A New Approach for Shortest Path Routing Problem by Random Key-based GA

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ABSTRACT

In this paper, we propose a Genetic Algorithm (GA) approach using a new paths growth procedure by the random key-based encoding for solving Shortest Path Routing (SPR) problem. And we also develop a combined algorithm by arithmetical crossover, swap mutation, and immigration operator as genetic operators. Numerical analysis for various scales of SPR problems shows the proposed random key-based genetic algorithm (rkGA) approach has a higher search capability that enhanced rate of reaching optimal solutions and improve computation time than other GA approaches using different genetic representation methods.

Categories and Subject Descriptors

C.2.1 [Network Architecture and Design]

General Terms

Algorithms, Performance, Design

Keywords

Random Key-based Genetic Algorithm, Shortest Path Routing

1. INTRODUCTION

Shortest Path Routing (SPR) problem is the heart in network design problems. As the simplest network models, they capture many of the most salient core ingredients of network design problems [1][2].

The underlying topology of networks can be specified by the directed graph $G = (N, A)$, where a set N of n nodes and a set A of m directed arcs. Each arc $(i, j) \in A$ has an associated cost c_{ij}. The decision variable x_{ij} represents the link on an arc $(i, j) \in A$. The SPR problem is the minimum cost z from a specified source node s (or 1) to another specified sink node t (or n) and is formulated a shortest path model as the following integer programming formulation:

$$\textbf{min} \quad z = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij} \tag{1}$$

$$\textbf{s. t.} \quad \sum_{j=1}^{n} x_{ij} - \sum_{k=1}^{n} x_{ki} = \begin{cases} 1 & (i = 1) \\ 0 & (i = 2,3,\cdots,n-1) \\ -1 & (i = n) \end{cases} \tag{2}$$

$$x_{ij} = 0 \text{ or } 1 \quad (i, j = 1,\ 2,\ \cdots,\ n) \tag{3}$$

In this paper, we consider a SPR problem with a negative cycle that is an NP-complete problem. Observe that for any network containing a negative cycle W, the linear programming formulation has an unbounded solution because we can send an infinite amount of flow along W.

Ahn *et al.* (2002) [3] propose a GA for solving the SPR problem. Variable-length chromosomes have been employed. The crossover exchanges partial chromosomes (partial-routes) and the mutation introduces new partial chromosomes (partial-routes). Lack of positional dependency in respect of crossing sites helps maintain diversity of the population. Inagaki *et al.* (1999) [4] proposed an algorithm that employs fixed (deterministic) length chromosomes. Gen *et al.* (1997) [1] proposed *a priority-based encoding method* and recently Lin *et al.* [5] improved priority-based approach and proposed a new crossover operator, *weight mapping crossover* for solving the SPR problem. However, the nature of the priority-based encoding is a kind of permutation representation. Generally, this representation will yield illegal offspring by one-cut point crossover or other simple crossover operators in the sense of that some node's priority may be duplicated in the offspring.

2. RANDOM KEY-BASED GA

Random key encoding is a powerful method to represent permutations, particularly, because there is no infeasibility problem to deal with - traditional crossover operators produces only feasible offspring. Moreover, relative and absolute ordering information can be preserved after recombination. An example of generated chromosome and its decoded path is shown in Fig. 2, for the undirected network shown in Fig. 1. At the beginning, we try to find a node for the position next to source node 1. Nodes 2, 3 and 4 are eligible for the position, which can be easily fixed according to adjacent relation among nodes. The priorities of them are 1.24, 6.68 and 4.74, respectively. The node 3 has the highest priority and is put into the path. The possible nodes next to node 3 are nodes 4 and 6. Because node 4 has the largest priority value, it is put into the path. Then we form the set of nodes available for next position and select the one with the highest priority among them. Repeat these steps until we obtain a complete path, (1-3-4-7).

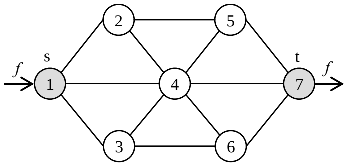

Fig. 1. A simple undirected network with 7 nodes and 12 edges

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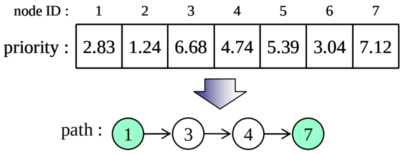

Fig. 2. Example of generated chromosome and its decoded path

1) Arithmetical Crossover: The basic concept of this kind of operator is borrowed from the convex set theory [1]. Generally, the weighted average of two vectors $v_j(i)$ and $v_k(i)$ of *j*th chromosome and *k*th chromosome, is calculated as follows:

$$v_j'(i) = \lambda_1 v_j(i) + \lambda_2 v_k(i), \quad \forall i$$

$$v_k'(i) = \lambda_1 v_k(i) + \lambda_2 v_j(i), \quad \forall i$$

where

$$\lambda_1 + \lambda_2 = 1, \quad \lambda_1 > 0 \text{ and } \lambda_2 > 0$$

2) Swap Mutation: selects two positions at random and then swaps the gene on these positions. In this paper, we adopt swap mutation for generating various offspring.

3) Immigration: The algorithm is modified to (1) include immigration routine, in each generation, (2) generate and (3) evaluate $popSize{\cdot}\mu$ random members, and (4) replace the $popSize{\cdot}\mu$ worst members of the population with the $popSize{\cdot}\mu$ random members (μ, called the immigration probability) [6].

The overall structure of proposed Random key-based Genetic Algorithm approach is given as follows:

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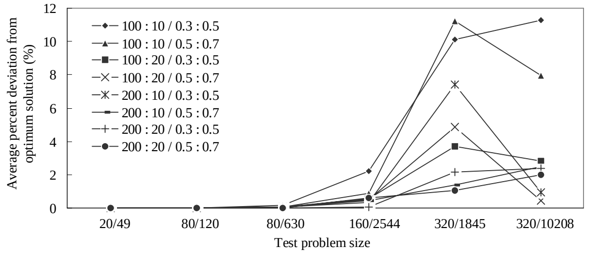
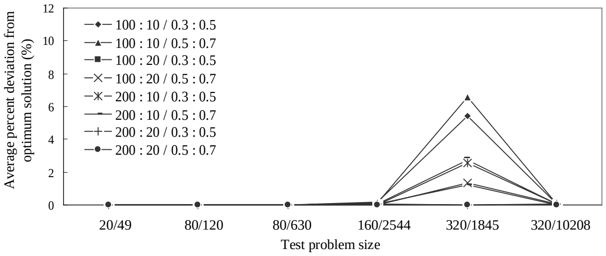
procedure: Random key-based GA for SPR Problem
input: network data ($A, V, \mathbf{c}$), GA parameters
output: best shortest path
begin
    $t \leftarrow 0$;
    initialize $P(t)$ by random key-based encoding;
    fitness $eval(P)$ by random key-based decoding;
    while (not termination condition) do
        crossover $P(t)$ to yield $C(t)$ by arithmetical crossover;
        mutation $P(t)$ to yield $C(t)$ by swap mutation;
        immigration to yield $C(t)$
        fitness $eval(C)$ by random key-based decoding;
        select $P(t+1)$ from $P(t)$ and $C(t)$
                by Roulette wheel selection;
        $t \leftarrow t + 1$;
    end
    output best shortest path;
end
```

3. EXPERIMENTS AND DISCUSSION

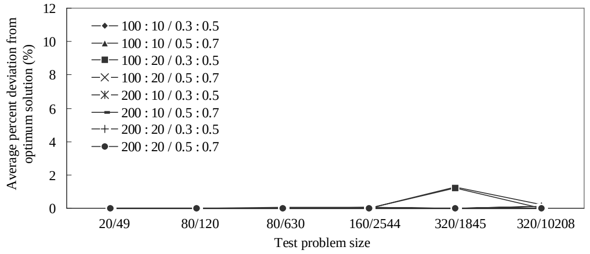
we show comparisons for the proposed approach based on random key-based encoding with Ahn *et al.'s* approach based on variable-length encoding method [3] and Gen and Lin's approach [5] based on priority-based GA on 6 shortest path problems [7]. Each test problem was run by 20 times for each GA approach. In addition, each test problem is divided into several numerical experiments to investigate the effects with different GA parameter setting and maximum generation *maxGen*=1000; Immigration rate, μ = 0.15 was employed.



(a) Ahn *et al.*'s GA



(b) Lin *et al.*'s priGA



(c) proposed rkGA

Fig. 3. Comparisons of percent deviation from optimum solution for combined different parameters setting.

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