Evaluation of the Constraint Method-Based Multiobjective Evolutionary Algorithm (CMEA) for a Three-Objective Optimization Problem

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Abstract

This paper presents a systematic comparative study of CMEA (constraint method-based multiobjective evolutionary algorithm) with several other commonly reported mulitobjective evolutionary algorithms (MOEAs) in solving a three-objective optimization problem. The best estimate of the noninferior space was also obtained by solving this multiobjective (MO) problem using a binary linear programming procedure. Several quantitative metrics are used to compare the noninferior solutions with respect to relative accuracy, as well as spread and distribution of solutions in the noninferior space. Results based on multiple random trials of the MOEAs indicate that overall CMEA performs better than the other MOEAs for this three-objective problem.

1 Introduction

With the recent emergence of interest in solving realistic multiobjective (MO) problems, numerous multiobjective evolutionary algorithms (MOEAs) have been reported in the literature (Deb 2001). While most of them have been successfully tested and evaluated for an array of two-objective test problems, little work is reported on solving MO problems involving more than two objectives. Building upon the study reported by Zitzler et al. (2001) for a three-objective problem, this paper compares and contrasts the performance of the constraint method-based evolutionary algorithm (CMEA) (Ranjithan et al. 2001) with those of SPEA-II (Zitzler et al. 2001), NSGA-II (Deb et al. 2000), and PESA (Corne et al. 2000). These results are also compared with the noninferior set obtained using an MO analysis with a binary linear programming procedure. An array of quantitative metrics is used to conduct a systematic performance comparison among the solutions generated by these MOEAs. In addition to several existing metrics that are extended from the original definitions for two objectives, a new metric is defined to evaluate the relative degree of dominance of one set of noninferior solutions over another.

The next section provides a brief background on CMEA. The subsequent section describes the performance metrics used in this study. Section 4 defines the test problem and a comparison of the results, followed by conclusions.

2 Background

The ϵ-constraint method, typically employed with traditional mathematical programming methods, generates the noninferior set for multiple objectives through iterative solution of the following single objective problem:

$$\begin{aligned} &\text{Maximize } Z_h(\mathbf{x}) \\ &\text{Subject to } g_i(\mathbf{x}) \leq \forall i = 1, 2, ..., c \\ &\qquad\qquad Z_l(\mathbf{x}) \geq Z_l^t \forall l = 1, 2, ..., k; l \neq h \\ &\qquad\qquad \mathbf{x} \in X \end{aligned} \tag{1}$$

The problem is assumed to be a maximization problem without loss of generality. Z_h is one of k objectives, Z_l^t is the constraint value for objective $l (l \neq h)$, $\mathbf{x} = \{x_j : j = 1, 2, ..., n\}$ represents the decision vector, X represents the decision space, c is the total number of constraints, and $g_i(\mathbf{x})$ is the i^{th} constraint. The value of Z_l^t is varied incrementally, making the search migrate from one noninferior solution to another.

The evolutionary multiobjective optimization algorithm CMEA combines the ϵ-constraint method for MO within an evolutionary computation framework (Ranjithan et al. 2001). Pareto optimality is achieved in an implicit manner by ensuring the population to migrate along the noninferior surface. A noninferior solution is generated by converging the population to the optimal solution to the above model corresponding to a set of values for Z_l^t. The population is then migrated gradually by incrementally changing the values of Z_l^t. (Please see Ranjithan et al. (2001) for more details.)

A comparison of the results show that the CMEA performs equally or better than SPEA-II, NSGA-II, and PESA for a range of two-objective test problems (Ranjithan et al. 2001, chapter 5 in Kumar 2002). Although the results reported so far have focused on two-objective problems, the underlying concepts and procedures of CMEA are equally applicable to higher order MO problems.

3 Performance Metrics

A spread metric (*Spread*) that determines in each objective space the maximum range represented by the noninferior solutions, and a coverage metric that represents the distribution of the solutions along the noninferior surface were introduced by Ranjithan et al. (2001). Using Figure 1 for illustration, *Spread* in objective Z_1 is the horizontal distance between C_1 and C_q, the two extreme points generated by the MOEA. Similarly, *Spread* in objective Z_2 is the vertical distance between C_1 and C_q. A higher value of the *Spread* metric indicates a better performance.

Two different estimates, $V1$ and $V2$, are defined to characterize the coverage of the noninferior space by the nondominated solutions set (NDS_{MOEA}) generated by an MOEA. Using the notations in Figure 1, $V1$ is defined as $Max\{d_h, \forall h \in \{0, 1, ..., q\}\}$, where d is the distance between adjacent solutions. $V1$ calculations include the set NDS_{MOEA} and the extreme noninferior solutions A and B, which represent the optimum solutions determined separately for each objective. $V2$ is defined as $Max\{d_h, \forall h \in \{1, ..., q-1\}\}$, which includes only the set NDS_{MOEA}. As $V1$ and $V2$ represent the largest gap between adjacent noninferior solutions in the objective space, they characterize how well the solutions generated by an MOEA are distributed to cover the noninferior space. Smaller values for $V1$ and $V2$ indicate a better performance.

Zitzler and Thiele (1999) introduced the C factor that compares two noninferior sets. This parameter can be used to show how the noninferior set of one algorithm dominates the noninferior set of another.

In addition to these metrics, this paper introduces an alternative performance metric called the D (dominance) factor that represents the degree of dominance of noninferior solutions produced by one algorithm over another. For illustration, the two-objective case in Figure 2 shows the noninferior sets NDS_{MOEA-1} and NDS_{MOEA-2} generated by two different MOEAs. To calculate the dominance factor of MOEA-1 over MOEA-2, a distance measure between a solution from the set NDS_{MOEA-1} and the solutions it dominates in the set NDS_{MOEA-2} is determined. In this example, the distance measure for solution i from the set NDS_{MOEA-1} is defined as

Table 1: Summary of Metrics Used for Performance Comparisons in This Paper

Metric	Description	Reference
C factor	Measure of relative dominance of solutions generated by one algorithm over another	Zitzler and Thiele (1999)
$V1$	Measure of the coverage including the known best extreme points	Ranjithan et al. (2001)
$V2$	Measure of the coverage excluding the known best extreme points	Ranjithan et al. (2001)
Spread	Measure of the maximum range covered by the noninferior solutions	Ranjithan et al. (2001)
D factor	Measure of the degree of dominance of solutions generated by one algorithm over another	Figure 2

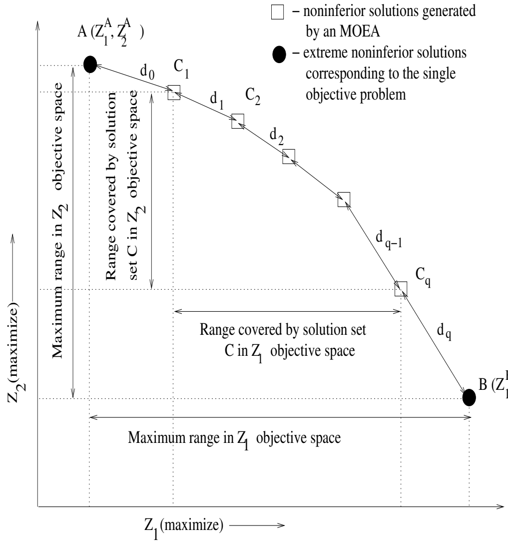

Figure 1: An Example of Two-objective Noninferior Tradeoff to Illustrate the Computation of metrics. d_i represents the distance between two adjacent solutions

di = Max $\{dij : j = 1, 2, ..., m\}$, where m is the number of solutions it dominates in the set NDS_{MOEA-2}. Then the following aggregate value $D_{1/2}$ is used to define the degree of dominance of MOEA-1 over MOEA-2.

$$D_{1/2} = \frac{\sum_{i=1}^{N} di}{N} \tag{2}$$

where N is the total number of solutions in the set NDS_{MOEA-1}. The corresponding value for $D_{2/1}$ can be computed similarly.

A summary of the performance metrics used in this paper for the comparison of different algorithms is shown in Table 1. These metrics, although described here for only a two-objective case, are extended for the higher dimensional MO problem presented in this paper.

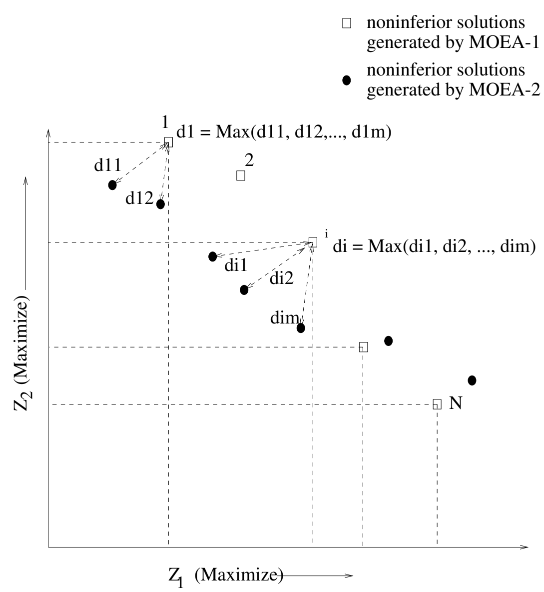

Figure 2: An Example of Two-objective Noninferior Tradeoff to Illustrate the Computation of D factor

4 Testing of CMEA for a Three-Objective Problem

The extended 0/1 multiobjective knapsack problem presented by Zitzler and Thiele (1999) is a constrained, binary, combinatorial search problem. This MO knapsack problem extends a single objective problem by incorporating a number of knapsacks that can be filled by items selected from a common collection of items. The goal is to choose the allocation of items in different knapsacks so that the payoff of each knapsack is maximized without violating the respective weight capacity constraint. This problem is defined mathematically as follows:

$$\begin{aligned} &\text{Maximize } Z_l(\mathbf{x}) = \sum_{j=1}^{n} p_{l,j} x_j \;\; \forall l = 1, 2, ..., k \\ &\text{Subject to } \sum_{j=1}^{n} w_{l,j} x_j \leq c_l \;\; \forall l = 1, 2, ..., k \\ &x_j \in \{0, 1\} \end{aligned} \tag{3}$$

where, $Z_l(\mathbf{x})$ is the total profit associated with the knapsack l, $p_{l,j}$ is the profit of placing item j in knapsack l, $w_{l,j}$ is the weight of item j when placed in knapsack l, c_l is the capacity of knapsack l, $\mathbf{x} = (x_1, x_2, ..., x_n) \in \{0, 1\}^n$ such that $x_j = 1$ if selected and $= 0$ otherwise, n is the total number of available items, and k is the total number of knapsacks.

In this paper, an instance of this multiobjective knapsack problem with three knapsacks ($k = 3$), each with 750 items ($n = 750$) is considered. In addition to solving this problem using CMEA, a binary linear programming solver (BLP), CPLEX®, was also used to generate a set of noninferior solutions by solving Model (1) iteratively. The problem was solved using CMEA (with a population of 100, binary tournament selection, uniform crossover, adaptive mutation, and 289 intervals in each objective) for 10 different random seeds, and the performance metrics listed in Table 1 are calculated based on these 10 runs. The results obtained using CMEA is compared with those obtained using SPEA-II, NSGA-II, and PESA. The performance metrics for SPEA-II, NSGA-II, and PESA are calculated based on 30 different sets of solutions reported by Zitzler et al. (2001).

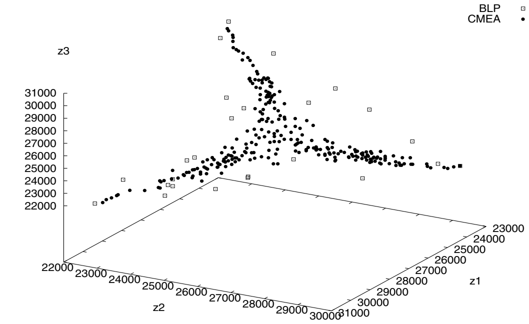

Figure 3: A Comparison of the Noninferior Sets Obtained Using CMEA and BLP

Figure 3 compares the noninferior solutions generated by CMEA and the BLP solutions. The CMEA solutions appear to represent most of the noninferior surface defined by the BLP solutions, which represent the best estimate of the noninferior front available for this problem. The extreme regions, associated with the three "tail-like" sections, are well represented by the CMEA solutions. Figures 4 to 6 compare the noninferior solutions generated by SPEA-II, NSGA-II, and PESA, respectively, with those generated by CMEA. The results from SPEA-II, NSGA-II, and PESA required approximately 576,000 function evaluations (Zitzler et al. 2001), and the CMEA solutions are based on approximately 594,000 function evaluations. While the solutions by SPEA-II, NSGA-II, and PESA solutions well represent the "center" region of the noninferior surface, they entirely miss the three tail regions.

The performance metrics listed in Table 1 were computed for the solutions generated by the MOEAs, and are compared in Figures 7 to 15. These graphs show for each metric the average and the range based on different random trials. The *Spread* metrics in the three objective

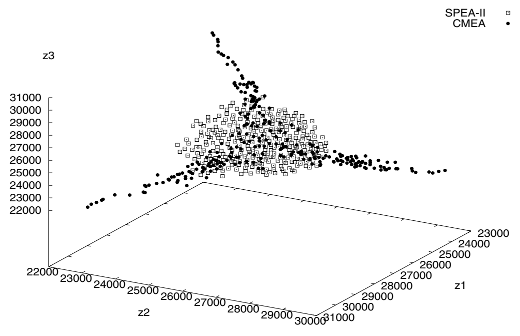

Figure 4: A Comparison of the Noninferior Sets Obtained Using CMEA and SPEA-II

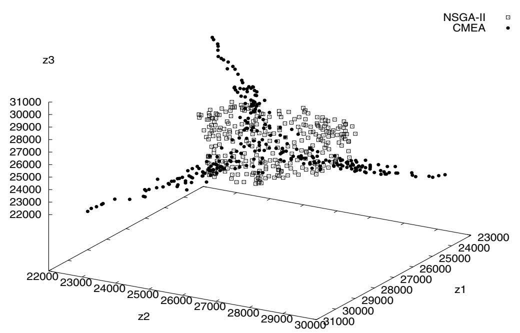

Figure 5: A Comparison of the Noninferior Sets Obtained Using CMEA and NSGA-II

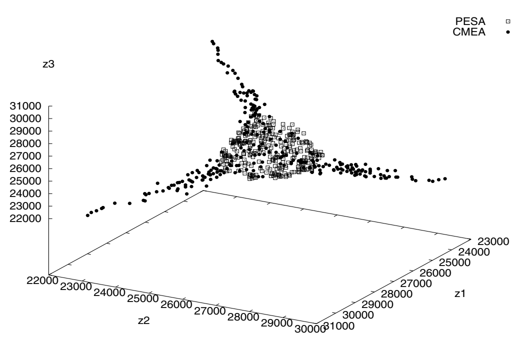

Figure 6: A Comparison of the Noninferior Sets Obtained Using CMEA and PESA

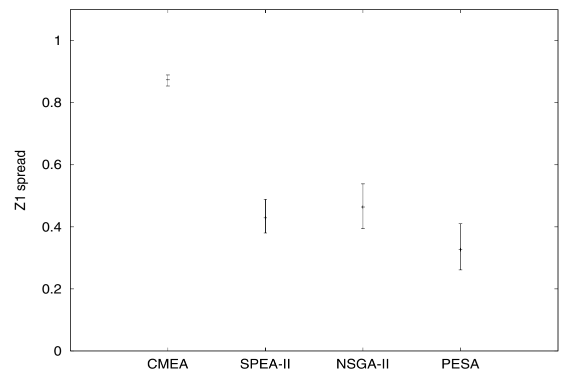

Figure 7: Z_1 spread comparison for CMEA, SPEA-II, NSGA-II, and PESA (a higher value indicates a better performance)

spaces are shown in Figures 7 to 9. These figures indicate that CMEA outperforms the other algorithms compared in this study. Figure 10 compares the coverage metric $V2$. While all MOEAs perform similarly, PESA shows a slight edge over the others. When comparing metric $V1$ in Figure 11, CMEA clearly outperforms the other MOEAs. This confirms the observations (from Figures 4 to 6) that SPEA-II, NSGA-II, and PESA solutions cover only the central region of the noninferior surface while the CMEA solutions are broadly distributed.

Comparisons of C factors are shown in Figures 12 and 13. This further confirms the conclusions drawn above about coverage based on the $V1$ and $V2$ metrics. Comparisons of D-factor are shown in Figures 14 and 15. By collectively interpreting the values of $D_{\text{CMEA/another MOEA}}$ and $D_{\text{another MOEA/CMEA}}$ for each MOEA compared, it can be concluded that CMEA solutions dominate those of the other MOEAs to a higher degree than *vice versa*. Alternatively, it implies that when CMEA solutions dominate solutions of another MOEA, they dominate them to a higher degree than when solutions of another MOEA dominate solutions of CMEA.

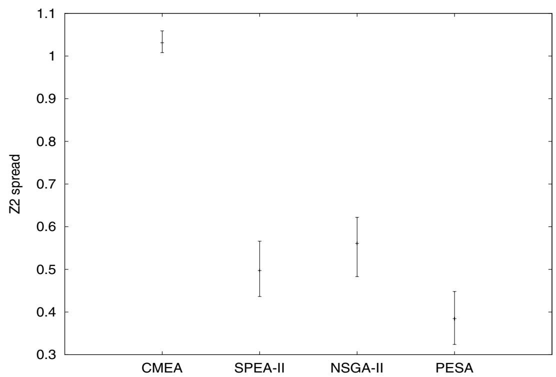

Figure 8: Z_2 spread comparison for CMEA, SPEA-II, NSGA-II, and PESA (a higher value indicates a better performance)

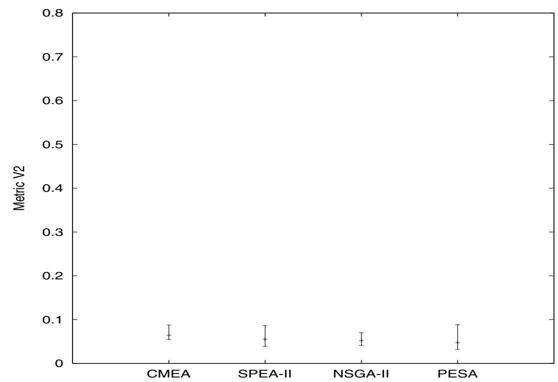

Figure 10: Coverage metric $V2$ comparison for CMEA, SPEA-II, NSGA-II, and PESA (a lower value indicates a better performance)

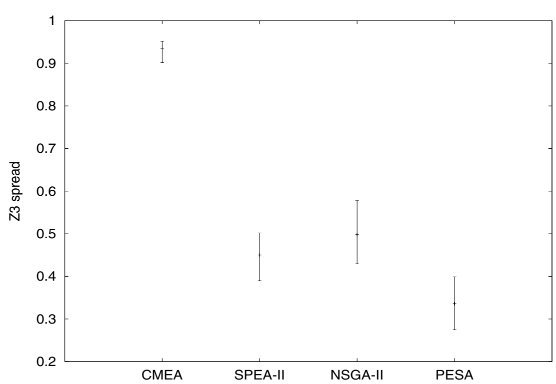

Figure 9: Z_3 spread comparison for CMEA, SPEA-II, NSGA-II, and PESA (a higher value indicates a better performance)

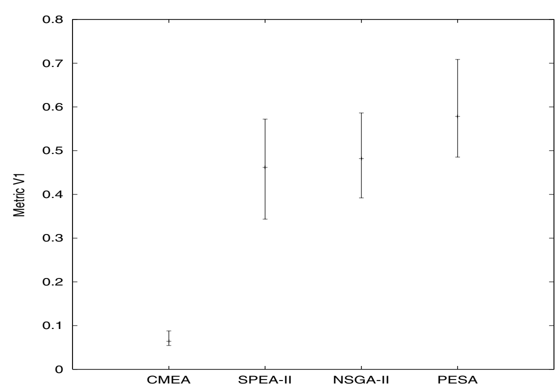

Figure 11: Coverage metric $V1$ comparison for CMEA, SPEA-II, NSGA-II, and PESA (a lower value indicates a better performance)

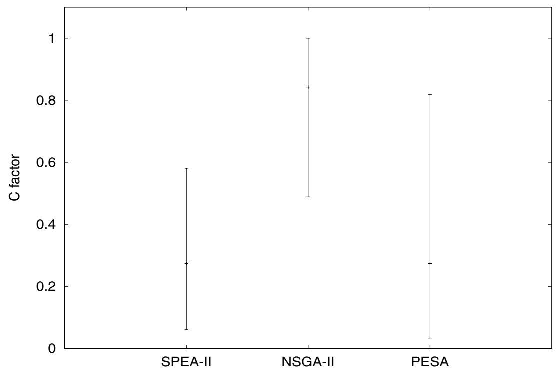

Figure 12: Comparison of C factor: CMEA solutions over SPEA-II, NSGA-II, and PESA solutions

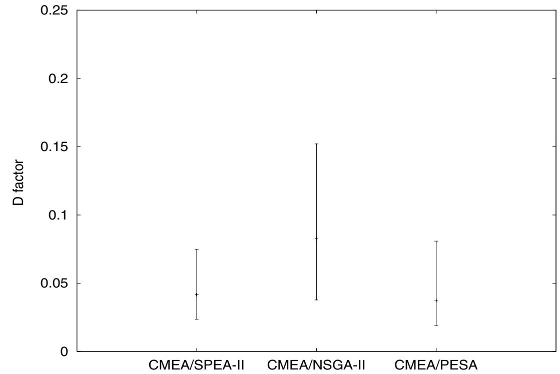

Figure 14: Comparison of D factor: CMEA solutions over SPEA-II, NSGA-II, and PESA solutions (a larger value indicates a higher degree of dominance of CMEA over another MOEA)

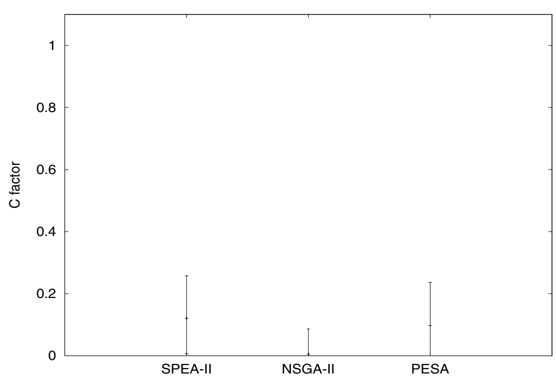

Figure 13: Comparison of C factor: SPEA-II, NSGA-II, and PESA solutions over CMEA solutions

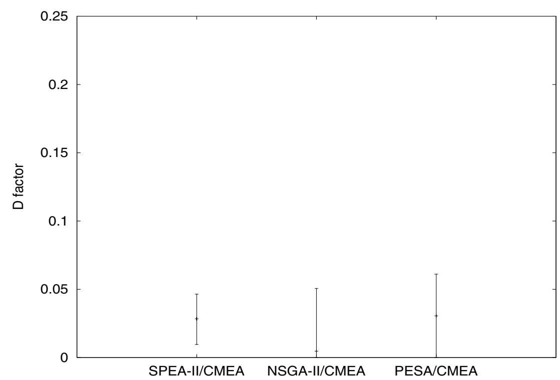

Figure 15: Comparison of D factor: SPEA-II, NSGA-II, and PESA solutions over CMEA solutions (a larger value indicates a higher degree of dominance of another MOEA over CMEA)

5 Conclusions

Comparing the noninferior solutions obtained using SPEA-II, NSGA-II, and PESA with those obtained using BLP (which is the best available estimate of the noninferior surface), CMEA performs relatively well in finding noninferior solutions that are close to the best available estimation, as well as in covering most of the noninferior surface for the thee-objective extended knapsack problem. When comparing the solutions obtained by the different MOEAs tested in this study, CMEA performs better than all others with respect to the spread of solutions in the noninferior space. While CMEA solutions are able to cover a broader portion of the noninferior surface, the other MOEAs generate a high density of solutions in the central portion of the noninferior surface. In the context of accuracy or degree of dominance, the CMEA solutions dominate those generated by the other MOEAs relatively more frequently.

While this study provides a systematic comparison of several MOEAs for only one three-objective optimization problem, further testing and evaluation studies are needed. Similar to the large array of test problems used in two-objective MO optimization, additional three-objective test problems reflecting different problem complexities need to be defined and be used in further comparative studies of these MOEAs. Also, the scale-up implications of higher number of objectives on the computational needs of the different MOEAs need to be investigated.

6 Acknowledgments

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