Improving the Scalability of Dynastically Optimal Forma Recombination by Tuning the Granularity of the Representation

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1 DYNASTICALLY OPTIMAL RECOMBINATION (DOR)

Let $x$ and $y$ be two individuals from a solution space $\mathcal{S}$. A recombination operator $X$ can be defined as a function $X : \mathcal{S} \times \mathcal{S} \times \mathcal{S} \rightarrow [0, 1]$, where $X(x, y, z)$ is the probability of generating $z$ when recombining $x$ and $y$ using $X$. Clearly,

$$\forall x \in \mathcal{S}, \forall y \in \mathcal{S} : \sum_{z \in \mathcal{S}} X(x, y, z) = 1 \quad (1)$$

The *Dynamic Potential* of $x$ and $y$ is defined as

$$\Gamma_{(x,y)} = \{ z | \forall \xi \in \Xi : z \in \xi \Rightarrow (x \in \xi) \land (y \in \xi) \} \quad (2)$$

where $\Xi$ is the set of basic formae.

A recombination operator is said to be transmitting iff

$$\{ z | X(x, y, z) > 0 \} \subseteq \Gamma_{(x,y)}.$$ 

Now, let $\phi : \mathcal{S} \rightarrow \mathcal{R}^+$ be the target function (minimization is assumed). DOR is a transmitting recombination operator for which:

$$\text{DOR}(x, y, z) > 0 \Rightarrow \forall w \in \Gamma_{(x,y)} : \phi(w) \geq \phi(z) \quad (3)$$

Thus, no other solution in the dynamic potential is better than any solution generated by DOR. According to this definition, the use of DOR implies performing an exhaustive search in a small subset of the solution space. Such an exhaustive search can be efficiently done by means of a subordinate A*-like mechanism.

DOR uses optimistic estimations $\hat{\phi}(\Psi)$ of the fitness of partially specified solutions $\Psi$ (i.e., $\forall z \in \Psi : \hat{\phi}(\Psi) \leq \phi(z)$) for directing the search to promising regions. These solutions are incrementally constructed using the formae to which any of the parents belong. More precisely, let $\Psi^0 = \mathcal{S}$. Subsequently,

$$\Psi^{2j+1}_{i+1} = \Psi^j_i \cap \Sigma(\Psi^j_i, x), \quad (4)$$

$$\Psi^{2j+2}_{i+1} = \Psi^j_i \cap \Sigma(\Psi^j_i, y) \quad (5)$$

are considered. Whenever $\hat{\phi} < \hat{\phi}(\Psi)$ (where $\hat{\phi}$ is the fitness of the best-so-far solution generated during this process), the macro-forma $\Psi$ is closed (i.e., discarded), hence pruning dynastically suboptimal solutions. Otherwise, the process is repeated for open macro-formae. Each $\Sigma(\Psi, w)$ is termed a *construction unit*. These construction units are defined as

$$\Sigma(\Psi, w) = \cap_{1 \leq i \leq g} \xi_i, \ w \in \xi_i, \quad (6)$$

and their structure depends on the problem considered. The parameter $g$ is called the *granularity* of the representation. It can be seen that the size of the set of solutions in which DOR searches is $O(2^n/g)$, where $n$ is the dimensionality of the representation.

The minimal value of $g$ for a given representation is termed the *basic* granularity (e.g., $g = 1$ when the representation is orthogonal). If the computational complexity of DOR is too high for this basic granularity, $g$ can be increased so as to make DOR combine larger portions of the ancestors.

Experimental results on the Brachystochrone design problem and the Rosenbrock function show a nearly-linear relation between the granularity of the representation and the reduction of the computational effort. Furthermore, it is shown that intermediate granularity values are better since low $g$ is computationally prohibitive and high $g$ reduces the chances for information interchange during recombination. This is verified on orthogonal and non-orthogonal separable representations exhibiting epistasis [Cotta et al., 1999].

References