





# Multi-Objective Genetic Algorithms With Concepts from Statistical Thermodynamics

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## Abstract

Obtaining the fullest possible representation of solutions to a multi-objective optimization problem has been a major concern in Multi-Objective Genetic Algorithms (MOGAs). This is because a MOGA, due to its nature, usually produces several clusters of solutions that does not cover the whole range of Pareto frontier. This poster paper introduces an overview of a new approach, one that aims at obtaining a Pareto solution set with maximum possible coverage and uniformiy. The proposed algorithm is based on an application of the concepts from statistical theory of gases (i.e., entropy) to a MOGA.

## 1 INTRODUCTION

One can find an analogy in the statistical theory of gases, which can be taken advantage of to improve the coverage and uniformity of the solutions obtained by a MOGA. When an ideal gas undergoes an expansion to fill in an enclosure, the molecules move randomly (i.e., without having any a priori information about the enclosure) to fill it in and achieve a homogenous and uniform equilibrium state with maximum entropy. This anology can be applied to the evolution of individuals in a MOGA to achieve uniformly distributed solutions with maximum possible coverage over the Pareto frontier. Indeed, the stochastic and evolutionary nature of statistical thermodynamics makes it a favorable platform for simulating the dynamics of the genetic algorithms (Shapiro et al. , 1995).

## 2 PROPOSED APPROACH

As we mentioned before, if we can simulate the behavior of the ideal gas and apply it to a MOGA to control the expansion of individuals over the Pareto frontier, the final state will be a uniformly distributed Pareto set, instead of a set of clustered points. As such, the objective would be to modify a MOGA, so that it enables an expansion of a sample of individuals in lateral directions (i.e., normal to

the evolution direction), according to a statistical gas model, while at the same time GA operators are applied.

To achieve this objective, first we assign a velocity to each member of the population, according to the ideal gas velocity distribution model. Then we define a transverse-expansion hyper-surface normal to the evolution direction of the population. The lateral direction vectors are tangent to this hyper-surface. An expansion operator is applied at each iteration of the process. This operator is to enhance each individual (or a certain percentage of the population) and the magnitude of enhancement is proportional to its assigned velocity, along the assigned direction vector in the variable space. Since this operator is applied at each iteration, it simulates the gradual movement and expansion of gas molecules with different velocity magnitudes and directions. Constraint handling is also very similar to that of ideal gas. When a gas molecule collides with the boundary of an enclosure, it is reflected back into the enclosure without a change in its speed, but the movement direction will be different and follows the reflection law in a mirror. Similarly, an inequality constraint in MOGA is a hyper-surface in the variable space and can be treated as a wall or a boundary. This constraint handling aspect of the approach makes this MOGA a feasible-direction type search method.

## 3 CONCLUSION

In this poster paper, we presented an overview of a new approach to multi-objective genetic algorithms, according to an analogy from the statistical thermodynamics. This MOGA expands while it also evolves a sample of population as it approaches the Pareto frontier in order to achieve maximum uniformity and coverage of the solution set.

## 4 REFERENCES

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## Self-Adaptive Evolutionary Pattern Search Algorithms

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### Abstract

Convergence analyses of evolutionary pattern search algorithms (EPSAs) have shown that these methods have a weak stationary point convergence theory for a broad class of unconstrained and linearly constrained problems. We describe how the algorithmic framework for EPSAs can be adapted to allow each individual in a population to have its own mutation step length (similar to the design of evolutionary programs and evolutionary strategies). We call these self-adaptive EPSAs, since each individual's mutation step length is adapted independently. The update rule for the step length parameter can be quite simply stated, and we describe how pattern step mutation can be integrated into a variety of standard formulations of evolutionary algorithms.

### 1 Introduction

Evolutionary pattern search algorithms (EPSAs) are a class of real-valued evolutionary algorithms (EAs) that adaptively modify the mutation *step length* during optimization. The principle motivation for the development of EPSAs is to better understand how to adapt the mutation step length in real-coded EAs on general classes of nonconvex multimodal problems. We use the term *step length* to describe the absolute step scale for methods like EPSAs as well as scale factors for random variables that are added to perform mutation in methods like evolutionary strategies and evolutionary programs.

Recent analyses of EPSAs [2, 1] have examined their

convergence behavior on problems of the form

$$\begin{aligned} \min \quad & f(x) \\ \text{subject to} \quad & x \in \Omega = \{x \in \mathbf{R}^n \mid l \leq Ax \leq u\}, \end{aligned} \quad (1)$$

where  $l, u \in \mathbf{R}^m \cup \{\pm\infty\}$  and  $A \in \mathbf{Q}^{m \times n}$ . The definition of EPSAs faithfully captures the basic algorithmic framework of generic EAs while satisfying the conditions of the convergence theory. Minimal assumptions are made on the stochastic competitive selection, and standard recombination operators can be applied in these EAs. Perhaps the most salient difference between EPSAs and other real-valued EAs is that EPSAs have an absolute step length that is used to generate mutation steps for all members of the population. By contrast, commonly used methods like EPs and ESs generate mutation steps using a step length parameter that is independently adapted for each member of the population.

We describe how the formulation of EPSAs can be refined to allow the step length parameter to be independently adapted for each member of the population. Conceptually, each individual in these EPSAs adapts its own step length parameter, so we call these *self-adaptive EPSAs*. We describe a variety of EA formulations that fall within the framework of self-adaptive EPSAs, and we note how these different EAs ensure that the convergence theory for self-adaptive EPSAs is applicable.

### References

- [1] W. E. HART, *Evolutionary pattern search algorithms for unconstrained and linearly constrained optimization*, IEEE Trans Evolutionary Computation. (to appear).
- [2] ———, *A convergence analysis of unconstrained and bound constrained evolutionary pattern search*, Evolutionary Computation, 9 (2001), pp. 1–23.