India and Pakistan, a classic "Richardson" Arms Race: A Genetic Algorithmic approach.

Tim Hackworth

Department of Computer Science, Birkbeck College,
University of London, Malet Street, London, WC1E 7HX
tim_hackworth@bcs.org.uk

Abstract

The mid-1998 troubles between India and Pakistan are used to demonstrate that a real-valued Genetic Algorithm (GA) can find workable solutions to Richardson's Theory of Arms Races. This theory, developed in the late 1940s, needs such a big search space that it was never very useful — until the arrival of GAs. Rough 'starter' values for the rate-constants in Richardson's equations are gleaned from International Monetary Fund statistics. The resulting GA found, first, that very small changes in percentage defence expenditure made all the difference between stability and instability**;** second, that although there are dangerous pockets of potential instability in the developing arms race, nevertheless there are large areas of stability as well**;** and third, that it is possible to predict when instabilities will occur. Properly refined, such predictions could give early warning of 'flashpoints' and might even be used to prevent an arms race from escalating into open war. The conclusion includes comments on the practicalities of using GAs in this type of application.

1. INTRODUCTION.

The ability to analyse stability and equilibrium is vital to all models of the real world and is based on principles established in the eighteenth century by Le Chatelier and Gibbs. More recently, however, the credit for pioneering the application of mathematical logic to international politics belongs to the meteorologist Lewis Fry Richardson DSc FRS (1881 - 1953). Richardson was convinced that the understanding which grew from the systematic analysis of the events which were known to lead to war would contribute more to advancing the cause of peace than the intuitive and emotive reasoning of statesmen, politicians, soldiers and diplomats. His principal work on this subject is *Arms and Insecurity* published posthumously in 1960. This contains his *Theory of Arms Races.*

2. RICHARDSON'S THEORY OF ARMS RACES.

Let the annual defence expenditures of India and Pakistan respectively be US \$ x billion and US \$ y billion respectively. Richardson's theory says that defence expenditures will increase at the following rates:

$$dy/dt = ax - by + h \quad \text{and}$$
$$dx/dt = cy - fx + g \qquad (1)$$

where a and c are called *defence coefficients*, b and f are the *fatigue and expense coefficients*, while g and h are *grievances* when positive, *goodwill* when negative. Richardson [1960] had assumed **:**

- that in a two-nation arms race each country would attempt to increase its armaments over the other
- that economic factors impose constraints that tend to diminish the rate by an amount proportional to the size of the existing friendly forces, and
- a nation will build arms motivated by ambition and hostility even if the other nation poses no threat to it.

Richardson also showed that stability occurs when (in Equations 1) the product ac < the product bf **;** more interestingly, instability occurs when

$$ac > bf \qquad (2)$$

Any potential solution is required **:**

- To establish arms race contours for India and Pakistan.
- Having established a particular value for one side, to determine the corresponding value for the other.
- To seek the 'fittest ' solution which in context means minimising the defence expenditures.

The original Richardson equations were further developed and extended by Mayer-Kress [1989] who decided to work in terms of finite steps, (ie deriving x_{t+1} from x_t etc, rather than in rates of growth, *dy/dt*) and came up with the following 3-agent solution **:**

$$x_{t+1} = x_t + (k_{11} (x_s - x_t) + k_{23} (y_t + z_t)) (x_m - x_t)$$
$$y_{t+1} = y_t + (k_{22}(y_s - y_t) + k_{13} (x_t - z_t)) (y_m - y_t)$$
$$z_{t+1} = z_t + (k_{33}(z_s - z_t) + k_{12} (x_t - y_t)) (z_m - z_t) \quad \text{..........(3)}$$

Equations (3) assume that Nations Y and Z are allied together against X, and have neatly got rid of the grievances *g* and *h* on which it is never possible to put numerical values. Reducing Equations (3) to two agents gives **:**

$$x_{t+1} = x_t + (k_{11} (x_s - x_t) + k_{12}\, y_t) (x_m - x_t)$$
$$y_{t+1} = y_t + (k_{22}(y_s - y_t) + k_{21}\, x_t) (y_m - y_t) \quad \text{...........(4)}$$

where :

- x_t and y_t are the expenditures of the two countries on arms this year. [The initial settings of x_t and y_t are called x_0 and y_0 as they can be used as the basis for iteration]
- x_s and y_s are the intrinsic arms expenditures (how much each country spends on defence irrespective of competitive spending by its neighbours). These are essentially the 'standing costs' of the Armed Forces.
- x_m and y_m respectively represent the economic constraints on countries X and Y, i.e. the fraction of the country's total resources (known as the Gross Domestic Product (GDP) available to be spent on arms.
- *k*... are the four rate-constants. These have the dimensions of $(\text{time})^{-1}$, and are largely a measure of how fast a nation can react to changing threats.

3. SETTING UP THE GA: ESTABLISHING A BASIS OF FACT.

Equations (4) contains a lot of constants and variables **;** initially, we know nothing about any of them, so we are in no position to set up a GA. Table 1 is based on International Monetary Fund Statistics [1998] and Hunter [1992-8] and establishes a basis of fact for the GA, enabling us to derive rough 'starter values' for the rate-constants *k.* All Table 1's figures are shown as billions of US dollars, using the conversion rates from Indian rupees or Pakistani rupees applicable during the month when the various figures were published. The US dollar has held more or less steady over the past few years, so this act of currency conversion takes account of local inflation. Hence like can be compared with like.

A set of (almost empirical) formulae were based on Equations (4) **:**

$$x_{t+1} = x_t + ((k_{11} / 10 * 0.46 * x_t) + (k_{12} / 50 * y_t)) * G_x / 1000;$$
$$y_{t+1} = y_t + ((k_{22} / 10 * 0.21 * y_t) + (k_{21} / 50 * x_t)) * G_y / 1000; \quad \text{.......................................(5)}$$

and the GDPs and percentage defence expenditures for 1993 were inserted. By manipulating the constants and the scaling factors by hand, it proved possible to obtain a reasonable fit to the live data for subsequent years, see Figure 1 below. In particular, rough values could be put on the four rate-constants. It is stressed that the curves of Figure 1 represent the iterated and extrapolated output of Equations 5, and *not* the result of a curve-fitting exercise on the IMF data. This worked well**;** the fit was so good for Pakistan (y) that it was not thought necessary to seek another formula to accommodate India (x) where the fit in the middle reaches was not so good.

Equations (3) and (4) may be conceptually correct but they make little numerical sense. x_s will always be less than x_t (typically 80% of it) so their difference will be negative**;** $(x_m - x_t)$ is the difference between a percentage and money measured in billions of dollars. To make these equations dimensionally consistent and numerically usable they have, therefore, to be scaled. For instance, $(100\, x_m - x_t)$ results in reasonable figures, and this was used in Equations (6).

Equations (5) used too few genes and too many set values to form a GA, but they did generate some scaling factors and starter values. Using these in a pragmatic variant of Equations (4) we can now write some workable evaluation equations as a basis for a 'proper' genetic algorithm, which we call GA - 37a **:**

$$x_{t+1} = x_t + (k_{11} / 100 * (x_s / 100) * x_t + k_{12} / 100 * y_t) ((100 * x_m - x_t) / 85)$$
$$y_{t+1} = y_t + (k_{22} / 100 * (y_s / 100) * y_t + k_{21} / 100 * x_t) ((100 * y_m - y_t) / 85) \quad \text{.......................... (6)}$$

Equations (6) are broadly similar to Equations (4) but are distinct from them in two ways **:** In the chromosome all the rate constants k are defined as integers lying between 0 and 99 so have to be divided by 100 **;** and, second, while Equations (4) had a term in $k_{11} (x_s - x_t)$ it was better expressed, not as in Equations (6) [where it was $k_{11} * 0.46 * x_t$, implying that x_s is a fixed proportion of x_t] but as $k_{11} * (x_s / 100) * x_t$. Therefore x_s does have a say but its numerical effect is not crucial**;** much more important is the fact that it introduces two more genes into the genetic algorithm. A GA treats all its genes equally, no matter what scaling factors are ultimately put on them. [Mitchell, 1996] Equations (6) worked, and provided stable convergent solutions. The first result was that very small changes in defence expenditure have a marked effect on the stability of the outcome, and that there are 'pockets' of instability. See Figures 2, 3, and 4 in sequence**;** an **extra 0.1 %** defexp can make the system oscillate between stable / unstable / stable.

A PROBLEM OF INTERPRETATION

To be of practical use, the Richardson's conceptually accurate but rather vague equations need to be replaced by (or transformed into) something more precise and expressed in terms of the information available**!** It is all very well to be able to prove theoretically from Equations (1) that instability occurs when ***ac* > *bf*** and, later in this paper, it will be shown that the sign of the difference ***(a-c)*** indicates the direction of the curvature of the limit cycle and is crucial in predicting when instability is likely to occur.

Table 1. International Monetary Fund Statistics 1992 to 1998. All figures are billions of US dollars.

Year	**Category**	**I N D I A**			**P A K I S T A N**		
1992	GDP	219			38.79		
	Defence Expenditure		4.20			1.79	
	(Defexp **:** GDP) %			1.9%			4.6%
1993	GDP	195			35.25		
	Defence Expenditure		3.57			2.24	
	(Defexp **:** GDP) %			1.8%			6.3%
1994	GDP	204			33.69		
	Defence Expenditure		3.68			2.32	
	(Defexp **:** GDP) %			1.8%			6.8%
1995	GDP	213			33.97		
	Defence Expenditure		4.41			2.59	
	(Defexp **:** GDP) %			2.1%			7.6%
1996	GDP	202			31.56		
	Defence Expenditure		5.59			3.19	
	(Defexp **:** GDP) %			2.7%			10%
1997	GDP	202			29.37		
	Defence Expenditure		13.51			3.37	
	(Defexp **:** GDP) %			6.7%			11.5%
1998	GDP	190			29.03		
	Defence Expenditure		22.61			3.6	
	(Defexp **:** GDP) %			14%			12.4%
	k 'starter values'	**$k_{11} = 46$**	**$k_{12} = 6$**		**$k_{22} = 2$**	**$k_{21} = 2$**	

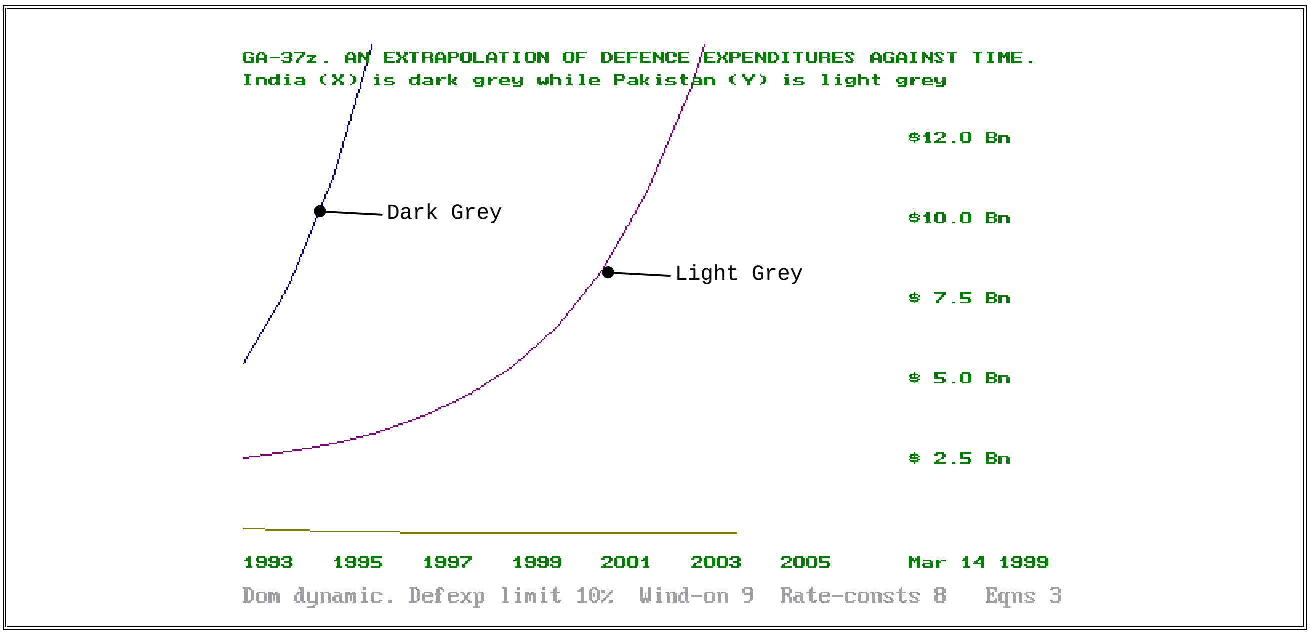

Figure 1 : The Arms Race Between India and Pakistan, from 1993 onwards.

But the relationship between the coefficients ***a, b, c*** & ***f*** and Equations (6) is never fully clear. To be valid, evaluation equations must always mirror Richardson's intentions.

4. THE GENETIC ALGORITHM

Ten genes representing the parameters k_{11}, k_{22}, k_{12}, k_{21}, x_s, y_s, x_m, y_m, x_t and y_t from Equations (6) above were set up to appear as one integer in a chromosome. A multiple population (typically 100) of these chromosomes were initially filled from a (constrained) random number generator. Every chromosome is a point in the search space of candidate solutions, so the genes from each one could be used in the two evaluation equations. The 'best' solution was chosen by a reciprocal 'least squares' fitness function

$$fitness = 10000 \Big/ \sqrt{x_{t+1}^2 + y_{t+1}^2} \text{ .... (7)}$$

whose role is to highlight those chromosomes which offer minimum values of x_{t+1} and y_{t+1} simultaneously.
The resulting fitnesses are sorted and normalised, and 'roulette-wheel' selection is then applied, giving the 'better' chromosomes (ie those with smaller fitness) a chance to reproduce more frequently than the poorer ones. Put formally, this is *weighted random pairing*,

[Haupt and Haupt, 1998] the weighting being by 'cost' — hence the linear normalisation without which superfit chromosomes might get too big a reproductive advantage. The reproduced chromosomes are then mutated, crossed-over once, reassembled, and then taken into use as the new generation replacing the old. In practice, if the figures converge at all, they do so quite quickly, so it never proved necessary to run more than twenty generations. The space being implicitly searched is enormous, in fact 10^{24}. Inasmuch that chromosomes representing the parameters of the problem are applied to the problem in the **evaluation equations**, rated by the **fitness function**, selected for 'parenthood' by virtue of their fitness, mutated, crossed-over, re-combined, applied again to the evaluation functions etc., hopefully getting better each generation, the algorithm (called GA-37a) is an entirely conventional GA, [Davis, 1991] except that the chromosomes are real-valued, i.e. the alleles are decimal and not binary. Although historically a 'conventional' GA [Holland, 1973] uses binary alleles, there is now sufficient experience of (and published material on) real-valued GAs [Eshelma and Shaffer, 1993] [Montana and Davis 1989] [Adewuya, 1996] [Michalewicz, 1992] to make further discussion unnecessary here.

"Generation" is used in four discrete contexts **:**

- the normal stage-by-stage completion of a GA
- annually, see Figure 1
- from budget date to budget date
- to mark the end of any definite stage (in which case generations may not be equally spaced in time).

REDUNDANCY IN CHROMOSOMES

When GA - 37a was first run, it was convenient to use an existing vector developed previously for a 3-Agent situation whose chromosome contained fifteen genes. As the current 2-Agent situation used only ten genes there was, therefore, some redundancy. Numbers were still generated for the five redundant genes (and mutated and crossed-over with all the others) but they were never used in the evaluation equations. Later, a fully-occupied, non-redundant ten-gene chromosome was developed specially, only to find that in use it was far less flexible than the redundant chromosome, and much more prone to *epistasis*, i.e. swapping the location of the genes in the chromosome made a difference to the results obtained. The Effects of Redundancy in Chromosomes is the subject of a separate paper [Hackworth, 1999] .

5. IS RICHARDSON'S STABILITY CRITERION BORNE OUT IN PRACTICE?

Inequality (2) above said that the system would be unstable if $ac > bf$, but stable if not. [Richardson 1960] Hence, according to the theory, the equality condition $ac = bf$ must signal a change. But does it ? It is not feasible to work this out for the two Equations (6) — because neither can be made explicit for x_t and y_t — but it can be done for Equations (5). Using the same a, c, b, f nomenclature as in Equations (1) and Inequality (2) and setting the defence expenditures for both sides to a maximum of **6.5 %** the transforms become **:**

$a = k_{21} \times G_y = 2 \times 6.5 = 13$ **;**
$c = k_{12} \times G_x = 6 \times 6.5 = 39$; $\quad$ So $ac = 507$
$b = 1 + (k_{22} \times 0.21 \times G_y) = 1 + (2 \times 0.21 \times 6.5) = 3.73$
$f = 1 + (k_{11} \times 0.46 \times G_x) = 1 + (46 \times 0.46 \times 6.5) = 138.5$
$bf = 512$, hence $ac < bf$, situation stable (Fig 2)

Using similar reasoning, for 'defexp' set to a maximum of **6.6 % :**
$a = k_{21} \times G_y = 2 \times 6.6 = 13.2$ **;**
$c = k_{12} \times G_x = 6 \times 6.6 = 39.6$ **;** $\quad$ So $ac = 523$
$b = 1 + (k_{22} \times 0.21 \times G_y) = 1 + (2 \times 0.21 \times 6.6) = 3.7$
$f = 1 + (k_{11} \times 0.46 \times G_x) = 1 + (46 \times 0.46 \times 6.6) = 140.6$ $\quad bf = 518$ so $ac > bf$, situation unstable (Fig 3)

Hence Richardson's stability criterion does work for the India / Pakistan arms race.

6. STABILITY.

A new program GA - 37e puts GA - 37a **inside** two additional for-loops, so that it is possible to examine the behaviour of the GA as any two parameters (such as *defexp%* and x_0 , *defexp%* and any of the rate-constants k, or defexp%India and defexp%Pakistan) are varied 25 times, over the whole of their likely range. The output is a square matrix of 625 tiny coloured squares, each one representing a whole family of iterations. The question is simple. Is each family of iterations (i.e. each tiny square) stable (light grey) or is it not (dark grey) ?

To obtain the 'Stability Diagram' at Figure 5, two decisions had to be made **:**

- How were each of the 625 families of iterations to be represented? Rather than using the stability condition of the square at the extreme right-hand end of the last generation — which, in the circumstances was more likely than not to be 'unstable' if the parameters were stretched — it was decided to take a 'bottom-line consensus' approach, i.e. the number of stable and of unstable squares in the last row (the last generation) were accumulated and a 'majority vote' was taken.
- What dictated whether a particular iteration had become unstable ? It was noted that in a stable, convergent series x_t or y_t rarely exceeded 3000. It was therefore decided that if at any time $|x_t|$ or $|y_t|$ exceeded 5000 the iteration would be deemed unstable. The decision was imprecise, but it worked.

Figure 5 is a Stability Diagram generated by GA - 37e. The rows, the percentage defence expenditures (defexp% India), vary from 5.8% to 8.2% in 25 steps of 0.1. The columns, (defexp% Pakistan), vary in the same way. The other rate constants are 'free', i.e. are chosen by the GA.

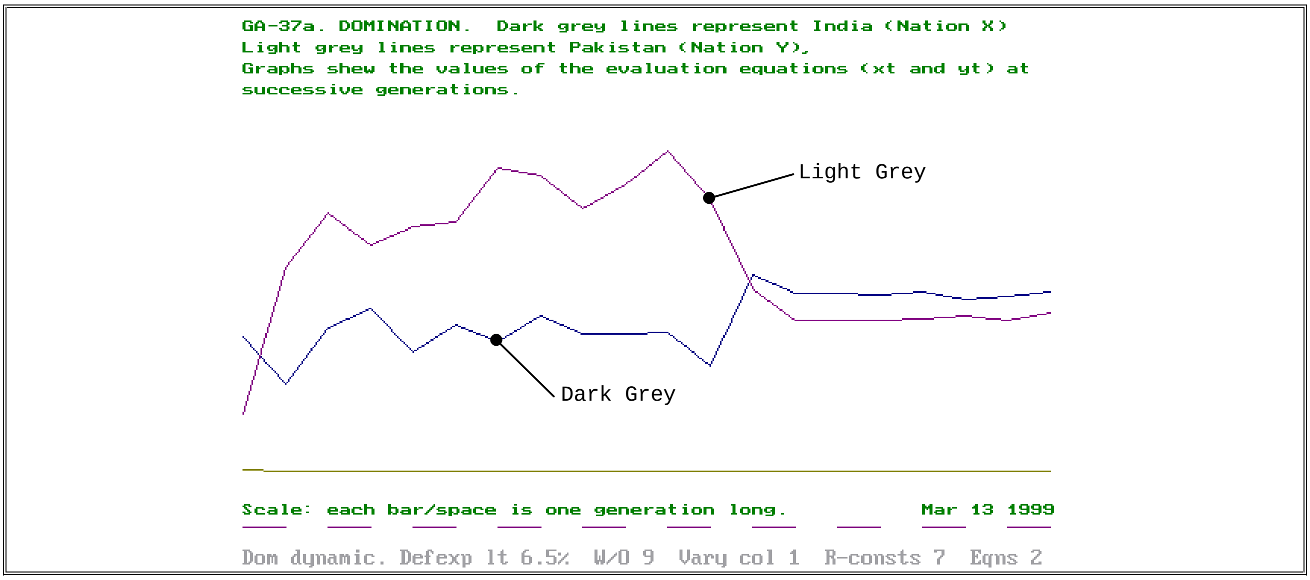

Figure 2 : Defence expenditure limit **6.5 %.** Only just stable! Now see Figure 3 below.

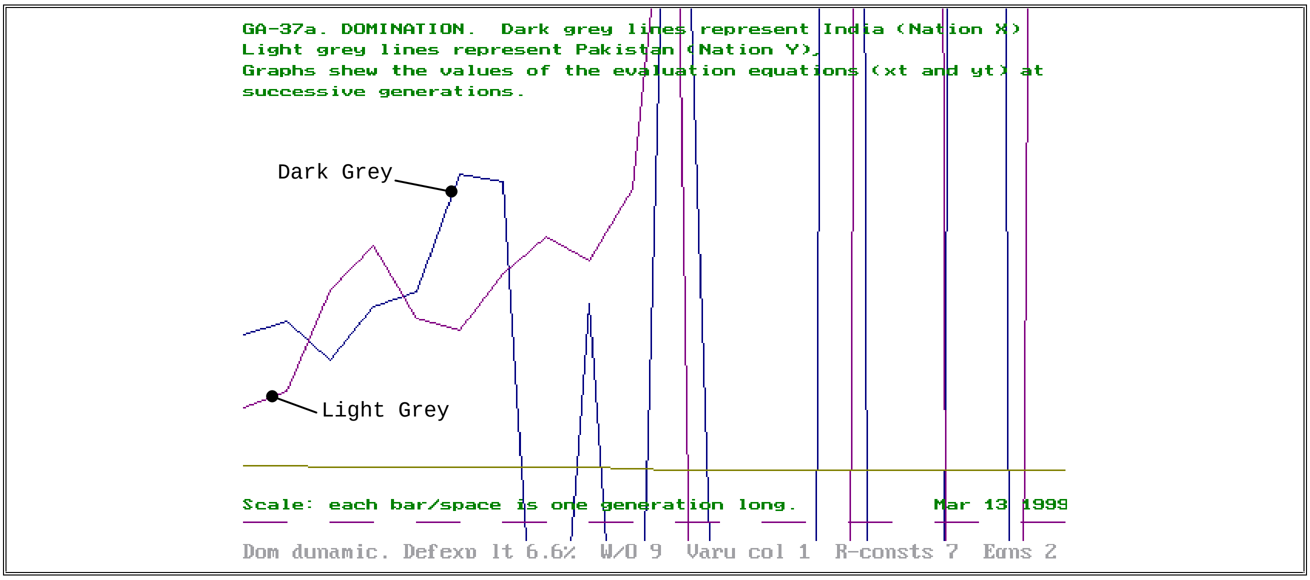

Figure 3 : Defence expenditure limit **6.6** % Too far!

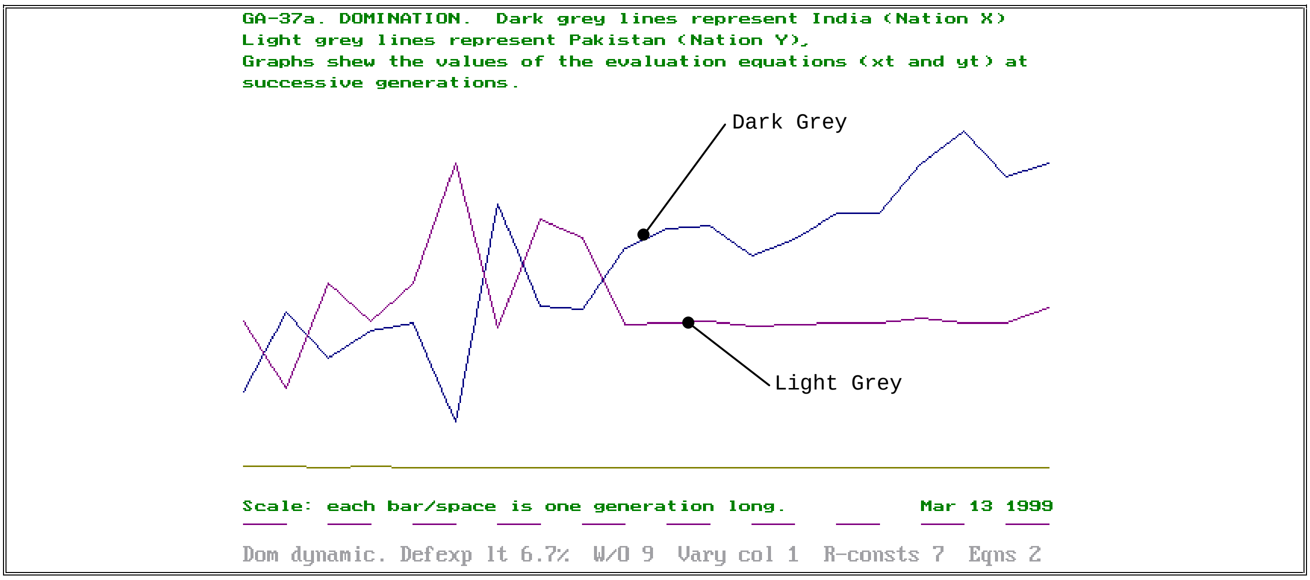

Figure 4. Defence Expenditure Limit **6.7 %** Calming down after the explosion of Figure 3.

One had been led to expect [Forrest and Mayer-Kress, 1996] diagonal 'walls' i.e. there would be large well-defined, contiguous and clearly-separated areas of dark grey and light grey. Walls never did occur**;** despite running GA-37e varying a large number of parameters (including all the rate-constants in turn) the effect was always 'patchy', as if there was a significant noise problem.

It only then became apparent that the 'patchiness' of dark grey was not (or not significantly) due to noise, but to definite regions of instability some of which were quite local. See, in sequence, Figures 2, 3 and 4 which plot the values of Evaluation Equations (6) x_t and y_t against the generations. A quiet period was working up to the violent instability of Figure 3, and then quietening down again, the difference of defence expenditure between each of the diagrams being only 0.1%. Thus we have stable states lying either side of unstable ones, a phenomenon well known to chemical engineers.

There are no general areas of stability in the India / Pakistan situation**;** the system lurches from one instability to another with periods of respite in between. One cannot therefore say simplistically that war will be averted if, say, India's percentage defence expenditure drops below xx %, of if rate-constant k_{21} is more than yy, or if Pakistan's defence budget exceeds US $zz billion. It is more complex than that.

7. CANARD EXPLOSIONS — A DIGRESSION

It was suggested[1] that instabilities in the model of a certain industrial chemical process — the Edblom-Orbán-Epstein ("EOE") Reaction — bear a striking resemblance to those modelled by the two-agent Richardson arms race equations. The EOE reaction involves very rapid changes from stable to unstable states in a liquid mixture of ions used in the manufacture of plastics, and often results in what is called a 'canard' explosion. Such behaviour is most unwelcome commercially, and, understandably, much effort has been put into preventing it. The EOE reaction, as modelled by Peng, Gáspár and Showalter [1991] on previous work by Benoit et al [1981] and later developed by Brøns and Bar-Eli [1994] is characterised by two non-linear ordinary differential equations which look remarkably similar to Richardson's equations (1), i.e. they are of the form *dy/dt* = ..., *dx/dt* = ..., and contain rate-constants and a stack of dependent variables.

By analysing the path of what Peng called the 'limit cycle' $d\{ dy/dt \ / dx/dt \} / dt = 0$.......... ....(8) Peng et al showed that explosions **always** occurred just after [a Hopf bifurcation where] the direction of curvature of the limit cycle had changed. Furthermore, such changes were quite easy to forecast.

[1] Professor George Loizou, personal communication.

8. APPLYING CANARD EXPLOSION THEORY TO RICHARDSON'S EQUATIONS.

Let us now apply these 'canard' arguments to Richardson's equations for arms races between nations, superficially a very different application from processes in the manufacture of plastics. It can be shown that the curvature of the limit cycle, $d\{ dy/dt \ / dx/dt \} / dt = 0$ in Richardson's Equations (1) has the sign of $(a - c)$. If $a > c$ the limit cycle revolves counter-clockwise **;** if $\boldsymbol{a < c}$, clockwise. Put another way, if $\boldsymbol{a}$ overtakes c numerically, there will be a change of sign (and of direction of curvature). The mathematics indicate that instability does not occur at the precise point of change of sign, but just after it.

9. CAN INSTABILITY IN RICHARDSON'S EQUATIONS BE PREDICTED?

Can these changes of sign be used to **predict** the approach of an unstable point? It was decided to modify GA - 37a to calculate the approximation $\boldsymbol{a} \ (\approx k_{12} * x_m)$ and $\boldsymbol{c} \ (\approx k_{21} * y_m)$ and to print $(a - c)$ but only for the fifteen best chromosomes on display. The modified program was called GA - 37aa.

In context, the significance of the difference $(a - c)$ lies not in its value but in the number of changes of its sign at each iteration. Figure 6 plots the **number of negative signs in** $(a - c)$ (maximum 15) at increasing values of defexp%. Each instability is prefaced by a sudden plunge from a high number of minus signs to a small number of minus signs. For $\boldsymbol{(a - c)}$ to be negative $c > a$, and for it to be positive $a > c$. Analysing this and other charts, if a overtakes c very rapidly (normally denoted by a change of ten or more signs from minus to plus at one step) then instability is imminent**.** [Curiously, $\boldsymbol{c}$ overtaking $\boldsymbol{a}$ does not seem to have the same destabilising effect]. A lesser number of sign changes, say eight, does not have this effect. Perhaps a more useful indicator of impending active hostilities would be a high level of minus signs**;** unless it is above ten there is no likelihood of instability.

10. USING GENETIC ALGORITHMS IN THIS TYPE OF APPLICATION.

- Any GA attempting to model the real world must be based on some live data from that world. At least some of the multitude of variables must be fact.
- The real-valued GA used was fairly standard. Both crossover and mutation were varied dynamically. Crossover started high but reduced while mutation started low but increased, albeit neither by much**;**
- once convergence starts then the search for better solutions is led by mutation.

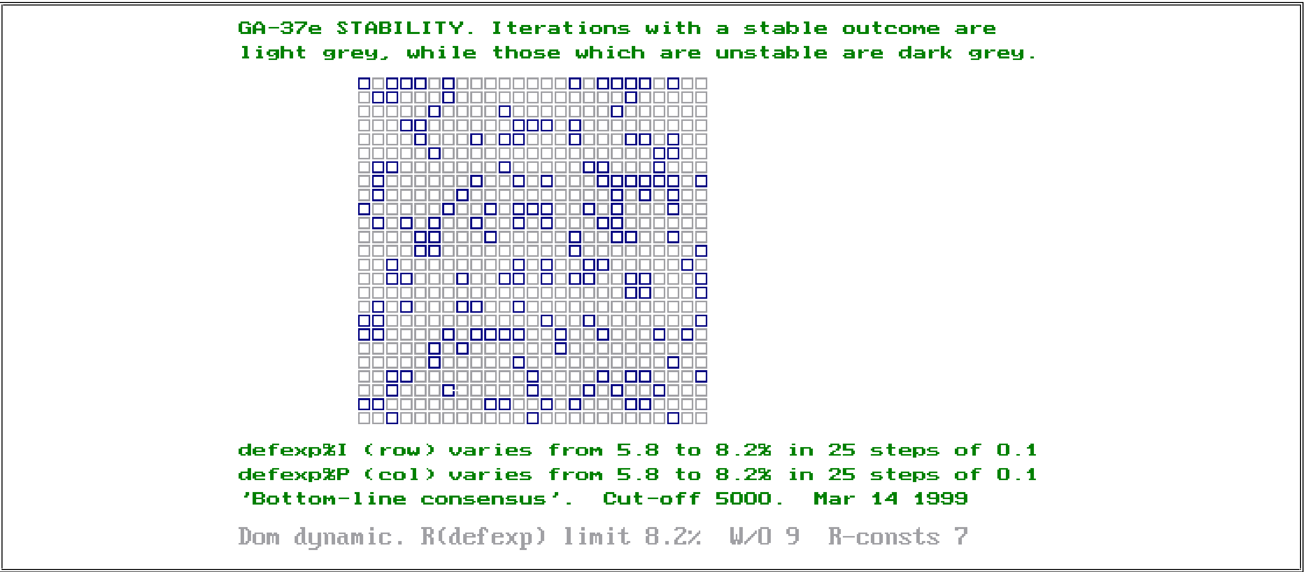

Figure 5 : A Stability Diagram.

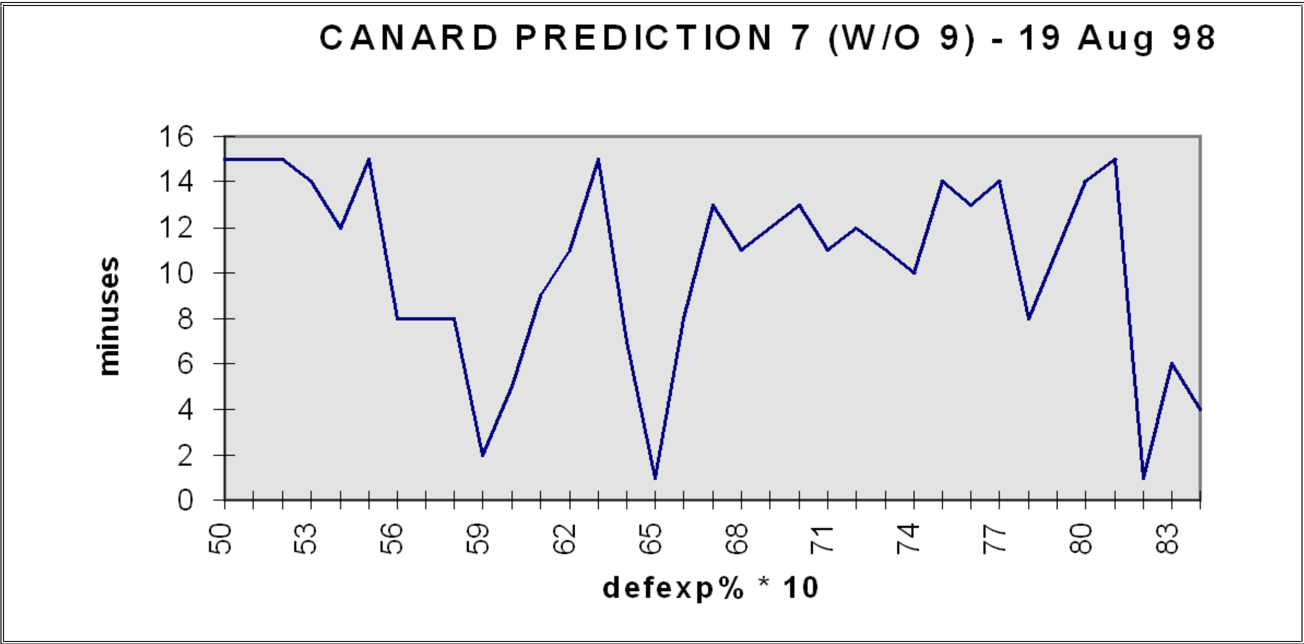

Figure 6 : Instabilities actually occur at 6.5 - 6.6% and at 8.2 - 8.3%

- GAs must never be *forced*, but they can be *coaxed*. A good way to coax is to constrain the initialising random number generator to a likely limit for each parameter. If it is known that a certain parameter never exceeds, say, 45 then to set *random(99)* is a waste of both time and resources. In other words, the search space should be controlled and it should be feasible.
- Some of the genes were integers, some were real numbers, some had two digits and others three. When concatenated in a chromosome, however, it all *looked like* one big integer! The downside of this is that at crossover, some chromosomes get split mid-gene. No noise was apparent from these loose 'bin-ends'.
- Different forms of crossover were tried, including *Uniform Crossover* [Syswerda, 1989] but, on balance, single crossover appeared to be best for this application.

11. CONCLUSIONS.

This paper set out to investigate the arms race between India and Pakistan by making use of a GA to search the large spaces needed by Richardson's Arms Race Theory. It used real-world IMF data to generate the constants and scaling data needed in two workable evaluation equations. The resulting real-valued GA did vindicate the theoretically-derived stability criteria. It also found that, far from the expected 'seas' of stability and instability divided by clearly marked 'walls' that there were large areas of stability separated by quite local but violent pockets of instability. A comparison was made with the Peng model of canard explosions and it was found that the two models are very similar**;** the advantage of this approach was that the mathematics of the canard model — and in particular changes in the direction of curvature of the path of the limit cycle — allowed instabilities (i.e. explosions) to be **predicted**. It would seem that the lessons of the canard model **can** be applied successfully to arms races. Known and likely instabilities between India and Pakistan seem always to be prefaced by a sudden change in the limit cycle. This change is very simple and is expressed by the sign of **(a - c),** two coefficients from Equations (1). If the sign of **(a - c)** of two-thirds of the GA's population changes in successive iterations from minus to plus, ***then it seems that an instability will occur at the next timeframe.***

For this hypothesis to be useful in the real, political world we need better means of deriving **a** and **c**. The simple expressions used here are too crude, besides which the information is not readily available or easily updated on a daily basis. Something more pragmatic, some better transform, is needed.

Some would argue that it is not valid to extrapolate from mechanistic cause and effect (the certainty that certain concentrations of ions will explode) to the vagaries of human interaction. Nations go to war, not because of their percentage defence expenditures but because their leaders believe that they should, and that they will be backed in that decision by their people. Equally, those same leaders like to believe that they have free will and are not pre-destined or pre-programmed automatons. On the other hand, the humorist Frank Muir has claimed that mankind has as much freedom of decision as a plastic duck in a Jacuzzi! The truth may lie somewhere in between these extremes. Had the First World War not been 'waiting to happen', as historians now claim, the assassination of Arch-Duke Ferdinand by Bosnian nationalists at Sarajevo would never have plunged the world into four years of appalling bloodshed. Sarajevo was only the final detonator, the last straw. It has been shown in this paper that the situation between India and Pakistan contains large areas of stability and only pockets of instability. No matter how belligerent their leaders, no matter how much sabre-rattling is done, ***it will not physically be possible to start a war when all other parameters are stable.*** The danger comes when the parameters are unstable, for at that point almost anything — such as one inflammatory speech — can trigger a conflict. It follows that it ***is*** worth predicting the pockets of instability providing it is appreciated (and hoped) that a trigger may never materialise and the potentially unstable situation may quietly revert into stability.

AFTERNOTE

Crucial to this paper was the selection of the 'least squares' fitness function at Equation 7. Do India and Pakistan really want to minimise the combined cost of their armaments? If the answer is 'yes', then both governments are indulging in sabre-rattling and vote-catching for internal purposes. To launch a nuclear war against one's immediate geographical neighbour, and to risk fallout from one's own weapons, would be suicidal. Assuming that there was a serious risk of conflict in 1998, Figure 6 shows that, on present forecasts, the next flashpoint is not likely for another seventeen years, which means that it may not happen at all.

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