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# Prediction of Silicon Content of Hot Metal Using Fuzzy-GA Regression

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## Abstract

The use of fuzzy regression model is generally recommended for industrial processes in which both input and output parameters are fuzzy in nature. However, the application of fuzzy regression (using linear programming) to optimize the regression coefficients in our problem of silicon prediction in blast furnace hot metal showed large deviation from actual values at lower and upper bounds, in spite of the fact that both correlation coefficient and standard deviation for entire data were acceptable. Analysis showed that linear programming procedure used in fuzzy regression is unable to take care of this anomaly. Therefore, in the present work, a fuzzy-GA model has been developed and it has been found that performance of Fuzzy-GA regression model is far superior to simple fuzzy regression. The spread of fuzzy coefficients obtained by fuzzy-GA regression is reduced significantly with the use of GA in the global search for optimized coefficients. It is recommended that simple fuzzy regression be replaced with fuzzy-GA regression for control and prediction in industrial unit operations which generally show a fuzzy behaviour.

## 1 INTRODUCTION

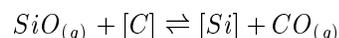
In an integrated steel plant, the silicon content of hot metal is an important parameter for control and operation of blast furnaces as well as of oxygen steel-making converters. In an earlier work (Singh, Srid-

har and Deo, 1996), several artificial intelligence techniques including artificial neural networks and fuzzy neural networks, were used to predict the silicon content on the basis of plant operational data. The best correlation coefficient (R) and standard error ( $\sigma$ ) obtained were 0.86 and 0.09 mass%, respectively.

In the present work, to start with, a non-linear regression was carried out on the same data set using artificial neural network with 7-1 architecture and with no hidden layer present. The correlation coefficient and standard error obtained were low, 0.55 and 0.075 mass%, respectively, and were unacceptable. Application of fuzzy regression and fuzzy-GA regression has substantially improved the accuracy of prediction; the best correlation coefficient and standard error with fuzzy-GA regression are now 0.997 and 0.01 mass%, respectively. The fuzzy-GA regression model, developed in this work for the first time, can be easily applied to any other unit operation showing fuzzy behaviour.

## 2 BLAST FURNACE OPERATION AND DATA

Blast furnaces at Visakhapatnam Steel Plant, Visakhapatnam, India, produce liquid iron (hot metal) containing 4% carbon and 0.5-0.8% silicon approximately. The raw materials charged in the blast furnace are iron ore, coke, sinter and limestone. Silica ( $SiO_2$ ) present in the ore, coke and sinter gets reduced to silicon ( $Si$ ) essentially by the carbon present in the coke. The exact mechanism of  $SiO_2$  reduction is complex and still not clearly understood. For example, at high temperatures  $SiO_2$  can get gasified to  $SiO$  vapours or form silicon carbide.  $SiO$  vapours become unstable at low temperatures and get reduced by carbon.



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Several mechanisms of silica reduction have been proposed (Batra, 1992 and Singh, Sridhar and Deo, 1996), in literature. Empirical equations have also been proposed which include the contribution of various operating factors (Niwa et al. 1990).

$$[Si] = \frac{KK_f P_{SiO_2} H^*}{W^{\frac{2}{3}}}$$

$$K_f = 2.116 \times 10^8 e^{\frac{-29260}{T_{HM}+273}}$$

$$H^* = 0.4257\gamma + 0.01537C_R + 0.00185T_{FT} - 0.00244T_{HM} + 0.0234MgO + 25.92$$

where,

$K$  is a constant,

$T_{HM}$  is the dropping metal temperature at the hearth,

$C_R$  is ore/coke ratio,

$T_{FT}$  is the theoretical flame temperature,

$MgO$  is the  $MgO$  content in the slag,

$W$  is the amount of hot metal tapped,

$P_{SiO_2}$  is the partial pressure of  $SiO$  gas, and

$\gamma$  is the heat flux ratio.

These coefficients vary from furnace to furnace and need to be adjusted.

In the present work, typical data for Blast Furnace No.1 at Visakhapatnam Steel Plant were obtained for a period of 182 days of continuous operation. After intensive study and analysis of literature (Batra, 1992 and Singh, Sridhar and Deo, 1996), the important operational parameters were identified. Coke rate (1), Hot blast temperature (2), Cold blast temperature (3), Blast pressure (4), Slag rate (5), Top pressure (6) and Slag basicity (7) are chosen as the independent variables, and the mass%-Silicon (8) in the hot metal as the dependent variable. The typical values of the variables are given in Table 1. These data were subjected to fuzzy regression for prediction of mass%-Silicon of hot metal using both linear programming (LP, conventional method) and Genetic Algorithm (GA, new solution procedure developed in this work).

### 3 FUZZY REGRESSION

Regression is a statistical technique which is used to model the relationship between one dependent ( $y$ ) and on or more independent ( $x_i$ ) variables. So,

$$y = f(x_1, x_2, \dots, x_n)$$

In a linear regression model, the degree of contribution of each variable to the output is represented by the coefficients ( $A_i$ ) of these variables,

$$y = f(x, A)$$

Table 1: Typical Data For BF1 At Visakhapatnam Steel Plant, Visakhapatnam, India

Cok Rt	H B Temp	C B F1	Bl Pr	Slg Rt	Top Pr	Slg Bas	% Si
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
530	1050	4250	3.00	319	1.70	1.03	0.46
595	1050	4250	3.09	354	1.64	1.10	0.67
501	1050	3850	2.65	299	1.39	1.04	0.67
505	1050	3700	2.59	302	1.39	0.97	0.62
500	1050	3750	2.50	301	1.20	1.00	0.69
525	1050	3900	2.70	313	1.39	0.94	0.68
525	1050	3900	2.84	319	1.64	0.97	0.70
519	1050	4200	3.00	310	1.70	0.95	0.63
599	1050	4300	3.00	368	1.75	0.95	0.51
513	1050	4100	2.95	303	1.70	0.94	0.55
513	1050	4200	2.90	306	1.60	0.93	0.58
517	1050	4400	3.90	309	1.70	0.94	0.46
506	1050	4300	3.00	302	1.70	0.97	0.63
536	1050	4400	3.20	320	1.79	0.93	0.66
550	1050	4400	3.20	330	1.79	0.95	0.69
520	1050	4450	3.09	311	1.79	0.98	0.58
508	1050	4500	3.20	303	1.85	0.94	0.52
501	1050	4800	3.20	300	1.85	0.98	0.45
555	1050	4800	3.20	331	1.85	0.99	0.46
513	1050	4600	3.20	306	1.79	1.02	0.47
501	1050	4500	3.20	299	1.79	1.04	0.50
549	1050	4900	3.29	328	1.85	0.93	0.57
552	1050	4900	3.29	329	1.85	0.92	0.43
527	1050	4800	3.29	314	1.85	0.94	0.46
483	1050	4800	3.20	288	1.89	0.94	0.43
576	1050	4800	3.20	344	1.79	0.98	0.42
554	1052	4400	3.20	320	1.79	0.93	0.41
500	1045	4400	3.20	298	1.79	0.95	0.62
532	1045	4750	3.09	318	1.79	1.00	0.47
509	1030	4700	3.09	305	1.79	1.01	0.42
529	1040	4600	3.09	316	1.79	0.99	0.51
547	1030	4500	3.20	329	1.75	1.00	0.66
593	1015	4600	3.20	356	1.79	0.92	0.52
491	1005	4650	3.20	294	1.79	1.00	0.42
526	1025	4400	3.09	309	1.75	1.01	0.60

$$y = A_0 + A_1x_1 + A_2x_2 + \dots + A_nx_n$$

where,

$A_0, A_1, \dots, A_n$  are coefficients.

The difference between a conventional regression and a fuzzy regression is as follows. In conventional regression, the difference between observed and estimated/calculated value, called observational error, is considered to be a random variable. It is probabilistic in nature. In fuzzy regression the difference between observed and estimated/calculated values is assumed to be due to inherent ambiguity present in the system. The output value for a specific input values is thus assumed to be a range of possible values and not an exact or crisp value as it happens in case of normal conventional regression analysis. In other words, fuzzy regression is possibilistic in nature. Also coefficients used in fuzzy regression are fuzzy functions or numbers. A linear fuzzy regression model is of the form as

shown,

$$\tilde{y} = f(x, \tilde{A})$$

$$\tilde{y} = \tilde{A}_1 x_1 + \tilde{A}_2 x_2 + \dots + \tilde{A}_n x_n$$

where,

$\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_n$  are fuzzy coefficients and  $\tilde{\Pi}$  is a fuzzy number.

Fuzzy representation procedure is well described by Ross (1995). A fuzzy number, as shown in Figure 1, is represented as a center-point or mid-point ( $p_i$ ) and its spread ( $c_i$ ). Thus representation of any fuzzy number is  $(p_i, c_i)$ .

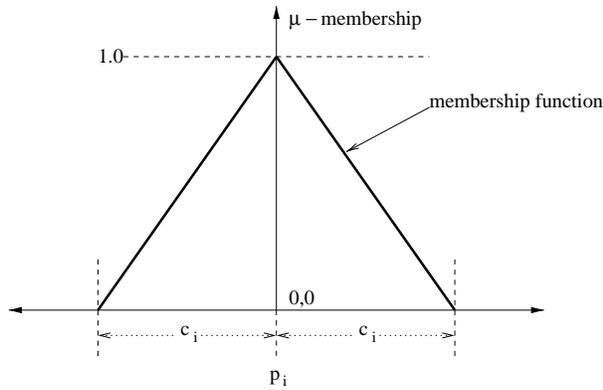


Figure 1: A Fuzzy Number Representation

We know that, mathematically,

$$\tilde{y} = \tilde{A}_1 x_1 + \tilde{A}_2 x_2 + \dots + \tilde{A}_n x_n$$

$$\tilde{y} = (p_1, c_1)x_1 + (p_2, c_2)x_2 + \dots + (p_n, c_n)x_n$$

$$\tilde{y} = \sum_{i=1}^n (p_i, c_i)x_i$$

the membership function for  $y$  as defined by Ross (1995), is,

$$\mu_y = \begin{cases} \max(\min[\mu_{A_i}]), & \{A \mid y = f(x, A)\} \neq \phi \\ 0, & \text{otherwise} \end{cases}$$

also, the generalized membership function could be written as,

$$\mu_{A_i} = \begin{cases} 1 - \frac{|p_i - c_i|}{c_i}, & (p_i - c_i) \leq x_i \leq (p_i + c_i) \\ 0, & \text{otherwise} \end{cases}$$

using above equation, we get,

$$\mu_{y_i} = \begin{cases} 1 - \frac{|y_i - \sum_{i=1}^n p_i x_i|}{\sum_{i=1}^n c_i}, & x_i \neq 0 \\ 1, & x_i = 0, y_i = 0 \\ 0, & x_i = 0, y_i \neq 0 \end{cases}$$

The objective of regression model is to determine the optimum parameters  $\tilde{A}^*$  such that fuzzy output set, which contains  $y_i$ , is associated with a membership value greater than  $h$ ,

$$\mu_{y_i} \geq h \quad i = 1, 2, \dots, n$$

As  $h$  increases the fuzziness of output increases. The corresponding values of spread and mid-point also vary. The value of  $h$  is set by the user; generally,  $h$  is taken to be 0.5.

In fuzzy regression the objective is to find the optimum values of the fuzzy coefficients which minimize spread of fuzzy output for all data sets. The objective function to be minimized is,

$$Obj = \min \left[ \sum_j^m \sum_i^n c_i x_{ij} \right]$$

which is subject to two constraints for each data (Figure 2), hence for  $n$  data points total number of constraints is

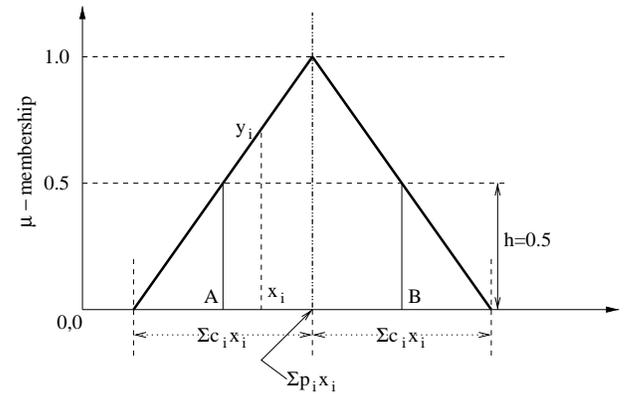


Figure 2: Representation Of Coefficient Values Using  $h = 0.5$

$2n$ , as defined below,

$$y_j \geq \sum_{i=1}^n p_i x_{ij} - (1-h) \sum_{i=1}^n c_i x_{ij}$$

$$y_j \leq \sum_{i=1}^n p_i x_{ij} + (1-h) \sum_{i=1}^n c_i x_{ij}$$

Thus the objective function to be minimized is,

$$c_0 + c_1(x_{11} + x_{12} + \dots + x_{1m}) + \dots + c_n(x_{11} + x_{12} + \dots + x_{nm})$$

subject to,

$$\begin{aligned}
 y_j &\geq p_0 + p_1 x_{1j} + \dots + p_n x_{nj} \\
 &\quad - (1-h)[c_0 + c_1 x_{1j} + \dots + c_n x_{nj}] \\
 y_j &\leq p_0 + p_1 x_{1j} + \dots + p_n x_{nj} \\
 &\quad + (1-h)[c_0 + c_1 x_{1j} + \dots + c_n x_{nj}]
 \end{aligned}$$

constraints for  $j = 1, n$  where  $n =$  number of data.

Fuzzy regression using linear programming has already been explained by Tanaka et al. (1982). They have used linear programming to find the values  $p_0, p_1, \dots, p_n$  and  $c_0, c_1, \dots, c_n$  for the regression model using the standard NAG routine E04MBF.

### 3.1 RESULTS OF FUZZY REGRESSION USING LINEAR PROGRAMMING

The coefficients obtained by LP are shown in Table 2.

For the plot of predicted versus the actual silicon of hot metal, the correlation coefficient and the standard error are 0.988 and 0.017 mass% respectively. Although LP gave a good overall prediction, at lower and upper bounds it showed large deviation from actual value, as shown in Figure 3.

Table 2: Fuzzy Coefficients Obtained Using LP

Variable	Mid-point	Spread
const	.1354046	.4812556
1	.6242060E-03	.0000000E+00
2	.3905879E-03	.0000000E+00
3	-.1596317E-03	.0000000E+00
4	.3661064E-01	.0000000E+00
5	.1032940E-04	.0000000E+00
6	.1233871	.0000000E+00
7	.1137497	.0000000E+00

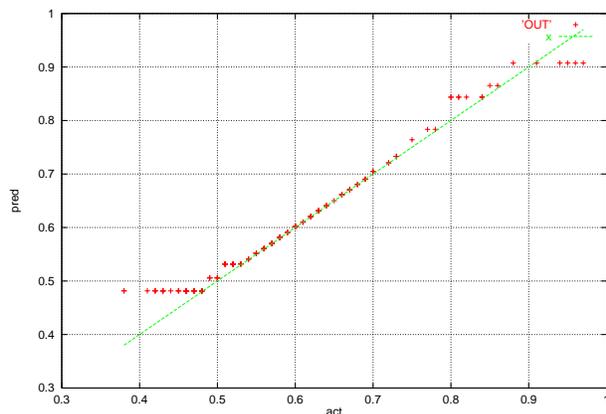


Figure 3: Fuzzy Regression Using LP; Predicted Versus Actual Silicon;  $R = 0.988$ ,  $\sigma = 0.017$  mass%

## 4 FUZZY REGRESSION USING GENETIC ALGORITHM

In the present work, Genetic Algorithm (GA) has been used to get the near-optimum values of fuzzy coefficients. The optimization problem to be solved by GA, however, is modified as,

$$Obj = \min / \max\{f(x)\}$$

subject to  $N$  inequality and  $M$  equality constraints,

$$g_i(x) \geq 0, \quad i = 1, 2, \dots, N$$

$$h_j(x) = 0, \quad j = 1, 2, \dots, M$$

The penalty function method (Deb, 1995) is used to incorporate the constraints. The objective function is changed to,

$$Obj = \min / \max\{f(x) + R * \langle g_i(x) \rangle^2 + R * i \{h_j(x)\}^2\}$$

where,

Bracket operator penalty,  $\langle \alpha \rangle = \alpha^2$ , when negative, 0, otherwise.

In the present work, Carroll's GA Fortran code (version 1.6.4) (1997) has been used with slight modifications.

After obtaining the values of coefficients the values of  $y$  obtained from the model are defuzzified to get crisp values. The maximum membership function method is used for defuzzification. The model returned value (which is a fuzzy number) is converted back to crisp value, such that the value is closest to the actual value, i.e., the value which has the maximum membership. In place of this, another method can be used which involves averaging, but it has not been found to be suitable for the present work.

### 4.1 RESULTS OF FUZZY-GA REGRESSION

In the first GA run itself no scatter was present at the ends (see Figure 4) in contrast to LP (see Figure 3). Also GA showed clustering of predicted data.

In the first GA run, micro-GA (or  $\mu$ -GA) was used, with following parameters:

```

total number of parameters evaluated = 16,
total number of constraints           = 364,
total binary string length            = 160
                                        (10 for each parameter)

```

```

population                             = 7,
(with 1 child strings kept for the next
generation)

```

```

mutation probability                    = 0.0001,
crossover probability                   = 0.8,
maximum generations                     = 5000,

```

Tournament selection, with uniform crossover and elitist selection is used.

```

Upper bound for all parameters         = 1.0

```

Lower bound for mid-points parameters = -1.0  
 Lower bound for spread parameters = 0.0

Best solution was found at generation 4763 and corresponding fuzzy coefficients are listed in Table 3.

Table 3: Fuzzy Coefficients Obtained Using GA

Variable	Mid-point	Spread
const	0.3118E+00	0.2151E-01
1	0.3763E+00	0.0000E+00
2	-.8798E-02	0.0000E+00
3	-.1075E-01	0.0000E+00
4	0.8416E+00	0.3597E+00
5	-.4643E+00	0.0000E+00
6	-.6383E+00	0.1564E-01
7	-.7224E+00	0.7234E-01

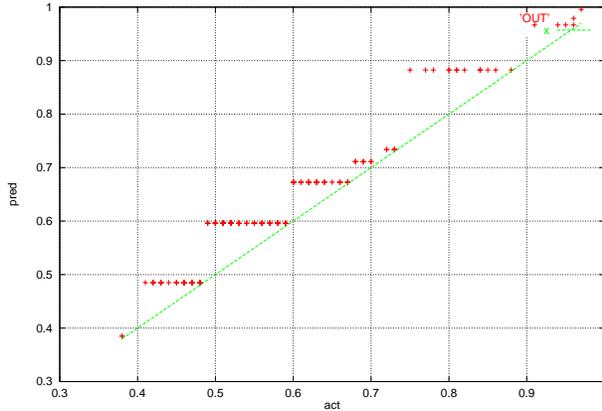


Figure 4: First GA Run; Predicted Versus Actual Silicon;  $R = 0.96$ ,  $\sigma = 0.03$  mass% (No Deviation At Ends Is Observed)

The correlation coefficient and the standard error were 0.96 and 0.03 mass% respectively.

The parameters in the second GA run were changed to further improve the results. The spread is also minimized, as shown in Table 4. Also, the scatter in actual versus predicted %Si plot is less (as shown in Figure 5) as compared to the 1st GA run results.

The GA parameters in the final run with  $\mu$ -GA were:

The total number of parameters evaluated, constraints, binary string length, and lower and upper bounds for all parameters were kept same as in the 1st run. The other parameters which were changed are listed below:

```

population                = 11,
(with 4 child strings kept for the next
                           generation)

mutation probability       = 0.00001,
crossover probability     = 0.9,
maximum generations       = 10000,
  
```

The best solution was found at generation 9547. The optimization fuzzy coefficients obtained are listed below in Table 4.

Table 4: Fuzzy Coefficients Obtained Using GA

Variable	Mid-point	Spread
const	0.9775E-03	0.3324E-01
1	-.1271E-01	0.0000E+00
2	0.1075E-01	0.0000E+00
3	-.9775E-03	0.0000E+00
4	-.1437E+00	0.0000E+00
5	0.6843E-02	0.0000E+00
6	-.7595E+00	0.9775E-02
7	0.9775E-03	0.2542E-01

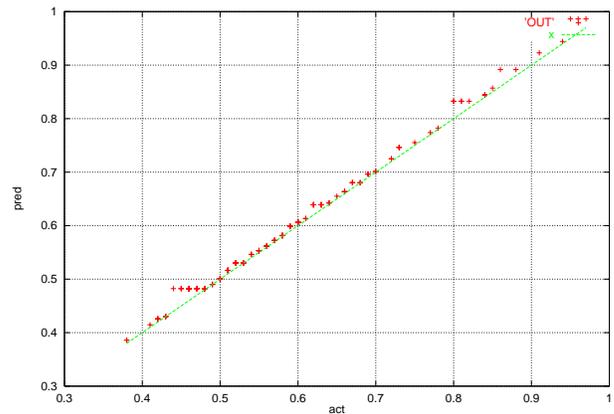


Figure 5: Final And Best Fuzzy-GA Result; Predicted Versus Actual Silicon;  $R = 0.997$ ,  $\sigma = 0.01$  mass%

The correlation coefficient and the standard error are 0.997 and 0.01 mass% respectively. Other statistical details pertaining to Figure 5 are provided below:

```

Mean of actual Si content
(independent variable) = .5814
Mean of predicted Si content
(dependent variable) = .5896
Standard deviation of
independent variable = .1194
dependent variable = .1210
Correlation coefficient = .9976

Regression coefficient = 1.0106
Standard error of coefficient = .0052
t-value for coefficient = 192.9625

Regression constant = .0020
Standard error of constant = .0031
t-value for constant = .6459

F-value = 37234.5202
  
```

## 5 COMPARISON BETWEEN FUZZY REGRESSION USING LP AND USING GA

Code”, <http://www.staff.uiuc.edu/~carroll/ga.html>

Control of silicon content of hot metal is important in blast furnace operation. Fuzzy regression was performed on operational data and the fuzzy coefficients have been calculated both by linear programming and GA. The objective was to reduce the spread over the whole data. LP gave a large scatter at lower and upper bounds (see Figure 3), which was unacceptable. With GA, not only the scatter at lower and upper bounds was minimized (compare Figure 3 and 5) but also the correlation coefficients (for GA 0.997 and for LP 0.988) and the standard deviation (for GA 0.01 mass% and for LP 0.017 mass%) were improved.

It is recommended that fuzzy-GA regression procedure, as developed in this work, may be applied to carry out regression on the normal non-fuzzy data set to get a regression model with fuzzy coefficients, specially to unit operations in steel industry, chemical industry and to other areas too, where a process is difficult to model and the relationship between dependent and independent variables is too complex and to some extent, possibilistic and/or stochastic in nature. Also this method can be used when data itself is of fuzzy nature. It is the experience of the authors that for iron and steel making processes artificial neural networks have not so far yielded as good results as obtained using fuzzy-GA regression.

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