Approaches Based on Genetic Algorithms for Multiobjetive Optimization Problems

Jose Aguilar
CEMISID. Dpto. de Computación
Facultad de Ingeniería
Universidad de los Andes
Av. Tulio Febres. 5101.
Mérida-Venezuela
email: aguilar@ing.ula.ve

Pablo Miranda
GIBULA
Facultad de Ingeniería
Universidad de los Andes
Av. Tulio Febres. 5101.
Mérida-Venezuela
email: dmiranda@ing.ula.ve

Abstract

In engineering, it is often necessary to formulate problems in which there are several criteria or objectives. It is unlikely that a solution that optimizes one of the objectives will be optimal for any of the others. Compromise solutions are therefore sought such that no other solutions are better in any one objective while remaining no worse in the others. These types of problems are known as either multiobjective, multicriteria, or vector optimization problems. The problem addressed in this paper concerns the proposition of different approaches based on Genetic Algorithms to solve multiobjective optimization problems. We use notions about population manipulation and Pareto theory to develop our approaches, and study the Left Ventricle 3D Reconstruction problem from two Angiographics Views to test them.

1. INTRODUCTION

MultiObjective (MO) optimization extends optimization theory by permitting multiple objectives to be optimized simultaneously. Multiobjective optimization has been used in economics and management science for years and has gradually crept in engineering (Azarm, 1998; Dozier et al, 1998; Evans, 1984; Goldberg, 1989; Takayama, 1974). Genetic Algorithms (GAs) have been recognized to be possibly well-suited to MO optimization since early in their development. GAs possess many characteristics desirable in a MO optimizer, most notably the concerted handling of multiple candidate solutions. MOs search and optimization is perhaps a problem area where Evolutionary Computation really distinguishes itself from its competitors. Several methods for adapting GAs for this purpose have been proposed. They fall into two categories (Azarm, 1998; Dozier et al, 1998; Evans, 1984; Fonseca et al., 1998; Fonseca et al., 1995; Goldberg, 1989; Rudolph, 1998; Takayama, 1974): firstly, there are methods that try to combine all the different objectives into one. Second, there are methods based on Pareto-ranking.

The problem addressed in this paper concerns the proposition of different approaches based on GAs to solve MOP. We use notions about population manipulation and Pareto theory to develop our approaches, and study the Left Ventricle 3D Reconstruction problem from two Angiographics Views to test them. Left ventricular function is one of the required parameters to advise an appropriate therapeutical technique for patients with cardiac diseases, therefore the research of procedures for quantifying this function is of special interest (Miranda et al., 1996; Navarro et al., 1998; Pellot et al., 1994; Toro et al., 1996). The left ventricular angiography provides both anatomical and functional information required for the evaluation of the cardiac diseases.

This work is organized as follows, in section 2 the theoretical bases of the MO optimization, GAs and the current evolutionary approaches to MO optimization are reviewed. Then, we present our optimization approaches based on population manipulation and Pareto theory. Section 4 presents the application of our approaches for the Left Ventricle 3D Reconstruction problem and results analysis. We compare our results with a method based on SA and a classical GA (using a weighted objective function) (Navarro et al., 1998; Toro et al., 1996). Conclusions are provided in section 5.

2. REVIEW OF THE PROBLEM

2.1 MULTIOBJETIVE OPTIMIZATION

Many real world problems involve multiple objectives, which should be optimized simultaneously. However, suitable solutions to the overall problem can seldom be found. Optimal performance according to one objective, often implies unacceptably low performance in one or more of the other objectives, creating the need for a compromise to be reached. That is, the simultaneous optimization of multiple objective functions deviates from single function optimization in that it seldom admits a single, perfect solution. Instead, MOPs tend to be characterized by a family of alternatives (multiple solutions) which are considered equivalent by the absence of information concerning the relevance of each objective relative to the others. The MOP is, without loss of generality, the problem of simultaneously minimizing (or maximizing) the *n* components f_k $\forall k=1, ..., n$ of a possibly nonlinear vector function *f* of a general decision variable *x* in a universe *U*, where $f(x)= (f_1(x), ..., f_n(x))$.

Mathematically, the MOP can be stated as follows: given a set of objective functions $f=\{f_1, ..., f_n\}$, find a point *x* such that *f* is minimized (or maximized). In all but the simplest cases, it will not be possible to find a solution to a MOP. In practice, we seek solutions such that no one component function can be improved without sacrificing another. In order to discriminate effectively between two points x_0 and x_1 it is important to impose some type of preference structure on *f*, which defines the relevance of each objective function in *f*. A candidate solution to the MOP, x_0, is said to dominate another candidate solution x_1, if x_0 preferred based on some preference structure *P*. In (Dozier et al, 1998), they define three preference structures for MO optimization: value function preference, Pareto preference, and lexicographic preference. In value function preference, a function *g* is defined on *f* such that $g(x_0)<g(x_1)$ if and only if x_0 is preferred to x_1. In the above case, x_0 is said to dominate x_1 and x_1 is said to the dominated by x_0. Another type of preference structure is known as lexicographic preference. In this type of preference an order is imposed on *f* and a point x_0 is said to dominate another point x_1 if $f_k(x_0)<f_k(x_1)$ and $f_i(x_0)=f_i(x_1)$ $\forall i=1, ..., k-1$.

Perhaps the most widely used preference structure used in evolutionary MO optimizers is the Pareto preference structure. Using Pareto preference, we say of two solutions $x_0,x_1 \in U$ that x_0 dominates x_1 if $\exists i \in \{1, ..., n\}$ such that $f_i(x_0)<f_i(x_1)$ and that $\forall j \in \{1, ..., n\}$, $f_j(x_0) \leq f_j(x_1)$. In others words, x_0 dominates x_1 if x_0 is better than x_1 for at least one objective function, and is no worse on any of the others. A solution is Pareto-optimal if it is not dominated by any other solution. Ideally, we would like to find the set of all Pareto-optimal solutions. That is, the problem usually has no unique, perfect solution, but a set of nondominated, alternative solutions, known as the Pareto-set. A set of points is said to be Pareto optimal if, in moving from a given point A to another point B in the set, any improvement in one of the objective functions from its current value would cause at least one of the other objective functions to deteriorate from its current value. The Pareto optimal set yields an infinite set of solutions, from which the engineer can choose the desired solution. Using Pareto theory, the optimization process is best viewed as a pareto optimal process seeking a consensus in which many objectives are balanced so that the improvement of any single objective will result in a negative impact on at least one other objective. A pareto optimal solution is not unique, but is a member of a set of such points which are considered equally good in terms of the vector objective. This space may be viewed as a space of compromise solutions in which each objective could be improved, but if it was, it would be improved at the expense of at least one other objective (Azarm, 1998; Dozier et al, 1998; Evans, 1984; Fonseca et al., 1998; Fonseca et al., 1995; Goldberg, 1989; Miranda et al., 1998; Rudolph, 1998; Takayama, 1974).

2.2 GENETIC ALGORITHMS

2.2.1 Introduction

GA, invented by J.H. Holland, emulates biological evolution in the computer and tries to build programs that can adapt by themselves to perform a given function (Goldberg, 1989). A GA follows an "intelligent evolution" process for individuals based on the utilization of evolution operators such as mutation, inversion, selection and crossover. Optimization is a major field of GA's applicability. They belong to the class of probabilistic algorithms, yet they are very different from random algorithms as they combine elements of directed and stochastic search. Because of this, GA's are also more robust than existing directed search methods. Another important property of such genetic based search methods is that they maintain a population of potential solutions, all other methods process a single point of the search space. The population undergoes a simulated evolution: at each generation the "good" solutions reproduce, while the "bad" solutions die. To distinguish between different solutions we use a cost function. The idea is to find the best local optimum, starting from a set of initial solutions (individuals), by applying the evolution operators to successive solutions so as to generate a new and better local minimum. The procedure evolves until it remains trapped in a local minimum or a given number of generations.

2.2.2 Genetic Algorithms and MultiObjective Optimization

GAs are often used to try to find optimal or near optimal solutions to problems. A number of adaptations to GAs have been proposed to deal with MO functions (Dozier et al, 1998; Fonseca et al., 1998; Fonseca et al., 1995; Goldberg, 1989; Miranda et al., 1998; Rudolph, 1998; Schaffer, 1985). Current MO evolutionary approaches ranging from the conventional analytical aggregation of the different objectives into a single objective function to a number of population based approaches, and the more recent ranking schemes based on the definition of Pareto-optimality.

a) Plain aggregating approaches: In most problems where no global criterion directly emerges from the problem formulation, objectives are often artificially combined, or aggregated, into a scalar function according to some understanding of the problem. This approach has the advantage of producing a single compromise solution. The problem is to determine an appropriate setting of the coefficients of the combining function. A lot of applications of this approach have been reported in the literature (Aguilar, 1995 1; Aguilar, 1995 2; Aguilar et al. 1998; Goldberg, 1989; Hidrobo et al., 1998 1, Hidrobo et al., 1998 2, Miranda et al., 1996; Mulhenbein et al., 1988; Navarro et al., 1998). The combination of various objective functions into a single fitness function might be done using a weighted sum method, by defining:

$$f(x) = \sum_{i=1}^{n} w_i f_i(x)$$

b) Population based: This approach recognizes the possibility of exploiting GA populations to treat noncommensurable objectives separately and search for multiple non-dominated solutions concurrently in a single GA run. That is, this approach attempts to promote the generation of multiple non-dominated solutions. VEGA (Vector Evaluation GA) is one of the main examples of this approach (Schaffer, 1985). In VEGA appropriate fractions of the next generation are selected from the whole of the old generation according to each of the objectives, separately. Crossover and mutation are applied as usual after shuffling all the subpopulation together. Shuffling and merging all subpopulation corresponds to averaging the normalized fitness components associated with each of the objectives. The resulting overall fitness corresponded, therefore, to a linear function of the objectives where the weights depended on the distribution of the population at each generation. This linear combination of the objectives explains why the population tended to split into species particularly strong in each of the objectives. Fourman proposes an approach where the selection is performed by comparing pairs of individuals, each pair according to one of the objectives (Goldberg, 1989). Objectives are assigned different priorities by the user and individuals are compared according to the objective with the highest priority. A second version, consists of randomly selecting the objective to be used in each comparison. A detailed discussion of other approaches which exploit GA population in order to search multiple nondominated solutions can be found in (Fonseca et al., 1995).

c) Pareto-rank-based approaches: Other methods have used the idea of pareto-ranking (Fonseca et al., 1995; Fonseca et al., 1998; Goldberg, 1989; Rudolph, 1998). The pareto-rank of an individual is defined to be the number of members of the population by which it is dominated. The idea is to then seek individuals with minimum pareto-rank. Pareto-based fitness assignment was proposed by (Goldberg, 1989), as a mean of assigning equal probability of reproduction to all non-dominated individuals in the population. More formally, the method consists of assigning the rank 1 to the non-dominated individuals and removing them from contention, then finding a new set of non-dominated individuals, ranked 2, and so forth. Fonseca and Fleming have proposed an individual's rank correspond to the number of individuals in the current population by which it is dominated (Fonseca et al., 1995). Non-dominated individuals are, therefore, all assigned the same rank. The algorithm proceeds by sorting the population according to the MO ranks previously determined. Tournament selection based on Pareto dominance has also been proposed (Dozier et al., 1998).

d) Niche induction techniques: Pareto-based ranking correctly assigns all non-dominated individuals the same fitness, but that does not guarantee that the Pareto set be uniformly sampled. When presented with multiple equivalent optima, finite populations tend to converge to only one of them. Goldberg et al. proposed to use fitness sharing to prevent genetic drift and to promote the sampling of the whole Pareto set by the population (Evans, 1984). This makes it unfavorable for a GA to generate individuals which are too similar. Fonseca and Fleming implement fitness sharing in the objective domain and provided theory for estimating the necessary niche sizes based on the properties of the Pareto set (Fonseca et al., 1995). They use niche formation techniques to promote diversity among preferable candidates, and progressive articulation of preferences is shown to be possible as long as the GA can recover from abrupt changes in the cost landscape.

3. GENETIC ALGORITHM APPROACHES

In this section, we present our MultiObjective Problems Resolution approaches based on GAs.

3.1 POPULATION BASED APPROACHES

3.1.1 Approaches based on subpopulations

a) First Approach (1A): In this approach we divide the population according to the number of objectives to optimize. That is, for a population of size *M* and with a problem with *n* objectives, we define *n* subpopulations with M/n individuals. Each subpopulation optimizes an objective function using a classical GA. Then, we define priorities to each objective, and we select each objective function according to its priority to evaluate all subpopulations a given number of generations. We use a partial replacement to sure a diversity in our final subpopulations. The general procedure is:

1. Divide the initial population in *n* subpopulations
2. Optimize each subpopulation using a different objective function and a classical GA
3. Rank objectives according to their priorities
4. Repeat until evaluate each objective
   Optimize all subpopulations using the same objective function according to their ranks

b) Second Approach (2A): In this approach we follow the same first two steps as above, that is, we define *n* subpopulations and we optimize them using a different objective in each one. Then, we select the best individuals of each subpopulation (individuals with the minimal value) and we create a new global population with them. Finally, we optimize this population choosing randomly a different objective function for each iteration. The general procedure is:

1. Divide the initial population in *n* subpopulations
2. Optimize each subpopulation using a different objective function and a classical GA
3. Create a new population with the best individuals of each subpopulation
4. Repeat a given number of iterations
   Choose randomly an objective function
   Optimize the population with this objective function and a classical GA

c) Third Approach (3A): In this approach we follow the same first two steps as above, that is, we define *n* subpopulations and we optimize them using a different objective in each one. Then, we select the best individuals of each subpopulation and we define the weight value for each objective as the number of optimal individuals in its subpopulation (individuals with the minimal value). Afterward, we define a global objective function as the combination of these objective functions using a weighted sum method. Next, we create a new global population with the best individuals of each subpopulation and we optimize it using the global objective function. The general procedure is:

1. Divide the initial population in *n* subpopulations
2. Optimize each subpopulation using a different objective function and a classical GA
3. Create a new population with the best individuals of each subpopulation
4. Define the global objective function
5. Optimize the new population using the global objective function and a classical GA

3.1.2. Approaches based on the global population

a) Fourth Approach (4A): In this approach we choose randomly an objective function as fitness function for each generation. To choose the objective function we can use two schemes: according a probability for each objective function that is minimized each time it is chosen, or using a tournament selection mechanism. The general procedure is:

1. Repeat a given number of iterations
   choose an objective function
   Optimize the population using this objective function and a classical GA

b) Fifth approach (5A): In this approach, the GA is run in two stages: the first one obtains the weight value of each objective function and defines a global objective function using a weighted sum method. The second one searches the optimal solution using the global objective function. To obtain the weight values, the population is evaluated with each objective function and the number of individuals with minimal value is the weight value of this objective function. The general procedure is:

1. Repeat *n* times
   Choose a new objective function
   Optimize population with this objective function
   Obtain the weight value for this objective function
2. Define the global objective function
3. Optimize population with the global objective function

3.2 PARETO BASED APPROACHES (PA)

This approach compromises the aggressive selection that will result from the total domination scheme, and the diversity that is maintained from the non-dominant random selection, using a "partial domination" Pareto-like optimality criteria. To implement this, the different objective functions are examined. Using a subpopulation for each objective function, the pareto-rank of an individual is defined to be the number of members of the subpopulation by which it is dominated. This approach is composed for two stages: in the first one we rank the individuals of each subpopulation, that is, the algorithm starts by sorting the subpopulations according to the MO ranks previously determined. Then, we use non-dominated individuals as reference individuals to apply crossover operator in each subpopulation.

Classification stage:
1. Divide the initial population in *n* subpopulations
2. Repeat for each subpopulation
   Define rank for each individual
   Select non-dominated individuals (reference individuals)

Optimization stage:

1. Select the best individuals of each subpopulation
2. Cross these individuals with reference individuals define in the previous phase
3. Migrate new individuals to each subpopulation

And, the general procedure is:

1. Repeat until system convergence
    1.1 Classification phase
    1.2 Optimization phase

4. RESULTS ANALYSIS

4.1 CASE OF STUDY

An interesting MOP is the Left Ventricle 3D Reconstruction Problem (Miranda et al., 1996; Navarro et al., 1998; Pellot et al., 1994; Toro et al., 1996). Left ventricular function is one of the required parameters to advise an appropriate therapeutical technique for patients with cardiac diseases, therefore the research of objective procedures for quantifying this function has special interest. The angiography is an invasive technique for the generation of medical images that allows visualization of coronary arteries and heart cavities after the injection of a contrast agent. In digital angiography the X-rays are first converted into visible light and the resulting image is acquired with a TV camera, this video signal can be digitized and stored in a computer system. The left ventricular angiography provides both anatomical and functional information required for the evaluation of the cardiac diseases. Computer analysis of those images aims to perform the qualitative and quantitative evaluation of the ventricular function. Currently, the development of several digital image processing techniques with application to digital angiography has become an interesting issue. One of these techniques corresponds to the 3D reconstruction from two angiographic views (Pellot, 1994).

The proposed reconstruction algorithm starts with the provided information from two preprocessed angiographic views, acquired simultaneously according to two mutually orthogonal directions. The algorithm works under the assumption of a homogeneous mixture of blood and contrast agent, in order to develop a binary reconstruction model. The 3D ventricular object is considered as a stacked bidimensional slice set and each slice is reconstructed from the two one-dimensional profiles corresponding to a pair of rows obtained from the segmented projections. That is, the 3D reconstruction becomes a set of multiple 2D reconstruction problems, where each slice is reconstructed based on its 1D densitometric profiles.

In general, the reconstruction problem from only the provided information from two orthogonal projections, is an ill defined inverse problem, because it is not possible to assure the existence, uniqueness and stability of the solution without including additional restrictions. Consequently, the solution must be regularized based on a priori information about the ventricular shape.

The proposed model works under the assumption that the acquired angiograms contain only information about the left ventricle. The intensity gray level of each pixel in the input images is related to the depth information in the left ventricle. This information is grouped into a matrix form for each image and they are denoted as Iy and Ix of size N1xN3 and N2xN3 respectively. The 2D reconstruction problem can be stated as follows: given two 1D projections array α_i and β_j with N1 and N2 elements, respectively, we want to reconstruct a 2D binary array of size N1xN2, denote $\{x_{ij}\}$ either with 0 or 1 values, such that this array will satisfy the projections

$$\sum_{j=1}^{N2} x_{ij} = \alpha_i \quad \forall i=1, ..., N1 \qquad \sum_{i=1}^{N1} x_{ij} = \beta_j \quad \forall j=1, ..., N2$$

and
$$\sum_{i=1}^{N1} \alpha_i = \sum_{j=1}^{N2} \beta_j$$

The proposed algorithm includes two stages: In the first stage an initial reconstruction is provided based on an ellipsoidal model or any other approximate reconstruction for each one of the 2D slices. During the second stage, the initial reconstruction is appropriately deformed in order to obtain the most probable slice form. Such deformation process is performed using our MO approaches based on GA which allow minimize the energy function that includes projections compatibility, connection and spatial regularity constraints. That is, the energy function is chosen for measures the degree of match between the given projections and the current slice reconstruction projections. The energy function is defined as:

$$U^z(k) = U_1^z(k) + U_2^z(k) + U_3^z(k)$$

where, z is the reconstructed slice
k is the iteration number

The first energy component corresponds to the slice fidelity with respect to the given projections. If the given projections are α'^z_i, β'^z_j and the current slice projections are denoted as $\alpha_i^z(k)$, $\beta_j^z(k)$, the first energy component U_1 for the slice *z* is estimated as:

$$U_1^z(k) = \sum_{i=1}^{N1}\left[\alpha_i^z(k) - \alpha_i'^z\right]^2 + \sum_{j=1}^{N2}\left[\beta_j^z(k) - \beta_j'^z\right]^2$$

The second component corresponds to the internal energy of the reconstructed slice. That is, the second energy component is a regularity term that restricts the number of plausible solutions to the smooth slice contours.

$$U_2^z(k) = \frac{1}{8}\sum_{i=1}^{N1}\sum_{j=1}^{N2}\left[8 - \sum_{m=i-1}^{i+1}\sum_{l=j-1}^{j+1} 8 - \delta\left(x_{ij}^{z}(k) - x_{ml}^{z}(k)\right)\right]$$

where, $\delta(*)$ is the Kronecker delta function, which is equal to one if $x_{ij}^{z}(k) = x_{ml}^{z}(k)$ and equal to zero if $x_{ij}^{z}(k) \neq x_{ml}^{z}(k)$, where $x_{ij}^{z}(k)$ represents the pixel value at position (i,j) for the reconstructed slice *z*, at the iteration *k*.

The third component corresponds to the energy of similarity between the current slice configuration and the adjacent slice previously reconstructed. That is, the third component considers the regularity constraint of spatial smoothing between adjacent slices. This term is obtained at the difference between the current contour and the previously reconstructed slice contour

$$U_3^z(k) = \sum_{i=1}^{N1}\sum_{j=1}^{N2}\left| x_{ij}^{z}(k) - x_{ij}^{z-1}\right|$$

In general, for each reconstruction slice the next procedure is executed:

1. The initial configuration is obtained of the slice
2. The optimal configuration of the slice is searched using our approaches.

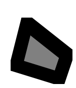
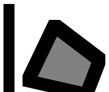
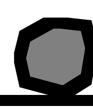
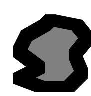
4.2 EXPERIMENTAL RESULTS

In order to evaluate our different MO approaches based on GA in the Left Ventricle 3D Reconstruction Problem, several tests were performed, including the slice reconstruction without considered the adjacent slice information and the reconstruction of a known 3D binary object, and by the performing the reconstruction from two real angiographic views appropriately preprocessed. The reconstruction was performed from the orthogonal projection corresponding to the row and column addition of several slices of the binary database. We compare our population-based approaches and pareto-based approach with the results obtained in (Navarro et al., 1998; Toro et al., 1996). The first work presents an approach based on a GA using a weighted sum method, and the second one presents an approach based on Simulated Annealing.

4.2.1 Isolated slice reconstruction

In this test, we use 60 elements for describing contour. The information of the adjacent slice was not considered and the reconstruction method was started with an elliptical initial approximation. In table 1, we see that our approaches give better results than previous works (Navarro et al., 1998; Toro et al., 1996). Particularly, our approaches give better results than (Navarro et al., 1998), because the classical GA used in that work not found solutions with a good combination between the different objectives. With respect to the results obtained in (Toro et al., 1996), PA, 1A, 2A and 4A given better results than it. The other approaches developed in this work (3A, 5A) do not give better results because we try to define a global objective function using the weighting objectives method, and this weights are obtained according to the number of individuals with optimal solution when GA converges (this is not a good criterion to define the weights).

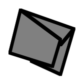
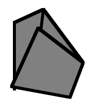
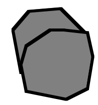
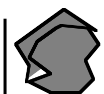
Table 1. Obtained Results for the Slice Reconstruction. In the upper row the Original Slices are shown, the Reconstruction Error for each Method are shown below

Original Slices						
Toro et al., 1996	4.3%	2.4%	1.5%	1.5%	1.9%	3%
Navarro et al., 1998	4.9%	2.8%	1.9%	2.7%	3.2%	1.9%
1A	4.3%	2.3%	1.4%	1.5%	1.8%	3%
2A	4.4%	2.4%	1.4%	1.6%	1.9%	3%
3A	4.7%	2.5%	1.6%	1.8%	2.2%	3.2%
4A	4.2%	2.3%	1.6%	1.5%	1.8%	3%
5A	4.6%	3%	2.1%	1.8%	2.6%	3.1%
PA	3.8%	2.3%	1.2%	1.1%	1.6%	2.6%

4.2.2 Tridimensional Reconstruction

In this test, 60 elements were used for representing each slice contour. An elliptical approximation is used to start the reconstruction method. We obtain the same quality of results than in the previous test, that is, PA, 1A, 2A and 4A give better results (table 2). PA gives the best results because it combines correctly the different objectives (it obtains Pareto optimal solutions).

Table 2. Obtained Results for the 3D Reconstruction. In the upper row the Original Slices are shown, the Reconstruction Error for each Method are shown below

Original objects						
Toro et al., 1996	10.3%	7.2%	4.3%	5.2%	3.3%	6.3%
Navarro et al., 1998	11.6%	8.3%	7.9%	7.1%	4.6%	8.8%
1A	9.8%	7.3%	4.1%	5%	3.4%	6.2%
2A	11%	8.1%	5.1%	5.4%	3.9%	6.9%
3A	10.9%	8%	5.6%	5.4%	3.7%	6.5%
4A	9.7%	7.6%	4.2%	5.4%	3.1%	6%
5A	9.9%	7.5%	4.2%	5%	3.5%	6.1%
PA	9.8%	7%	4%	5%	3%	6%

5. CONCLUSIONS

We have investigated the use of different versions of GAs in MOP. They have a number of advantages. First, they have a natural niching behavior that allows multiple pareto-optimal solutions to evolve. Second, there are some approaches (population based) faster than other (pareto-based approach). Third, they seem to produce a better spread and higher quality of result (at least for the Left Ventricle 3D Reconstruction Problem). Our approaches allow to solve the ambiguity of the problem by including a priori information in the form of constraints, an initial approximate reconstruction, and a MO optimization scheme. Previous approaches should define arbitrary the parameter weights of the energy function components, that controls the contribution of each objective into the global energy function. Directions for future research in MOP must include hybrid approaches using search strategies including the incorporation of fitness sharing, and adaptive representations on the approaches presented in this work.

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References

J. Aguilar (1995 1). Genetic Algorithms and Recurrent Random Neural Networks in Combinatorial Optimization problems. *Proceeding of the 8va Conferencia Nacional de Inteligencia Artificial y Sistemas Expertos.* 159-164. CNIASE95.

J. Aguilar (1995 2). A Hybrid Approach based on Genetic Algorithms and Simulated Annealing for Combinatorial Optimization Problems. *Proceeding of the XXI Latinoamerican Informatics Conference*. 1401-1412. CLEI.

J. Aguilar, A. Colmenares (1998). Resolution of Pattern Recognition Problems using a Hybrid Genetic/Random Neural Network Learning Algorithm, *Pattern Analysis and Applications*, 1 (1): 52-61.

A. Azarm (1996). Multiobjective Optimum Design. *Technical Report*. http://www.glue.umd.edu/~azarm/optimum_notes/multi.

G. Dozier, S. McCullough, A. Homaifar, E. Tunstel, L. Moore (1998). Multiobjective Evolutionary Path Planning via Fuzzy Tournament Selection. *Proceeding of the IEEE International Conference on Evolutionary Computation*, 684-689.

G. Evans (1984). Overview of techniques for solving multiobjectives mathematical programs. *Management Science*, 30 (11): 1268-1282.

C. Fonseca, P. Fleming (1998). Multiobjective optimization and Multiple constraint handling with evolutionary algorithms-part I: a unified formulation. *IEEE Trans. on systems, man and cybernetics*, 28(1): 26-37.

C. Fonseca, P. Fleming (1995). An overview of evolutionary algorithms in multiobjective optimization, *Evol. Compt.*, 3(1): 1-16.

D. Golberg (1989). *Genetic Algorithms in Search, Optimization and Machine Learning*. Addison Wesley.

F. Hidrobo, J. Aguilar (1998 1). Toward a Parallel Genetic Algorithm Approach Based on Collective Intelligence for Combinatorial Optimization Problems, *Proceeding of the IEEE International Conference on Evolutionary Computation*, 715-720.

F. Hidrobo, J. Aguilar (1998 2). Algoritmos Genéticos Paralelos en problemas de Optimización Combinatoria, *Revista Técnica de Ingeniería*, Universidad del Zulia, 21(1): 47-58.

P. Miranda, J. Aguilar (1998). Enfoques basados en Algoritmos Geneticos para evaluar Funciones Multiobjectivos. *Proceeding of the IV Jornadas Cientifico Tecnicas de la Facultad de Ingenieria, Universidad de los Andes, Venezuela,* 186-191.

P. Miranda, J. Aguilar (1996). Reconstruccion3D del ventriculo izquierdo usando Algoritmos Geneticos, *Technical Report, CEMISID, Universidad de los Andes*.

H. Mulhenbein, M. Georges-Schleuter and O. Kramer (1988). "Evolution algorithms in combinatorial optimization". *Parallel Computing*; 7(1): 65-88.

C. Navarro, R. Medina, J. Aguilar, P. Guillen (1998). Una Implementación Paralela de la Reconstrucción del Ventrículo Izquierdo usando Algoritmos Genéticos. *Proceedings of the IV Jornadas Científico-Técnicas de Ingeniería*, 174-179.

C. Pellot, A. Herment, M. Sigelle, P. Horain, H. Maitre and P. Peronneau (1994). A 3D Reconstruction of Vascular Structures from two X-Ray Angiograms using an adapted Simulated Annealing algorithm, *IEEE Transaction on Medical Imaging*, 13(1): 48-60.

G. Rudolph (1998). One Multi-Objective Evolutionary Algorithm and its Convergence to the Pareto Set. *Proceeding of the IEEE International Conference on Evolutionary Computation*, 511-516.

D. Schaffer (1985). Multiple objective optimization with vector evaluated GA. *Proceeding 1st Intl. Conference on Genetic Algorithm*, 93-100.

A. Takayama (1974). *Mathematical Economics*. Hinsdale II.

J. Toro, R. Medina, M. Garreau, H. Carrrasco, D. Jugo (1996). A Simulated Annealing Approach to Left Ventricle 3D Reconstruction from two angiographic views. *Caderno de Engenharia Biomédica*, 12(3): 131-141.