Optimal shape and location of piezoelectric materials for topology optimization of flextensional actuators

Ying Li [1] Xuemei Xin [2] Noboru Kikuchi[1] Kazuhiro Saitou[1]

[1] Department of Mechanical Engineering, University of Michigan, Ann Arbor, MI 48109-2125

[2] Department of Civil and Environmental Engineering, University of Michigan, Ann Arbor, MI48109-2125

Abstract

Piezoelectric actuator has been increasingly used in MEMS system due to its advantage of generality and flexibility. A flextensional actuator consist of a piezoceramic device, which can convert electrical energy into mechanical energy and vice versa, and a flexible mechanical structure, which can convert and amplify the output piezoceramic displacement in the desired direction and magnitude. A recent research in this area is optimizing the topology of the mechanical part while fixing the electrical part. In this research, the location and shape of the piezoelectrical component is optimization as an discrete problem. The mixed optimization problem has been solved by two-layer optimization procedure combining SLP and GA. The obtained optimal result is presented and discussed.

1 INTRODUCTION

Piezoelectric actuators are being increasingly used in various novel applications [1]. A piezoelectric actuator usually consists of two main components [2]: a mechanical part, which is a flexible structure, and an electrical part, which is the piezoelectric material block. One of the important issues of using piezoelectric actuator is to improve their performance for a certain amount of piezoelectric material, which is the goal of piezoelectric actuator design.

Design of piezoelectric actuators has been greatly advanced during the past ten years. Researchers are focusing on every aspect and every component in order to achieve the best performances, from piezo-ceramic composite design, optimal sizing and locating to topology optimization.

Topology optimization with homogenization method was proposed by Bensdϕe and Kikuchi to design the stiffest structure. This method is then applied to design compliant mechanisms[4][5] and composite materials. Since the mechanical part of flextensional actuators is actually a compliant mechanism, piezoelectric transducer [6] and thermal actuators [7][8] have been also designed using topology optimization technique. In this previous work of piezoelectric actuator optimization, topology and shape of the mechanical part of the actuator was designed, however, the location and shape of the piezoelectric material are fixed. This topological design optimization was able to generate effective mechanical structure that greatly improved the performances of the piezoelectric actuator.

There have also been design optimization techniques developed for the electrical part of piezoelectric actuator. The placement and size of piezo-material was optimized [10]. In a recent research, the distribution of piezoelectric material in the optical MEMS was optimized [11]. However the design of the flexible structure have not been taken into account.

Since the effectiveness of the actuator is decisively dependent on both mechanical and electrical part, it is desirable to design both. The work presented here is based on the topology optimization techniques and extends the design variables for shape and location of piezoelectric material in the extended design domain. Discrete optimization problem has been formulated in order to make the electrical part design consistent with the finite element model in topology optimization. Two layered optimization technique has been developed. An example is presented here to support the technique.

2 OPTIMIZATION TECHNIQUES

The use of fixed grid is the key point in topology optimization technique. Thus the finite element model does not change during the optimization process and excessive distortion to the finite elements can be avoided. In order to accommodate the two design parts in the same finite element model, the shape and location parameters are easily chosen as discrete variables, while the topology design parameters are still continuous. This requires the problem to be dealt as a mixed variable optimization problem. In order to utilize the existing topology optimization software, a specific two-layered optimization method is proposed to separate the continuous and discrete design variables in two optimization procedures, one over the other. That is:

$$\min_{a,b} f(x) = \min_{a}[\min_{b} f(x)]$$

Thus the optimization problem has been decomposed into two layers of optimization problems and the inner and outer optimzations are performed by homogenization design method and genetic algorithm respectively.

2.1 HOMOGENIZATION DESIGN METHOD

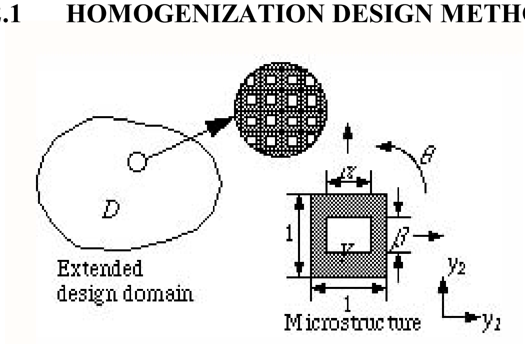


Figure 1: Homogenization Design Method

The topology optimization problems is formulated as a problem of finding the optimal distribution of material properties in an extended fixed domain. Where some structure cost function is maximized. Therefore the finite element model does not change during the optimization process. This technique is applied based on the homogenization method. To relax the optimization problem, a microstructure proposed by Bendsϕe and Kikuchi[1] is defined in each point of the domain which is a unit cell with a rectangular hole inside (Figure 1). The use of microstructure allows the intermediate materials rather than only void or full material in the final solution. The design variables are the dimensions α, β and the orientation θ of the micro-hole. In this sense the problem is to optimize the material distribution in a perforate domain with infinite microscope voids. The effective properties of the porous material, are calculated using the homogenization methods.

2.2 GENETIC ALGORITHM

Genetic algorithms are search algorithms based on the mechanics of natural selection and natural genetics [12]. It is a very efficient and robust method of discrete optimization.

The reason of choosing GA in this research is that GA searches a very large space and it exploits historical information to speculate on new search points. These make GA a speedy and efficient algorithm.

A genetic algorithm relies on the process of reproduction, crossover and mutation of notions to reach the global or "near global" optimum. Reproduction is a process by which the individuals are copied according to their objective function values. Crossover involves random mating of newly reproduced individuals in the mating pool. Mutation is the occasional random alteration of a string position. Mutation is necessary because although reproduction and crossover efficiently search and mix existing, occasionally they may result in loss of some infeasible solutions. High–performance notions are repeatedly tested and exchanged in the search for better and better performance.

GA is characterized by parameters p_c (crossover probability) and p_m (mutation probability).

2.3 OPTIMIZATION PROCEDURE

The two layered optimization procedure is shown in Figure 2. Topology optimization, the inner layer, contains a Sequential Linear Programming optimizer and finite element analysis and calculations as evalutation. A tolerance for design variables are specified as terminating criteria. If the value of objective is getting worse and also all of the variables have smaller than 10% change from their previous value, optimization is terminated and the current design is returned as the optimal. In the outer layer, the genetic alogrithm optimization is conducted by comercial optimization software iSIGHT5.5. GA parameters are automatically generated and updated internally during the process. It is observed during several experiments from different initial deisgn that after 150 iterations, GA does not produce a significantly better design, but will oscillate within a small range. Due to the charasteristic a maximum iteration number of 200 is chosen to get the best result within a short calculation time.

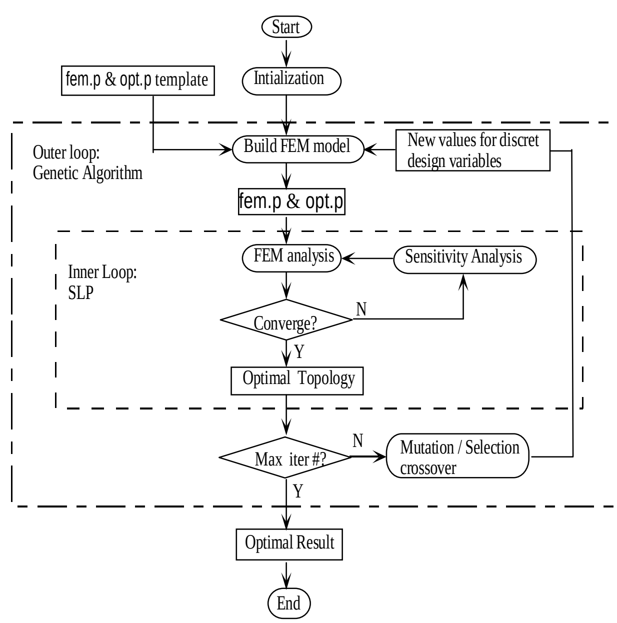


Figure 2: Flow char of the two- layered optimisation

The topology optimization part and discrete genetic algorithm optimization are connected through a program which generates topology optimization inputs (*fem.p* and *opt.p*) with the discrete variables generated by genetic algorithm optimization.

3 PROBLEM FORMULATION

The problem formulation is similar to that of the topology optimization of flextensional actuator as in[1]. In this work, for simplicity, the electrical part is fixed to be one-piece rectangular block align in horizontal direction with the dimensions and location being discrete design variables. The design problem and extended design domain is show in Figure 3.

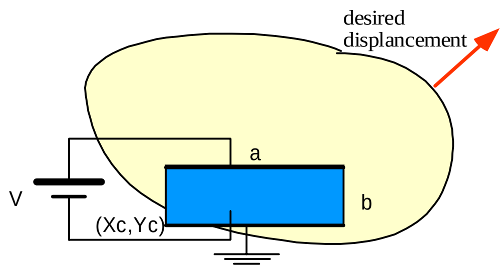


Figure 3: design problem

3.1 DESIGN VARIABLES

Continuous design variables are those of topology optimization:

$\alpha_i, \beta_i \in (0,1]$: Dimensions of the microscopic voids in homogenization design method;

$\theta_i \in [0,\pi]$: Orientation of the microscopic voids;

Discrete design variables are those of piezoelectric material block:

$a \in N, b \in N$: Dimensions of piezoelectric block;

$x_c \in N, y_c \in N$: Coordinates of the lower-left corner of piezoelectric block;

$V \in \{-1,1\}$: Direction of the applied voltage, which determines the displacement direction of the piezoelectric block for this giving polarization;

3.2 CONSTRAINTS

1.Side constraints of the microscopic voids:

$$0 \le \alpha_i \le \alpha_{\sup} < 1$$
$$0 \le \beta_i \le \beta_{\sup} < 1$$

where $\alpha_{\sup}$ and $\beta_{\sup}$ are upper limits for dimension of the voids. These prevent the existence of the zero stiffness, which may cause the ill posed stiffness matrix.

2.Volume constraint of the mechanical part:

$$\sum_{i=1}^{m}(1-\alpha_i\beta_i)\cdot V_e - V_{\sup} \le 0$$

Where V_e is the volume of a full element with no voids.

3. The total area of the piezoelectric block is assumed to satisfy:

$$a \cdot b \le A_0$$

4. The piezoelectric block must remain inside of the design domain:

$$x_l \le x_c \le x_u$$
$$y_l \le y_c \le y_u$$

where for different design domain, the boundary values are different. In some engineering case, they are defined by considering also the engineering feasibility.

5. Coupled equilibrium equations for three different loading cases:

$$\begin{bmatrix} \mathbf{K}_{uuu}^{H} & \mathbf{K}_{u\phi} \\ \mathbf{K}_{u\phi}{}^{T} & -\mathbf{K}_{\phi\phi} \end{bmatrix} \begin{bmatrix} \mathbf{U}^{(k)} \\ \mathbf{\Phi}^{(k)} \end{bmatrix} = \begin{bmatrix} \mathbf{F}^{(k)} \\ \mathbf{Q}^{(k)} \end{bmatrix}, k = 1,2,3$$

where (k) represents three different loading condition considered for objective function, shown is Figure 4.

$\mathbf{U}^{(k)}$ and $\mathbf{\Phi}^{(k)}$ are displacements (mechanical degrees of freedom) and voltage (electrical degree of freedom) respectively in k-th loading case. $\mathbf{K}_{uu}^{H}$ is the global stiffness matrix, calculated through homogenized elastic tensor.

$$\mathbf{c}^H = \frac{1}{|Y|}\int_Y \mathbf{c}(x,y)(\mathbf{I} - \boldsymbol{\varepsilon}(\chi))dY$$

where χ is the “characteristic deformation, and Y represents the unit cell microstructure. (Details refer to [4]).

Also, $\mathbf{K}_{\phi\phi}$ is the dielectric matrix and $\mathbf{K}_{u\phi}$ is the piezoelectric (coupled) matrix.

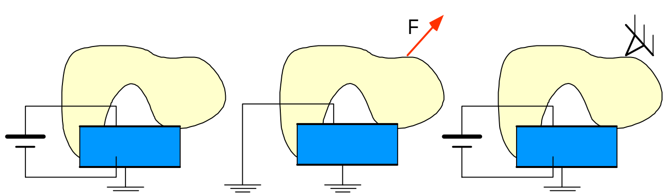


Figure 4: Loading cases for multi-objective function

The loading cases are as shown in Figure 4: 1) only voltage is applied at the piezoelectric material; 2) only a dummy load is applied at the point of desired displacement and 3) the voltage is fixed and dummy load is applied. Case (1) and (2) formulate the mutual mean compliance to meet the kinematics requirement and Case (3) formulate the mean compliance to meet the stiffness requirement.

3.3 OBJECTIVE FUNCTION

This mixed variable problem has the same objective function as the topology optimization problem [4]. For completion, we repeat the formulation of the multi-objective problem. Considering the three loading cases in Figure 4.

The objectives are:

1. Maximize the mutual mean compliance:

$$MMC = \begin{bmatrix} \mathbf{U}^{(1)} \\ \mathbf{\Phi}^{(1)} \end{bmatrix}^T \begin{bmatrix} \mathbf{K}^H_{uuu} & \mathbf{K}_{u\phi} \\ \mathbf{K}_{u\phi}{}^T & -\mathbf{K}_{\phi\phi} \end{bmatrix} \begin{bmatrix} \mathbf{U}^{(2)} \\ \mathbf{\Phi}^{(2)} \end{bmatrix}$$

which is the kinematics requirement or flexibility requirement.

2. Minimize the mean compliance:

$$MC = \begin{bmatrix} \mathbf{U}^{(3)} \\ \mathbf{\Phi}^{(3)} \end{bmatrix}^T \begin{bmatrix} \mathbf{K}^H_{uuu} & \mathbf{K}_{u\phi} \\ \mathbf{K}_{u\phi}{}^T & -\mathbf{K}_{\phi\phi} \end{bmatrix} \begin{bmatrix} \mathbf{U}^{(3)} \\ \mathbf{\Phi}^{(3)} \end{bmatrix}$$

which is the structural requirement or stiffness requirement.

The multi-objective function is formulated as:

$$\text{Maximize} \quad f = \frac{MMC}{MC}$$

4 EXAMPLES

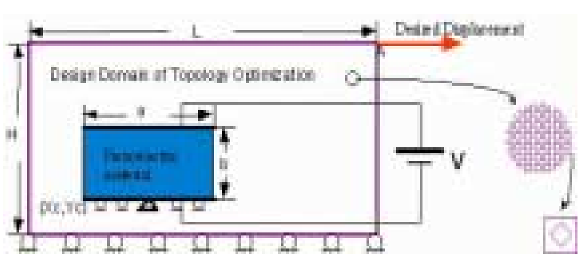


Figure 5: Design domain of the example

A specific flextensional actuator design problem is chosen as a follow-up to a previous design problem. The structure layout are chosen to be both the mechanical part and the electrical part inside of a specified design domain as illustrated in Figure 5, are optimized

Nishiwaki, et al[9] gave an example design problem by fixing the piezoelectric part at the central bottom of the design domain. The topology optimization result is shown in Figure 6 and values of inputs and outputs are shown in Table 1.

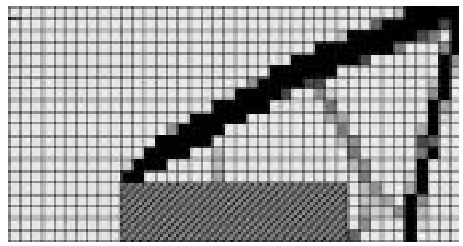


Figure 6: Previous result

In this two-layered optimization problem, the location and dimensions of piezoelectric part are optimized with genetic algorithm, while the mechanical part is optimized with previous topology method. A finite element method is used to calculate the objective function

The following parameters are used in this problem:

$$L = 40, H = 20;$$
$$A_0 = 100;$$
$$x_0 = 0, y_0 = 0;$$

However, it should be noticed that it is not necessary to assign units to the parameters and variables, since the topology optimization is applied on linear elastic fictitious material. Assigning units to the parameters and variables does not have any physical meaning.

	Optimal result	Previous result
A	22	20
B	2	5
x_c, y_c	11,8	5
V	-1 (-)	1 (+)
Objective	0.66	0.24
Displacement at desired point	0.16	0.13

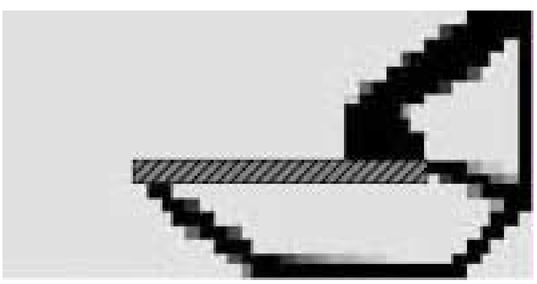
Table 1: Optimal result compared with the previous

Figure 7 shows the optimal configuration after 200 iterations. Table 1 compares the values of variables and objectives of the optimal design obtained in this research and the original optimal design from topology optimization only.

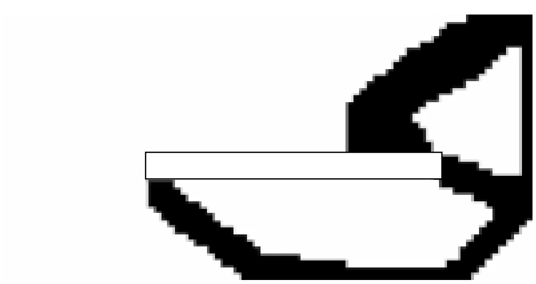
In the original design, the dimensions of the piezoelectric block is 20*5 and is located at (11,1), while in the optimized design, the size of the piezoelectric block is reduced to 22*2, and the location is moved up to (11,8). With the optimized design, the objective function value is increased by 175%, and the displacement is increased by 33%.

It is obvious the optimal result is an acceptable improvement to the previous result. Also, by allowing more freedom to the design, we can expect less stress concentration in the sense that difference of stresses at different point is smaller.

Genetic algorithm has shown its advantage of handling discrete design variables. If pure mutation is used to try different design, it needs a total of more than 576,000 topology optimization iterations, which take more than 400 days to finish. By genetic algorithm a feasible local optimal solution can be obtained within 2 days. If more computational time are allowed, a better global optimal can be possibly obtained.



(a)



(b)

Figure 7: Optimal configuration (a) material distribution (b) threshold result

To verify the result, an image processing technology is applied to the optimal material distribution to obtain a finite element model of a solid structure. Then the finite element analysis is conducted with ABAQUS. The structure is deformed in the desirable manner as shown in Figure 8.

Figure 8: Deformation verification from ABAQUS

5 CONCLUSIONS

In this research, optimal piezoelectric actuator design was achieved by giving more design freedom than the topology optimization. The two-layer optimization procedure has successfully collaborated two optimization techniques and provided consistently improved design. Genetic Algorithm has worked in an efficient manner.

The technique presented here can be also applied to three dimensional design optimization problems. The same methodology can be utilized in designs of other actuator and structures, which contains two or more different materials. For instance, the future design of thermal actuators, bi-material compliant mechanisms and MEMS can be possibly benefited from this research. Furthermore, other discrete design variables can be added, such as material types and actuation type, and other design objectives can be considered for the design of economic and environmental conscious mechanisms and devices.

Acknowledgment

The authors are grateful to Engineous Inc for generously providing the iSIGHT licence. Since this work is based on course project of ME558 (Discrete optimization methods) in Fall 2000 at the University of Michigan, the authors thank for this opportunity from College of Engineering, University of Michigan. Supports and concerns from Computational Mechanics Lab and Discrete Design Optimization Laboratory at University of Michigan are also appreciated.

References

[1] C. Near (1996) Piezoelectirc actuator technology. *Proceeding of SPIE* **2717** 246-258

[2] K. Onitsuka, A. Dogan, J. Tressler, Q. Xu, S. Yoshikawa and R. Nownham (1995) Metal-ceramic composite transducer, The Moone. *Journal of Intelligent Material Systems and Structures* **6** 447-455

[3] M. Bendsφe and N. Kikuchi (1988) Generating optimal topologies in structural design using a homogenization method. *Computer Methods in Applied Mechanics and Engineering* **71** 197-224

[4] O. Sigmund (1997) On the design of compliant mechanisms using topology optimization *Mechaincs of Structures and Machines* **25** 493-524

[5] S. Nishiwaki, M. Frecker, S. Min and N. Kikuchi (1998) Topology optimization of compliant mechanisms using homogenization method. *International Journal for Numerical Methods in Engineering.* **42** 535-559

[6] E. Silva , N. Kikuchi (1999) Design of piezoelectric transducer using topology optimization. *Smart Materials and Structures* **8** 350-364

[7] J. Jonsmann, O. Sigmund, S. Bouwstra(1999) Compliant electro-thermal microactuators. *Proceeding of the 1999 12th IEEE International Conference on MEMS*

[8] Y. Li, B. Chen and N. Kikuchi (1999) Topology optimization for mechanisms of thermal actuation with Eco-materials. *Proceeding of the 1999 4th International Conference on Eco-materials*

[9] S. Nishiwaki, E. Silva, Y. Li and N. Kikuchi (1998) Topology optimization for flextensional acutators *AIAA conference* St. Louise

[10] J. Main, E. Garcia and D. Howard (1994) Optimal placement and sizing of paried piezoactuators in beams and plates. *Smart Material and Structure* **3** 373-381

[11] Gabbert, U. and Weber, C-T (1999) Optimization of piezoelectric material distribution in smart structures. *Proceedings of SPIE- The International Society for Optical Engineering* **3667** 13-22

[12] D. Goldberg (1989) Genetic algorithms in search, optimization and machine learning Addison-Wesley Publishing