Well-Separated Switching Instances for Approximating the Optimal PWM using Quantum-Inspired Evolutionary Algorithm

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Abstract. The selected harmonic elimination (SHE) pulse-width modulation (PWM) aims to select the switching instances (angles) in such a way that a waveform with a particular characteristic is obtained and a certain criterion is minimized. The algorithms in the literature so far do not give consideration for separation of the consecutive switching angles computed from the equations, which is however a very important issue for practical applications.
In this paper, a new algorithm is proposed to solve this nonlinear problem under the constraint that any two consecutive solutions are well separated from each other. The algorithm first transforms the nonlinear equations to a polynomial problem, then uses the Quantum-inspired Evolutionary Algorithm (QEA) to find the roots with the necessary amount of separation. Other than the standard QEA, our QEA favors localized search which is more suitable in our case. Since the obtained switching angles are reasonably distant from each other, they can be directly applied for inverters to alter directions without need to manually adjust the angles as with other methods. Essentially, our method computes the best possible tradeoff between the maximum error for system performance and the minimum distance between consecutive switching angles. The polynomial is ill-conditioned and our algorithm is robust.
Keywords: Quantum-inspired evolutionary algorithm, Well-separated switching angles, Harmonic elimination, Pulse-width modulation.

1 Introduction

The *pulse-width modulation* (PWM) technique can effectively reduce the harmonics content of inverter output waveform and possesses evident merits in improving frequency, efficiency, dynamic response speed [9]. Therefore, PWM has extensive applications and many related techniques such as [1, 2] have been proposed. The *Selected Harmonics Elimination* PWM (SHE-PWM) [7] is one of the optimal PWM techniques. It can generate the output waveform of higher quality through eliminating specific lower order harmonics. The basic idea is to set up the notches at the specially designated sites of PWM waveform and then the inverter alters directions many times per half-cycle to control the inverter's output waveform appropriately. Suppose we use two switching instances (angles) to denote every notch. Then the switching instances can be determined through solving a set of transcendental equations. There are a lot of solutions in the literature so far, however, one of the main shortcomings in them is that the obtained angles can be clustered since the equations are highly ill-conditioned. However in real application it is not always possible for the switch to alter directions without enough time delay due to constraints of the available hardware. Even when it is possible, doing so will increase the switching loss and the switching damage probability of the inverter bridges, so will impair the inverter and the whole equipment. Therefore, we have to manually adjust the solution, however, once again due to ill-conditioned property of the equations, even very small perturbation to the solution can lead to significant changes for the evaluation of equations, and thus harm the reliability of the whole system!

In this paper, a new algorithm for solving the SHE-PWM problem under the constraints of well separation between switching angles is proposed. This new numerical process can be divided to three steps: transform the original transcendental equations to a polynomial equation, then transform it into a constraint optimization problem and use a Quantum-inspired Evolutionary Algorithm (QEA), which favors localized search by our implementation, to solve the optimization problem. Our algorithm generates the best possible solution which satisfies the requirement for the minimum tolerable distance between consecutive switching angles and is scalable for a variety of application environments.

2 Numerical Transformation of the SHE-PWM problem

A periodical PWM waveform with n notches per half-cycle can be represented using Fourier series expansion as $f(t) = \sum_{n=1}^{\infty}[a_n \sin(n\omega t) + b_n \cos(n\omega t)]$ where $\omega = 2\pi/T$. Owing to the property of odd quarter-wave symmetry, the coefficients of Fourier series are $b_n = 0$ for all n and $a_n = \frac{4}{n\pi}\sum_{k=1}^{n}(-1)^{k-1}\cos(n\alpha_k)$ for odd n and 0 for even n, where $0 < \alpha_1 < \alpha_2 < \ldots \alpha_n < \pi/2$. Note that a_n is the amplitude of an n-th harmonic component of the waveform f. To set selected harmonics of a full-bridge PWM inverter output voltage to desired values, we need to solve the following set of nonlinear equations:

$$\sum_{i=1}^{n}(-1)^{i-1}\cos(k\alpha_i) = h_k \tag{1}$$

where $h_k = k\pi\alpha_k/4E$, $k = 1, 3, 5, \ldots$ and E is the inverter DC bus voltage. Applying the result in [5], we can transform the above nonlinear equation system to a monic real-coefficient algebraic monic polynomial problem $p(x)$ of which all the roots are real. Precisely, $p(x)$ can be written as a Toeplitz system

$$\begin{bmatrix} g_n, & \cdots, g_1 \\ & \cdots \\ & \cdots \\ g_{2n-1}, & \cdots, g_n \end{bmatrix} \begin{bmatrix} (-1)p_1 \\ \cdots \\ \cdots \\ (-1)^n p_n \end{bmatrix} = - \begin{bmatrix} g_{n+1} \\ \cdots \\ \cdots \\ g_{2n} \end{bmatrix} \tag{2}$$

where p_i is the i-th order coefficient in polynomial $p(x)$. From it, we can obtain p_i using Toeplitz Matrix algorithm. We are ready to describe the algorithm for solving $p(x)$. Recall that we need the obtained roots x_i (actually the corresponding angles

$\arccos x_i$) well separated from each other, we can therefore cast the problem for computing roots of a polynomial into an optimization problem as follows:

$$\min(\sum_{i=1}^{n} |\sum_{j=0}^{n} p_j x_i^j| + \rho \exp^{\frac{\sum_{k=1}^{n-1} \Phi(k)}{\sigma}}) \quad (3)$$

$$\Phi(k) = \begin{cases} |\arccos x_k - \arccos x_{k+1} - \theta| & : \ |\arccos x_k - \arccos x_{k+1}| < \theta \\ 0 & : \ otherwise \end{cases}$$

where x_i is the i-th root of the monic $p(x)$ and $0 < x_1 < x_2 < x_3 < ... < x_n < 1$, and ρ and σ are two parameters for the penalty function. θ is a pre-specified minimum tolerable gap between consecutive angles according to the hardware. A reasonable value for θ is $\frac{\pi}{4n}$ since the average gap for consecutive angles is $\frac{\pi}{2n}$. Note that setting ρ to 0 will compute the exact solution to $p(x)$. Due to the properties of $p(x)$, usually the better the approximation to the roots, the smaller the minimum distance between consecutive roots. Therefore, our optimization problem is essentially to compute the best possible tradeoff between the maximum error in the system and the minimum distance between switching angles.

3 Solving the SHE-PWM by Localized Quantum-Inspired Evolutionary Algorithm

The *Quantum-inspired Evolutionary Algorithm* (QEA) [3] is a relatively new evolutionary computing algorithm, which is characterized by the principles of quantum computing including concepts of qubits and superposition of states. QEA can imitate parallel computation in classical computers. The recent results on QEA include [4, 8, 6]. QEA is chosen in our case since it can treat the balance between exploration and exploitation more easily compared with the conventional GAs. In addition, QEA can efficiently conduct search for the global optimum in the search space with only a small number of individuals.

As the smallest unit of information, a *qubit* is a quantum system whose states lies in a two dimensional Hilbert space. Note that a qubit can be in "1" state, "0" state or simultaneously in the both (*superposition*). The state of a qubit can be represented as $|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle$ where α and β specify the probability of the corresponding states, and $|\alpha|^2 + |\beta|^2 = 1$. A *quantum gate* is a unitary transformation that acts on a fixed number of qubits to change their states. Inspired by the quantum computing, QEA uses the Q-bit representation for the probabilistic representation. An m Q-bits representation is defined as $q = [\kappa_1|\kappa_2|\dots|\kappa_m]$ where $\kappa_i = (\alpha_i, \beta_i)^T$ and $|\alpha_i|^2 + |\beta_i|^2 = 1$.

Recall that we expect the resulting roots well-separated from each other, we therefore would also expect that each root should be more or less close to the value of $i/n, i = 1, 2, \dots, n$, respectively. Suppose an initial solution consists of $i/n, i = 1, 2, \dots, n$, we then need an algorithm which favors localized search for each root instead of the one that will perturb the root too much. Motivated by localized search, the main structure and operations of our QEA are as follows.

Other than the standard QEA, we keep in the population at generation t both the Q-bit individuals, $Q(t) = \{q_1^t, q_2^t, \dots, q_n^t\}$ and the corresponding solutions, $S(t) =$

$\{s_1^t, s_2^t, \ldots, s_n^t\}$, where n is the size of the population, and q_i^t is a Q-bit individual of length m: $q_j = [\kappa_{j1}^t|\kappa_{j2}^t|\ldots|\kappa_{jm}^t]$. Each Q-bit individual $q_j^t, j = 1, 2, \ldots, n$ corresponds to a solution $s_j^t, j = 1, 2, \ldots, n$ in the population, and is used for deciding whether to add or subtract a small (random) amount to s_j^t to form the new solution $s_j'^t$. By comparing s_j^t and $s_j'^t$ as well as their fitness values, we then appropriately update q_j^t (see below). After all solutions generate descendants, the best n out of the $2n$ solutions ($\{s_j^t\} \bigcup \{s_j'^t\}$) are selected to form the set of solutions $S(t+1) = \{s_j^{t+1}\}$ for the new population. Note that we also need to select those q_j^t, which corresponds to s_j^{t+1}, to form $Q(t+1) = \{q_j^{t+1}\}$. If both s_j^t and $s_j'^t$ are selected, then two q_j^t will exist in the new population. If none of s_j^t and $s_j'^t$ gets selected, nor does q_j^t. The details are elaborated below.

(1) In the initialization step, all α_i^0 and $\beta_i^0, i = 1, 2, \ldots, m$, of all $q_j^0, j = 1, 2, \ldots, n$, are initialized to $1/\sqrt{2}$, such that each $q_j^0, j = 1, 2, \ldots, n$ can represent the linear superposition of all 2^m states, namely $|\Psi_{q_j^0}\rangle = \frac{1}{\sqrt{2^m}}(|00\ldots0\rangle + |00\ldots1\rangle + |11\ldots1\rangle)$. By the above discussion, we initialize each solution as the set of values (in sorted order) uniformly distributed between $[0, 1]$.

(2) Generate new solutions $s_j'^t$ from s_j^t as described above. Note that we restrict α, β to be positive, to avoid the ambiguity.

(3) Update q_j^t. We apply a quantum gate, i.e., a unitary transformation $U(\Delta\theta)$ to obtain new q_j^t, namely $\kappa_{ji}^t = U(\Delta\theta)\kappa_{ji}^t$ for each κ_{ji}^t in q_j^t. The $U(\Delta\theta)$ given in [3] is

$$U(\Delta\theta) = \begin{bmatrix} \cos(\Delta\theta) & -\sin(\Delta\theta) \\ \sin(\Delta\theta) & \cos(\Delta\theta) \end{bmatrix} \tag{4}$$

where $\Delta\theta$ is defined in our case as $\mathrm{sign}((f(s_j'^t) - f(s_j^t))(s_j'^t - s_j^t))\pi/50$ and $f(\cdot)$ is the fitness function. The sign of $\Delta\theta$ is not hard to decide, e.g., if $s_j'^t$ is fitter than s_j^t and $s_{ji}'^t < s_{ji}^t$, we should try to increase the possibility for decreasing s_{ji}^t. Recall that we use $|\alpha_{ji}^t|^2$ (precisely, the predicate: $\mathrm{random}[0, 1) > |\alpha_{ji}^t|^2$) to decide whether to add a small amount to s_{ji}^t, therefore, we should increase α_{ji}^t and $\Delta\theta$ is negative.

(4) The best n out of $2n$ solutions and their corresponding Q-bit individuals are selected to form the next generation.

(5) Repeat (2)-(4) until certain condition is met.

Since the solutions are only perturbed by very small values for obtaining the new solutions each time, our QEA exhibits good localized search ability. Each Q-bit individual starts from the the same value, and is successively updated to favor the specific direction for improving the solution vectors. Eventually when QEA converges to the optimum, Q-bit individuals will return to the same values, namely $1/\sqrt{2}$. Since the polynomial $p(x)$ and thus Problem (3) are highly ill-conditioned (see below), the amount for addition and subtraction in step (2) is set to be progressively smaller once per 200 iterations, which is especially important when QEA is converging to the optimum.

4 Experiments

We consider the case where the number of switching angles is $n = 20$, and lower order harmonics are set to $h_1 = 0.6$ and all other h are set to 0. We first compute and sort the switching angles corresponding to the obtained roots of the monic (i.e., $\arccos x_i$ for each root x_i). The switching angles for four criteria: the exact angles (X.A.), well-separated angles (W.A.) by QEA , randomly perturbed angles (R.P.) from the exact angles, and manually perturbed angles (M.P.) from the exact angles, are shown in Table 1. We then compute the maximum error and average error for evaluation of the above angles to the equation (1). As the key motivation for this paper, the minimum distances between consecutive switching angles are also computed. These results are summarized in Table 2. The spectra of the resulting waveforms for X.A., W.A., R.P. and M.P. are shown in Figure 1.

It is necessary to make a few notes for these results. The values for R.P. are generated by perturbing the corresponding values for X.A. with very small random values in $[-0.01, 0.01]$, however, these lead to the significant error to the system: the maximum error is 142% as indicated by Table 2. The equation is clearly ill-conditioned. We observe from the values for X.A. that the minimum gap between consecutive switching angles occurs between the first two angles, therefore, we adjust them (generating values for M.P.) by simply deducting roughly 0.005 from the first one, and add the same amount to the second one, which extends the gap to 0.025 (other adjustments act similarly). We found that the evaluation of the (n-1)th equation in (1) gives 0.45 rather than the desired 0, which greatly harms the system's reliability! Indeed, it is almost impossible to adjust the switching angles from our experience to obtain good results since equations are highly ill-conditioned.

We next apply our QEA (generating values for W.A.) to find the best possible results also with the minimum distance 0.025, and the resulting maximum error is 6% compared with 45% by M.P, and the average error is 6% compared with 14% by M.P.. The resulting spectrum of W.A. is much better than the spectrum of M.P. (refer to Figure 1). In addition, we clearly see from the items for X.A. and W.A. in Table 2 that our method computes a good tradeoff between the maximum error and the minimum distance. By adjusting θ in (3), our method is scalable for a variety of application requirements.

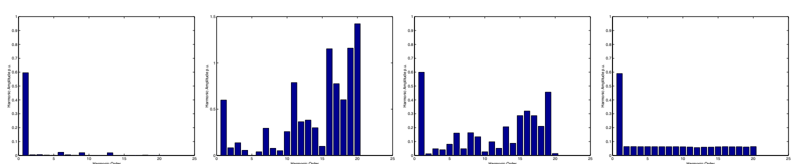

Fig. 1. The output spectrum from left to right is for X.A., R.P., M.P. and W.A.

5 Conclusions

A new robust algorithm is proposed to solve the optimal PWM problem under the constraint that any two consecutive solutions are well separated from each other. The algorithm first transforms the nonlinear equation to a polynomial equation, then uses the quantum-inspired evolutionary algorithm, which favors localized search by our implementation, to find the roots with certain criterion for separation. The most salient feature of the new method is that the obtained switching angles are reasonably distant from each other, and thus the solution can be directly applied to set amplitudes of several tens of harmonics without need to manually adjust the solution as with other methods. The experiments indicate the soundness of the method.

Table 1. Resulting angles (truncated at 10^{-4}) for the criteria of Exact Angle (X.A.), Well-separated Angle (W.A.), Randomly Perturbed angle (R.P.) and Manually Perturbed angle (M.P.)

A. No.	1	2	3	4	5	6	7	8	9	10
X.A.	0.1381	0.1544	0.2766	0.3088	0.4157	0.4634	0.5558	0.6182	0.6971	0.7732
W.A.	0.1287	0.1534	0.2681	0.3057	0.4083	0.4618	0.5507	0.6165	0.6928	0.7723
R.P.	0.1422	0.1510	0.2697	0.3010	0.4095	0.4639	0.5639	0.6243	0.6876	0.7705
M.P.	0.1335	0.1590	0.2766	0.3088	0.4157	0.4634	0.5558	0.6182	0.6971	0.7732
A. No.	11	12	13	14	15	16	17	18	19	20
X.A.	0.8401	0.9285	0.9852	1.0802	1.1321	1.2404	1.2809	1.3960	1.4340	1.5521
W.A.	0.8378	0.9277	0.9832	1.0838	1.1320	1.2399	1.2819	1.3962	1.4351	1.5522
R.P.	0.8310	0.9287	0.9758	1.0779	1.1338	1.2353	1.2729	1.3891	1.4407	1.5485
M.P.	0.8401	0.9285	0.9852	1.0802	1.1321	1.2404	1.2809	1.3960	1.4340	1.5521

Table 2. The Maximum Error, Average Error and Minimum Distance between consecutive angles for the values in Table 1. The error ratio is calculated with the requirement $0 < h < 1$.

Criterion	Exact Angle	Randomly Perturbed	Well-separated Angle	Manually Perturbed
Maximum Error	2%	142%	6%	45%
Average Error	0.5%	40%	6%	14%
Minimum Dist.	0.016	0.009	0.025	0.025

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