Solutions to Systems of Nonlinear Equations Via Genetic Algorithm

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Abstract

Solving systems of nonlinear equations is perhaps the most difficult problem in all of numerical computation. In this paper, a hybrid scheme is presented in which a genetic algorithm is used to locate efficient initial guesses which are supplied to a Newton method for solving a system of nonlinear equations. The hybrid scheme is tested on a specific example that is representative of this class of problems - one of determining the coefficients used in Gauss-Legendre numerical integration.

1 A SPECIFIC PROBLEM FROM NUMERICAL INTEGRATION

The basic problem of numerical integration (quadrature) is to approximate the definite integral of $f(x)$ over the interval $[a, b]$ by evaluating $f(x)$ at a finite number of sample points. An example quadrature formula is:

$$Q[f] = \sum_{j=1}^{N} w_j f(x_i) = w_1 f(x_1) + w_2 f(x_2), ..., w_N f(x_N)$$

with the property that

$$\int_a^b f(x)dx = Q[f] + E[f]$$

where $Q[f]$ is the approximation to the integral and $E[f]$ is the truncation error associated with the approximation. The values x_j are called the *quadrature nodes* and the values w_j are called *weights*.

A general Gauss-Legendre N-point formula can be developed that is exact for polynomial functions of degree $\leq (2N-1)$. The general N-point formula results in a system of nonlinear equations that must be solved to compute the value of the quadrature nodes (x_j) and their associated weights (w_j). The system of nonlinear equations that results from the general N-point formula is:

$$f(w_1, w_1, .., w_N, x_1, x_1, .., x_N) = \sum_{j=1}^{N} w_j x_i^{N-1} - \int_{-1}^{1} x^{N-1} = 0$$

for $n = 1, 2, ..., 2(N-1), 2N$. The resulting system of nonlinear equations is difficult to solve using traditional search techniques such as a Newton method. Here, a genetic algorithm is used to generate points in the search space from which to initiate a Newton search.

2 RESULTS

A hybrid search scheme combining a genetic algorithm with a Newton search was used to develop N-point Gauss-Legendre quadrature formulae.

The results are summarized the table below.

N	Successes in 100,000	Trials	Evals
4	792	126	900
5	181	552	900
6	39	2,564	1100
7	4	25,256	2200
8	1	85,132	2500

When considering this table, recall that a fitness function evaluation consisted only of computing the numerical value of $2N$ individual equations and taking the maximum of these values. Thus, a fitness function evaluation is actually quite inexpensive computationally. The number of fitness function evaluations listed in the table are the number of evaluations required such that the Newton method would converge to the solution of the system of nonlinear equations on this first guess. Notice in this table that the number of function evaluations required by the genetic algorithm increases polynomially with N as opposed to the exponential behavior depicted by the Newton method acting independently.