Study of Evolution in Genetic Algorithms by Eigen's Theory Including Crossover Operator

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Abstract

A theory representing the dynamics of infinite population Genetic Algorithms is given by using Eigen's evolution model. The effect of crossover is included to study the evolution of infinite population GAs. We present a theory of GA dynamics, in which we give discrete time equations for GA evolution including the mutation and crossover processes. The Walsh analysis of allele frequencies provides a very powerful tool for studying the evolutionary processes by mutation and crossover.

In previous papers [1, 2], we gave a model for describing the evolution of allele frequencies in infinite population GAs. In the first paper [1], we applied Eigen's model for describing evolutionary processes of GAs with selection and mutation. In this system, the combined effect of selection and mutation is represented by the selection-mutation matrix. In the second paper [2], we modified differential equations to a system of difference equations for simulating generational GA processes and gave a procedure for solving equations.

We consider the infinite population model of the Simple Genetic Algorithms with selection, mutation and crossover. We treat binary strings of length l as chromosomes having one gene with $n = 2^l$ alleles. Let $x_i(t)$ be the relative frequency of the i-th allele at generation t. The change of the distribution in the mutation process is expressed by the mutation matrix M

$$x_i(t+1) = \sum_{j=0}^{n-1} M_{ij} x_j(t) \qquad (i = 0, \ldots, n-1), \quad (1)$$

The solution of the equations (1) is

$$\boldsymbol{x}(t) = M^t \boldsymbol{x}(0). \quad (2)$$

We can obtain an explicit form of the solution

$$\boldsymbol{x}(t) = \frac{1}{n} \sum_{j=0}^{n-1} (1-2p)^{|j|t}\, \tilde{x}_j(0) \mathbf{w}_j. \quad (3)$$

Here p is the mutation rate and $|j|$ is the number of bit ones in the integer j. The components of $\mathbf{w}_j$ are the jth Walsh functions.

The evolution equation of crossover is given as

$$x_i(t+1) = \sum_{j=0}^{n-1} \sum_{j'=0}^{n-1} C_{i,jj'} x_j(t) x_{j'}(t).$$

We have the evolution equation for the Walsh coefficients $\tilde{x}_k(t)$ in the crossover process

$$\begin{aligned} \tilde{x}_k(t+1) &= \frac{1}{n} \sum_{k=i\oplus j} \sum c_{ij}\, \tilde{x}_i(t)\, \tilde{x}_j(t) \\ &= \frac{1}{n} \sum_{i=0}^{n-1} c_{i,i\oplus k}\, \tilde{x}_i(t)\, \tilde{x}_{i\oplus k}(t), \end{aligned}$$

where $i \oplus j$ is bitwise exclusive-or on integers, and the crossover coefficient c_{ij} for uniform crossover is

$$c_{ij} = n \prod_{k=1}^{l} \left\{ \frac{1-i_k}{2} + \frac{1-j_k}{2} \right\}. \quad (4)$$

References

[1] H. Furutani, "Application of Eigen's evolution model to infinite population genetic algorithms with selection and mutation ," *Complex Systems*, **10** (1996) 345–366.

[2] H. Furutani, "Analytical solutions for infinite population genetic algorithms on multiplicative landscape," In W. Banzhaf et al. (Eds.) *Proceedings of the Genetic and Evolutionary Computation Conference, GECCO-99* (Morgan Kaufmann), **1** (1999) 204–211.