Using Genetic Algorithms For Adaptive Function Approximation and Mesh Generation

Rida E. Moustafa
Computational Sciences and Informatics
George Mason University
Fairfax, VA 22030

Kenneth A. De Jong
Computer Science Dept.
George Mason University
Fairfax, VA 22030

Edward J. Wegman
Computational Sciences and Informatics
George Mason University
Fairfax, VA 22030

Abstract

We have developed GA-based search procedures for continuous function approximations. We have designed algorithms that perform both adaptive mesh refinement and selection of interpolations functions, both of which are necessary to achieve highly accurate function approximations while keeping the number of mesh points at a minimum. We demonstrate the feasibility of the approach by testing it on a variety of functions.

1 Introduction

Developing effective techniques for approximating functions is an important and well-studied area. One of the difficult issues is how to deal with the fact that some portions of functions have large functional variation while other portions are relatively smooth and easy to approximate. A uniform mesh approach requires that the fixed inteval between the node points on a mesh be sufficiently small to deal with the worst case functional variation, resulting in many unnecessary mesh points. Adaptive function approximation attempts to deal with this by varying the distance between mesh points in different portions depending on the functional variation.

In addition, the choice of which interpolation function to use in between mesh points is an important factor in both the cost and accuracy of the approximation. A uniform approach uses the same interpolation function on all mesh elements while an adaptive approach selects on an element by element basis.

Adapting both the mesh and the interpolation functions simultaneously is a difficult and computationally expensive process. It looks ideally suited for an evolutionary computation approach. We have developed several GA-based algorithms to do so, and in this paper we present our initial results.

2 Summary of Results

In our study we used the Lagrange polynomials for approximations and a fitness function defined as $Error = |f(x) - p_n(x)|$, and we tested our ideas on several cases. First, we followed the so called $h - refinement$ in which we simply increase the number of points in the mesh and let the GA find the optimal location of these mesh points. In the second case, we use the GA to perform $h - p - refinement$ in which both the number of mesh points and the degree of the interpolation polynomial are selected to achieve higher approximation accuracy. Figure 1 provides a simple illustration of how the number and distribution of mesh points changes as the degree of the interpolation polynomial changes from 1 to 2 to 3 for the test function $f(x) = sin(4x)e^{-2x}$. Notice how the adaptive mesh refinement concentrates the mesh points in regions of high functional variance with fewer points required for higher degrees. Our results at the moment are promising and agree with our expectations, but still need more testing and comparison with other methods. For more details, see http://www.galaxy.gmu.edu/ rmoustaf/publications.

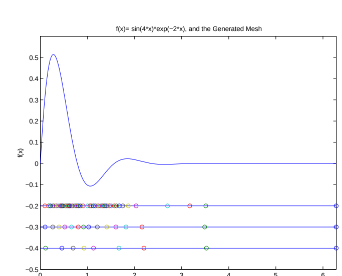

Figure 1: Results with polynomial degrees (1,2,3).