An Ant Colony Approach for The Steiner Tree Problem

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An instance of the *Steiner tree* problem consists of:

1. A graph $G(V, E)$, where V is a set of vertices (or nodes) and $E \subset V \times V$ is a set of edges.
2. A weight associated with each edge, where the weight is a mapping,

$$w : E \rightarrow \Re^{+}$$

3. A set of terminal nodes, $T \subseteq V$.

The problem is to compute a minimum Steiner tree, i.e. an acyclic subset $S(V_S, E_S)$ of G, with $T \subset V_S$, such that the vertices included in T are all mutually reachable, with the smallest possible cost function.,

$$w(S) = \sum_{e \in E_S} w(e). \qquad (1)$$

We propose an ant colony approach to compute minimal Steiner trees (Dorigo, Gambardella, 1997).

One ant is placed initially at each of the given terminal vertices that are to be connected. In each iteration, an ant is moved to a new location via an edge, determined stochastically, but biased in such a manner that the ants get drawn to the paths traced out by one another. Each ant maintains its own separate list of vertices already visited to avoid revisiting it. When any ant collides with another ant, or even with the path of another, it merges into the latter. An ant m, currently at a vertex i, selects a vertex j not in its tabu list $T^{(m)}$, to move to, only if $(i, j) \in E$. In order to ensure that the ants merge with one another as quickly as possible, we define a potential for each vertex j in V, with respect to an ant m as follows,

$$\psi_j^{(m)} = \min_k \{d(j,k)\}, \qquad (2a)$$

where,

$$k \in \bigcup_{m' \neq m} T^{(m')}. \qquad (2b)$$

Here, m' is any other ant, and $d(j,k)$ is the shortest distance from the two vertices, j and k. The potential of a vertex is, therefore, a measure of the minimum possible additional cost required to join it with any of the partially completed trees. Our algorithm tries to place ants with lower potentials, but it also considers the actual cost of moving an ant from its present location to the other vertices. We define the desirability of a vertex with respect to an ant m, currently in vertex i as,

$$\eta_j^{(m)} = \frac{1}{w(i, j) + \gamma \psi_j^{(m)}}. \qquad (3)$$

The quantity γ is a constant. The ant's position may be updated using the following equation,

$$p_{i,j} = \frac{[\tau_{i,j}]^{\alpha} [\eta_{i,j}^{(m)}]^{\beta}}{\sum_{k \notin T^{(m)}} [\tau_{i,k}]^{\alpha} [\eta_{i,k}^{(m)}]^{\beta}}. \qquad (4)$$

Here, τ_{ij} and p_{ij} are the trail intensity of edge (i, j) and the probability of an ant using that edge to move. Trail updating rules and parameters were borrowed from known work (Dorigo, Gambardella, 1997). The results obtained are shown in Table 1.

Table 1: Results from ten test runs

Graph Size			Tree Weight *w*(*S*)		
|V|	|E|	|T|	Avg	Best	*w**
50	100	13	61	61	61
50	63	9	82	82	82
50	63	25	138	138	138
75	150	19	89	88	88
100	125	25	235.3	235	235
100	200	50	225.5	224	218

REFERENCES

M. Dorigo, L. M. Gambardella, Ant Colony Systems: A Cooperative Learning Approach to the Traveling Salesman Problem, *IEEE Transactions on Evolutionary Computation*, 1(1): 53-66, 1997.