A Hybrid Genetic Algorithm for Optimal Hexagonal Tortoise Problem

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Hexagonal tortoise problem, which was first introduced by an old Korean mathematician Suk-Jung Choi (1646-1715) [1], is one of the most complicated tortoise problems. Since there has been no general heuristic to solve the hexagonal tortoise, an exhaustive search has been considered to be the only way for solving the problem to the optimum [4]. Recently a number of heuristics for the problem have been proposed [2], but they are too specific to be used for finding diverse instances.

The problem is to assign the consecutive numbers 1 through n to the vertices in a graph, which is composed of a number of overlapping hexagons, so that the sum of the numbers on each hexagon is the same. Figure 1 shows an optimal solution to the hexagonal tortoise problem with 30 numbers. The term "tortoise" comes from the fact that the overall shape of the graph resembles the *theca* of a turtle. We will take an optimization version of the problem that minimizes the difference among the hexagonal sums.

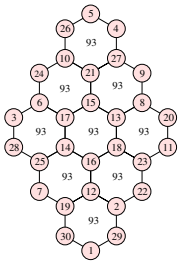

Figure 1: An optimal solution to the hexagonal tortoise with 30 numbers

We used a steady-state GA and encoded a solution as a matrix using hexagon-to-rectangle transformation

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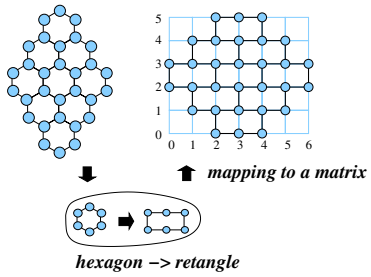

Figure 2: An example transformation

as shown in Figure 2. The fitness of a chromosome was given by the standard deviation of the hexagonal sums. The GA used the two-dimensional geographic crossover [3] and adopted an iterative improvement algorithm.

We attacked five problem instances with 30, 48, 70, 96, and 120 vertices, respectively. We denote by IGA the proposed GA with the iterative improvement. IGA found the optimal solutions up to the 70-number problems. Multi-Start performed much better than pure GA but was not comparable to IGA. The iterative improvement is a critical engine of both Multi-Start and IGA; but the performance was poor as its own. Pure GA showed the worst results among them.

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