# GA Hard *Problems*

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## Abstract

We define GA-hardness and provide a method for analyzing the GA-complexity of the underlying problem in a way that can be related to classical complexity.

## 1 GA-HARDNESS

By applying the same techniques used in classical complexity theory, we develop a rigorous definition for GA-Hard *problems*.

**Definition 1:** Let R be a polynomial time computable, optimality-preserving transformation. A problem G is **GA-hard** for class C with respect to R if every problem A in C reduces in polynomial time to G via R, and G is in PO, and any GA for G requires more than polynomial time to converge for some instance unless PO=NPO. (In this case, PO and NPO are the optimization analogs of P and NP.)

## 2 MINIMUM CHROMOSOME LENGTH

We show that by using Ankenbrandt's [Ankenbrandt, 1991] convergence proofs and adding a polynomial bound for the encoding/decoding, we can define the complexity of a problem instance. By then applying our Minimum Chromosome Length (MCL) method we can describe the GA-complexity of a problem. A *desideratum* for a representation is to minimize the number of bits in a chromosome that still uniquely identifies a solution to the problem.

**Definition 2:** For a problem P, and $D_n$ the set of instances of P of size n, let MCL(P,n) be the least $l$ for which there is an encoding $e{:}S^l \rightarrow D_n$ with a domain dependant evaluation function g, where g and e are in FP (the class of functions computable in polynomial time).

## 3 MCL GROWTH

Complexity is typically based on the rate of growth of a fundamental unit of measure as a function of the input size. The MCL growth rate can be used to bound the worst case complexity of the problem for a GA.

**Theorem 1:** for any problem P, if $2^{MCL(P,n)} \in O(n^k)$ for some *k*, then P$\in$PO.

**Conjecture 1:** If P$\in$PO, and $2^{MCL(P,n)} \notin O(n^k)$, then P$\neq$NP.

**Conjecture 2:** If P$\in$PO, and $2^{MCL(P,n)} \notin O(n^k)$, then P is GA-hard for PO (using optimization preserving polynomial time reductions).

## 4 GA COMPLEXITY CLASSES

This discussion leads us to propose a new complexity class specifically for algorithms, such as GAs, which use a mapping from genotype to phenotype. This class represents the class of GA problems whose MCL growth rate is linear.

**Definition 3:** A problem P is in the class NPG if MCL(P,n)$\in$O(n).

For example, maximum clique (MC) is in NPG. Any problem in PO is also in NPG, since one can use an *1*-bit null representation, which one then ignores while solving the problem directly.

## 5 CONCLUSIONS

These developments allow researchers to categorize a complexity hierarchy specifically for GAs. In addition, the development of a definition for GA-hardness should help the stalled efforts to find a GA-hard problem. This is important because we can now begin to evaluate the usefulness of GAs against other approximation methods.

**References**

Ankenbrandt, C.A. (1991). An Extension to the Theory of Convergence and a Proof of the Time Complexity of Genetic Algorithms, FOGA1, Morgan Kaufman.