$ with update orders $\pi_{\text{LHS}} = (1, 3, \ldots, 75, 2, 4, \ldots, 74)$ and $\pi_{\text{RHS}} = (1, 2, \ldots, 75)$. Here f is Wolfram rule 90.

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Applications

- A voting game: Consider the following simple model of an election. There are two candidates, say 0 and 1. There is only one voting box, so only one person can vote at a time. Each voter has some initial preference and will vote as the majority (Maj) of his friends will do or already has done. The vertices of the graph Y are the voters and there is an edge between two vertices if the corresponding voters are friends. This can be accurately described by the SDS $(Y, \mathbf{Maj}, \pi)$ where π is the voting order.
- Other applications include traffic studies and telecommunication.

The Mathematics of SDS

- How many SDS are there for a given graph Y and a given set of local functions? Answer: $|\mathrm{Acyc}(Y)|$.
- The fixed points of an SDS are independent of the update ordering. A somewhat surprising and important result.
- Modeling dynamical systems $F : \mathbb{F}_2^n \to \mathbb{F}_2^n$ with SDS using evolutionary techniques and GA approaches. Studying the number of linear orders mapping into partial order with GA techniques.
- Invertibility of SDS: The inverse of an SDS when defined is again an SDS over the same graph. This is not true for parallely updated cellular automata.

References

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