# Dominance Hierarchies and Social Diversity in Multi-Agent Systems

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## ABSTRACT

In this study, we investigate self-organizing social hierarchies in multi-agent systems. Agents occupy the nodes of a small-world network and interact exclusively with other agents in their local neighbourhood. Here, the interactions represent competition for a limited resource. Monte-Carlo simulations show that the changes in a network's structure can alter the steady-state attributes for fixed reward/penalty mechanisms. The results suggest that the expected phase transition from a homogeneous to a hierarchical society depends on: (a) the relative strengths of the feedback mechanisms employed, (b) the underlying communication topology, and (c) whether previously dominated agents are replaced in the population by agents with higher social status. A key contribution of this paper is the coherent picture painted of the relationship between social differentiation and spatial structure in a multi-agent system.

## Categories and Subject Descriptors

I.2.11 [**Artificial Intelligence**]: Distributed Artificial Intelligence—*Multi-agent systems, Coherence and coordination*

## General Terms

Experimentation

## Keywords

multi-agent systems, self-organization, hierarchy, small-world network, socio-physics

## 1. INTRODUCTION

Self-organizing hierarchies are a common phenomenon in many natural systems (eg. human culture, primates, insect colonies and other animal groups) [5, 6, 7, 11]. A number of studies have suggested that social stratification is not a random process [1, 2, 5, 8]. Repeated interactions between individuals typically generate asymmetries in the "social status" of the population. Existing power relationships and domination enable some individuals or social sub-systems to take advantage of others. Conflicts amplify such differences, especially when "winner-loser effects" occur [5, 8]. That is, if the winners of previous conflicts are more likely to win future conflicts, and the losers of previous conflicts are less likely to win in the future.

Increasingly, agent-based models are being used to explore the basic principles underpinning emergent properties in complex systems. One study that has recently gained a great deal of attention is the minimalist agent-based model introduced by Bonabeau and co-workers [2] in order to explain social stratification in natural systems. The model was further refined by Stauffer and colleagues [18, 19, 20, 21, 22], who introduced a feedback mechanism for determining the probability of an agent's rise or fall in the social hierarchy.

In the refined model (referred to as the B/S model), a population of agents diffuse across a regular lattice, where the carrying capacity of each site is one. Interactions between agents are restricted to the local neighbourhood. Consequently, agents must "fight" to gain control of (or occupy) a site. The outcome of the contest is determined by a function of the combatants' social status (or utility value). Winning agents are subsequently rewarded (increased utility), while the defeated agent is penalized (decreased utility). In addition, an agent's utility value decreases as a function of time. This relaxation mechanism may be thought of as a decline in social status due to inactivity.

As the number of interactions in the B/S model increases, the positive feedback mechanism amplifies small initial differences between individual agents and subsequently a heterogeneous social structure emerges. Monte-Carlo simulations have revealed a first-order phase transition between egalitarian societies for low densities and hierarchical societies for high densities, [2, 12, 19, 21]. This diversity originates as a consequence of agent experiences and their interaction with their environment.

A defining characteristic of the B/S model is the fact that the probability of interaction between agents was determined by the lattice topology and corresponding nearest neighbour (von Neumann neighbourhood) diffusion rule. However, work in the complex networks domain (eg. [9, 23]) suggests that by varying the underlying network topology (connections between individuals), the social hierarchy that emerges may be significantly different [11, 15]. For example, Gallos [10] has examined the impact that scale-free networks have on the B/S model. Interestingly, it was reported that the expected phase transition in social diversity occurred at a very low population density (typically less than $d = 0.1$), decreasing towards zero as the network size increased.

In recent years, a number of studies have appeared examining the widespread presence of the so-called "small–world" pattern of social, economic and technological networks (see [9, 23] for an overview). Small–world networks typically have a large overlap of neighbourhoods (clustering or groups of collaborators), and yet only relative short paths (node-to-node distance) connecting any two individuals in the network. These studies raise a number of questions, for example: What impact would various small–world topologies have on the emergent hierarchies? How do the environmental factors (reward mechanisms) shape the social hierarchy? At what population density does the expected phase transition from homogenous to heterogeneous social structure occur in small-world networks? Is it possible for lower-ranked individuals/groups to challenge the current social hierarchy?

In this study, we begin to address these questions. We extend the B/S model and investigate how the combination of Watts-Strogatz small–world interaction networks and alternative feedback mechanisms shape the emergent agent hierarchy. Here, the agents may not be aware of every other agent in the population. However, the assumption is that they do interact with a large number of agents. The spatial structure provides the basis for stable, repeated social interactions where an agent's relative social status may be different in alternative neighbourhoods. Our aim is to paint a coherent picture of the relationship between population density, spatial structure and social diversity in a multi-agent system. Specifically, we show that the transition point at which the population converges into dominance hierarchies is dependent upon the relative number of the short-range and long-range connections in the network.

The remainder of the paper is organized as follows: In the next section we describe our model in detail. The section starts with a description of the B/S model and a brief review of small-world networks. This is followed by the implementation details of the base network model and the dynamic hierarchy model. Computer simulation results are presented in the third section. In section 4, we discuss the findings in a more generic multi-agent system / self-organization context. The paper concludes with a summary and implications of this work as well as identifying future research directions.

# 2. MODEL

The proposed model is an extension of the B/S model. Here, agents inhabit a virtual world and their behaviour is restricted to competition for control of nodes (resource) in the network. As such, the agents have limited decision making capabilities (to fight for the resources or move within their local neighbourhood), however, these capabilities can be easily extended. The effects of winning and losing contests are self-reinforcing, thus may be thought of as "dominance interactions." The intensity of coupling is a function of both the underlying network topology and the total number of agents occupying nodes in network.

## 2.1 B/S model

In the B/S model, the multi-agent system consists of $n$ interacting agents $A = \{a_1, a_2..., a_n\}$ mapped onto a 2-dimensional lattice, $L \times L$. Each agent $a_i$ is characterized by a time dependent variable $h_i(t)$ representing its utility (or social status). Initially, all agents have $h_i(t) = 0$, which represents the egalitarian situation. As the model is iterated, agents diffuse randomly across the network via nearest neighbour rules.

If agent $a_i$ attempts to move onto a site occupied by agent $a_j$, a fight between the two agents occurs (the carrying capacity of a given node is one). The attacker agent $a_i$ will defeat agent $a_j$ with some probability:

$$q_{ij}(t) = \frac{1}{1 + exp[\sigma(h_j(t) - h_i(t))]} \tag{1}$$

The root-mean-square fluctuations in the winning probabilities, $\sigma$, measures the inequalities in the population at time $t$. The value of $\sigma$ is calculated by averaging over all interactions occurring at the current time step $t$ :

$$\sigma^2 =< q^2_{ij}(t) > - < q_{ij}(t) >^2 \tag{2}$$

where $< ... >$ is the mean value at time $t$. Note: in equation (1), the value of $\sigma = 1$ at time $t = 0$.

If agent $a_i$ wins the contest, it exchanges positions with agent $a_j$. Otherwise, the current lattice positions are maintained. After each contest, the agent's utility value, $h_i(t)$, is updated. The winner's status is typically increased by 1 and the loser's status decreases by 1 (or more generally, a penalty function $f$ is used, where $f \geq 1$). Therefore, agent heterogeneity is amplified.

Counter balancing the competition with feedback, a utility relaxation mechanism is also employed [12]. That is, after each time step all agents update their utility according to the following rule:

$$h_i(t)' = h_i(t) \times (1 - \mu) \tag{3}$$

where $0 < \mu < 1$. Consequently, if agents do not interact with other agents (engage in contests) it is not possible to maintain their social status, thus differences are absorbed.

The value of $\sigma$ acts as an order parameter of the system and can be used to describe the status of the population at time $t$. Small values of $\sigma$ indicate equal status, while large $\sigma$ values indicate a hierarchically organized population. In such hierarchies, there is a bias towards agents with higher social status (utility) winning the contests.

Simulations experiments have revealed a phase transition in society status at a critical population density (where the density $d = n/(L \times L)$) [2]. Stauffer and colleagues [20, 22] report that this phase transitions occurs at $d_c \approx 0.32$ when $f = 1$. For higher values of $d$, a non-zero social inequality $\sigma$ was obtained, while for lower values of $d$ the value $\sigma$ approached zero.

## 2.2 Small-world networks

A small–world network is typically described as a transition from a regular to random network [23]. In these networks, each link is re-wired with some probability $\lambda$. The effect of re-wiring is the substitution of some short-range

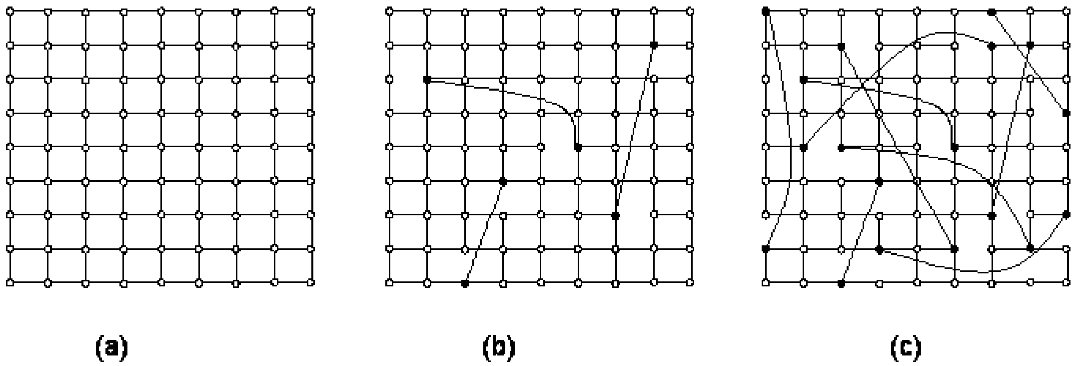

Figure 1: Small–world networks. (a) $\lambda = 0$, a regular lattice. (b) $\lambda = 0.1$, some of the links have been re-wired resulting in a small–world network. (c) $\lambda = 0.2$, additional re-wiring has occurred. As $\lambda$ approaches 1, a transition to a random network will occur.

connections with long-range connections (see Figure 1). The regular lattice ($\lambda = 0$) and the random graph ($\lambda = 1$) represent the two extreme models. Regular networks are highly clustered with relatively high shortest average path lengths. On the other hand, random networks are rather homogeneous, that is, most of the nodes have approximately the same number of links. Random networks have relatively short average path lengths and tend to have low clustering.

## 2.3 Base network model

The functionality of the base network model is similar to the B/S model. However, in our model, the agents are mapped to the nodes of alternative small–world networks. When an agent attempts to move across the network, they are restricted to their local neighbourhood. Here, the magnitude of $\lambda$ determines the number of long-range connections that are available. The inevitable contests that arise are the direct effect of the agents attempting to gain control of the limited resource – evolutionary processes at work.

A number of different scenarios were investigated using the base network model:

- the value of $\lambda$ was systematically incremented from 0 to 1 in steps of 0.1 (A regular lattice, $L = 60$, with von Neumann neighbourhood was used as the starting point)
- for each network, the population density $d$ was increased from 0 to 1 in steps of 0.02
- for all scenarios, the agent that won the contest had its utility value increased by 1 unit
- for each network–density combination, two penalty values were examine, $f = 1$ and $f = 2$
- the relaxation factor was set to $\mu = 0.1$ for each configuration

## 2.4 Dynamic hierarchy model

In the dynamic hierarchy model, we introduce an extension to the base network model (described above), where agents that are continually defeated in competition are replaced by a clone of one of their nearest neighbours. Typically, stratified systems are stable for long periods, but they are not immune from change [5]. Even in a strongly hierarchical society, “revolutions” can happen and thus new structure can emerge. Given the inherent clustering in small-world networks, the replacement of an individual with a low social status by an agent with a higher social status introduces the possibility of significant changes in the trajectory of the evolving population.

We assume that agents die with some probability at each iteration of the model and are replaced by other agents. An additional parameter was introduced to implement this version of the model – a *cull* threshold. Here, the cull threshold = 4. That is, if $h_i(t) < 4$, the agent $a_i$ was replaced by a copy of agent $a_j$ selected from the local neighbourhood. A range of mechanisms were examined for selecting the replacement agent $a_j$ (eg. roulette wheel, random, elitists). Based on similar work reported in Kirley [13], a modified roulette wheel–elitist approach was used.

# 3. SIMULATIONS AND RESULTS

The main goal of this study was to investigate the impact that different small-world networks would have on the phase transition in social diversity of the multi-agent system. Therefore, the behaviour of $\sigma$ (root-mean-square fluctuations in the winning probabilities) for different scenarios was the most important statistic considered. Fluctuations in this value provide insights into the distribution of utility values, $h_i(t)$, and corresponding emergent hierarchies within the population of agents. A related statistic also reported is the *power* distribution [22]. The power of each agent is represented by normalized utility values on a scale -1 to +1. The power value was calculated using the ratio $h_i(t)/H$, where $H$ is the maximal value $|h_i(t)|$ at time $t$

Extensive computational simulations were carried using the base network model and the dynamic hierarchy model. All simulations were performed with $N$=3600 nodes in the network[1]. The results reported below were averaged over

[1]Additional simulations using population sizes ranging from $N = 100$ were carried out. The results are not reported, as

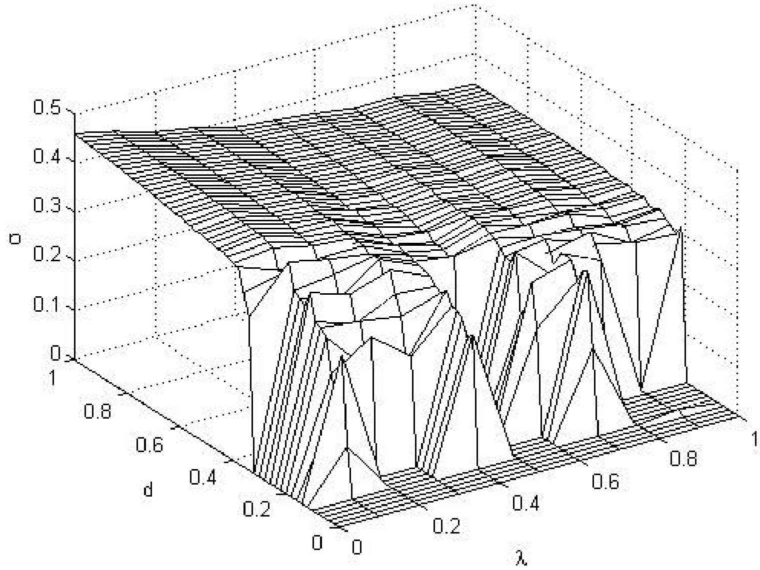


**Figure 2: Network re-wiring probability ($\lambda$) *vs* density ($d$) *vs* root-mean-square fluctuations in the winning probabilities ($\sigma$). Here, the loosing penalty $f$ was set to 1. The plot records data averaged over 20 independent trials.**

twenty independent trials of $10^5$ iterations. An iteration is defined as $n = d \times N$ randomly selected agents attempting to move at time $t$.

## 3.1 Spatial structure

In the first set of experiments, we have examined the effects of altering the network re-wiring probability, $\lambda$, on social stratification. In the early stages of a simulation run, only a small number of agent-agent interactions will have transpired. Consequently, population differentiation will not have occurred to any great extent and hierarchies will be difficult to identify. However, as the number of iterations increases, the emergent hierarchy will be qualitatively more stable (as measured by $\sigma$).

In Figure 2, we plot $\lambda$ vs $d$ vs $\sigma$ when $f = 1$ at equilibrium. The curves are to guide the eye. An inspection of the plot illustrates that the general behaviour is similar for all $\lambda$ values. As $\lambda$ increases, there is a corresponding drop in the value of $\sigma$ (at the phase transition) for all density values. Sudden jumps in the value of $\sigma$ were recorded as the density values increased (occurring in the range $d = 0.20$ to $d = 0.40$) for all values of $\lambda$, reminiscent of a first-order phase-transition. There is a distinct correlation between the developing hierarchy and the spatial structure. The transition point is shifted by the degree of spatial disorder of the underlying network.

The results provide further empirical evidence that social status appears to differentiate more clearly at high levels of interactions (based on population density). However, it is important to point out that the actions taken by agents of dominant and sub-ordinate groups not only results from their unequal positions, but also contributes to them. The impact of such actions is magnified as the level of order within the interaction topology decreases.

## 3.2 Feedback mechanisms

In the next set of simulation experiments, the focus shifts to investigating the impact of varying the penalty function, $f$, in the base network model. The reward for winning a contest was fixed at 1 unit, and the time relaxation factor was set to $\mu = 0.1$.

In Figure 3 and Figure 4 we plot $\sigma$ vs $d$ for three different small–world networks when $f = 1$ and $f = 2$ respectively. A comparison of the plots reveals similar trends in all three networks ($\lambda = 0$, $\lambda = 0.4$ and $\lambda = 1$) either side of the critical density value. As expected, there are differences in the corresponding critical density values for each of the networks, with the critical value being significantly smaller when $f = 2$, agreeing with the results reported by Stauffer and Sa Martins [22]. However, when $f = 1$ (Figure 3) the trajectory does not follow a pattern dictated by the magnitude of $\lambda$.

It is interesting to note that when $f = 1$ the results for $\lambda = 0.4$ appear to out-of-step with the regular lattice and random graphs. For small-world networks, the population density at the transition point appears smallest at mid $\lambda$ values. This may be attributed to the possibility that different highly clustered groups of agents are behaving in a similar way. According to the winner–loser effect, an individual's position in a hierarchy may not necessarily reflect its true "fighting ability" [6]. In practice, a strong differentiation in social status produces a cascade of unexpected effects. That is, because agent interactions are restricted to random local connections (defined by the underlying small-

there was no significance difference in the general trends observed

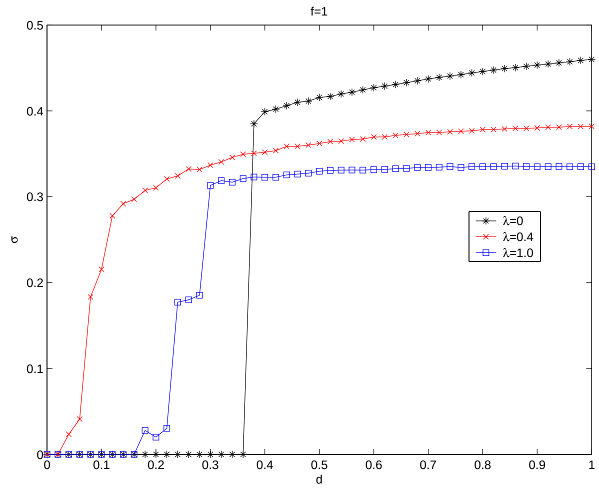


**Figure 3: Root-mean-square fluctuations in the winning probabilities ($\sigma$) *vs* density ($d$). Here, the loosing penalty $f$ was set to 1. The curves have been included to help differentiate between the different networks.**

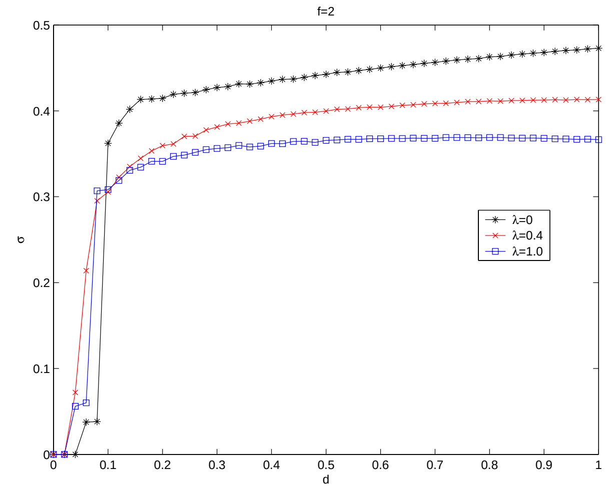


**Figure 4: Root-mean-square fluctuations in the winning probabilities ($\sigma$) *vs* density ($d$). Here, the loosing penalty $f$ was set to 2. The curves have been included to help differentiate between the different networks.**

world network), any particular history of victories (defeats) is strongly influenced by the dominant individual (the agent with greatest utility value). The dominant individual can "shut out" a number of other agents and keep them in a loosing state. This result suggests that different values of the average path length and clustering coefficient of alternative networks can lead to the establishment and persistence of different types of populations.

## 3.3 Dynamic hierarchies

In the final set of experiments, we examine how the systematic replacement of agents with lower social status impacts on the level of social diversity in the multi-agent system. Figure 5 plots $\sigma$ vs $d$ for three different small–world networks when $f = 1$ and the cull threshold $= 4$. A comparison with Figure 3 reveals similar trends. However, the critical density is significantly larger for both the regular lattice ($\lambda = 0$, $d \approx 0.65$) and random network ($\lambda = 1$, $d \approx 0.45$) , indicating that utility values were distributed more evenly across the population. Once again, the results for ($\lambda = 0.4$) are significantly different. (Note: the plot records data averaged over 20 independent trials). The introduction of a limited number of long-range connections resulting in the transition to a hierarchical state at smaller density values (in the range $d = 0.05$ to $d = 0.15$).

In the dynamic hierarchy model, when an agent's utility value falls below a threshold ($h_i(t) < 4$) they are replaced by a randomly selected agent with higher social status, drawn from the local neighbourhood. Therefore, it is reasonable to expect that the utility values may well be distributed more evenly than in the base network model. Here, the relative levels of clustering in alternative small-world networks is expected to impacts on the transition between homogenous and heterogeneous societies.

In Figure 6, the power distribution for alternative scenarios are plotted. Results for the regular lattice ($\lambda = 0$) with three different density values ($d = 0.1$, $d = 0.3$ and $d = 0.5$) with $f = 1$ and $\mu = 0.1$ are shown in Figure 6a. The distribution of power values does not reflect the phase transition shown in Figure 3. However, similar patterns are apparent for each of the density values – the plot simply shifted up the y-axis. This result is in agreement with similar trends reported by Stauffer and Sa Martins [22].

Figure 6b provides a contrast by plotting the small–world network with $\lambda = 0.4$, $d = 0.1$ and three different scenarios: $f = 1$, $f = 2$ and the cull threshold $= 4$. The distribution for $f = 1$ is symmetrical about the y-axis. When $f = 2$, the number of individuals with strong probability of winning (positive power values) is significantly lower than the number of agents that cannot easily win a fight. A similar pattern is evident in the cull model – albeit with the distribution shifted along the x-axis – indicating that there are a larger number of individuals with relatively higher social status.

# 4. DISCUSSION

The notion of interactions, which allows agents to find each other and then compete for a limited resource is a central point for the design of many multi-agent applications. Typically, the agents are autonomous entities and the repeated, constrained interactions between agents may result in self-organizing behaviour. That is, rich and complex global (macroscopic) properties can emerge from purely local (microscopic) interactions [5].

In this study, we have investigated dominance hierarchies and social diversity in a multi-agent system. Here, we have restricted the decision making capabilities of agents to relatively simple behaviours. An agent's social status represents an individual's overall ability to control or influence other agents. Our aim was to show how the "life experiences" of the agents and the interaction network impact on the transition between homogenous and hierarchical societies.

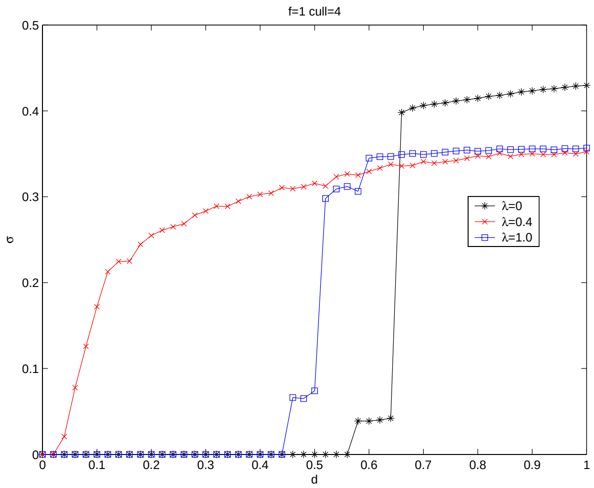


**Figure 5: Root-mean-square fluctuations in the winning probabilities ($\sigma$) *vs* density ($d$). Here, the cull threshold was set to 4. Once again, the loosing penalty $f$ was set to 1. The curves have been included to help differentiate between the different networks.**

Self-organization does not happen automatically in any multi-agent systems. The emergence of self-organization must take into account the dynamic interactions among individual agents and between these agents and the environment. Two basic mechanisms must be exploited – positive and negative feedback – for the agent population to self-organize [5]. Positive feedback consists of the amplification of some properties of the system that emerge from the random interactions between the individual components. In our model, this represents the reward allocated to agents for winning the contest to occupy a node in the network. Where as negative feedback acts as a regulatory mechanism, which exhausts the resources of the system. The time relaxation of an agent's social status serves this role in our model.

In the base network model, alternative social communication links based on small-world networks were introduced into the B/S model. The simulation results support the notion of a correlation between the hierarchical differentiation (measured by the $\sigma$ value) and the population density. Each simulation trial started from random meetings between equivalent individuals, yet a differentiated social structure emerged (as measured by $\sigma$). The trade-off between the positive and negative feedback mechanism guided the trajectory of the population. Specifically, we have shown how the unequal distribution of the social capital creates a system of stratification. That is, in the event of a contest, individuals higher up the "pecking order" have a greater probability of gaining control of the resources. Agents with similar utility values occupy specific levels in the hierarchy, and this rank order of social status typically remains fixed over a number of iterations. In economic terms, there is a corresponding level of social capital attached to a specific spatial structure.

Significantly, the critical density value tended to decrease as the level of disorder (measured by the $\lambda$ value) increased for particular combinations of feedback mechanisms. This was particularly the case when the penalty for losing was greater than the reward for winning. However, when the pay-off values were the same, mid-range $\lambda$ values tended to lead to even smaller transition critical density values. The restricted local neighbourhood topology forces individuals to interact, even at low densities. This suggests that the combination of pair wise interactions within local neighbourhoods and a limited number of long-range connections may in fact facilitate hierarchical control structures more readily.

We have also examined a scenario where agents with low social status were replaced. The substitute agents were randomly selected from the local neighbourhood. This update scheme reduces the relevant dominance levels of agent of higher social status, possibly leading to contest successes for the lower ranked agents, which in turn tends to equalize the social status of the population. This particular configuration provided the means for new hierarchies – *revolutionaries* – to emerge. Similar trends in the phase transition of social diversity were observer for alternative small-world networks. However, the transition critical density levels were a lot higher than the corresponding standard model. Analysis of the power distributions revealed significant difference in the number of agents with similar social status for differing reward/penalty combinations. Here, the small perturbations to the population structure affected the balance between the positive and negative feedback mechanisms, resulting in a significant increase in the population density level required to induce a phase transition in social structure.

For any multi-agent system, specifying the organisational structure is an essential first step in determining the interaction requirements. The identification of an agent's goals and roles within the system as a whole must be considered. For many systems, specialists and generalists agents may be pre-defined. While agent-based technologies provide a way to conceptualise large interconnected complex systems, centralized management techniques are not always appropriate and/or feasible. In contrast, decentralized and distributed management enables greater flexibility and robustness as resources are managed across many nodes in the network. For example, agent-based resource management systems for Grid computing or sensor network applications may require the agent to be organized into a hierarchy and to cooperate with each other to complete a complex task. For systems where adaptability is important, the social status of individual agents could be used to help forge alliances or coalitions and streamline negotiations between agents.

Recently, there has been increased interest in developing Self* systems [14]. That is, systems that exhibit self organization, self management, self configuration and self repair. While this aim may appear to be grandiose, there has been significant work reporting aggregation and coordination of tasks in the robotics domain [16, 17]. Successful applications of agent-based models, inspired by social insects have also been reported in limited domains (eg. [3, 4]). From an engineering point of view, a key advantage of self-organization models is the fact that the designer does not need to divide the desired behaviour into simple basic behaviours. In future work, we will look more closely at relevant environmental parameters and investigate the population dynamics as the agents learn from new experiences. In doing so, we hope to increase understanding of how to engineer agent-based self-organization applications on a large scale.

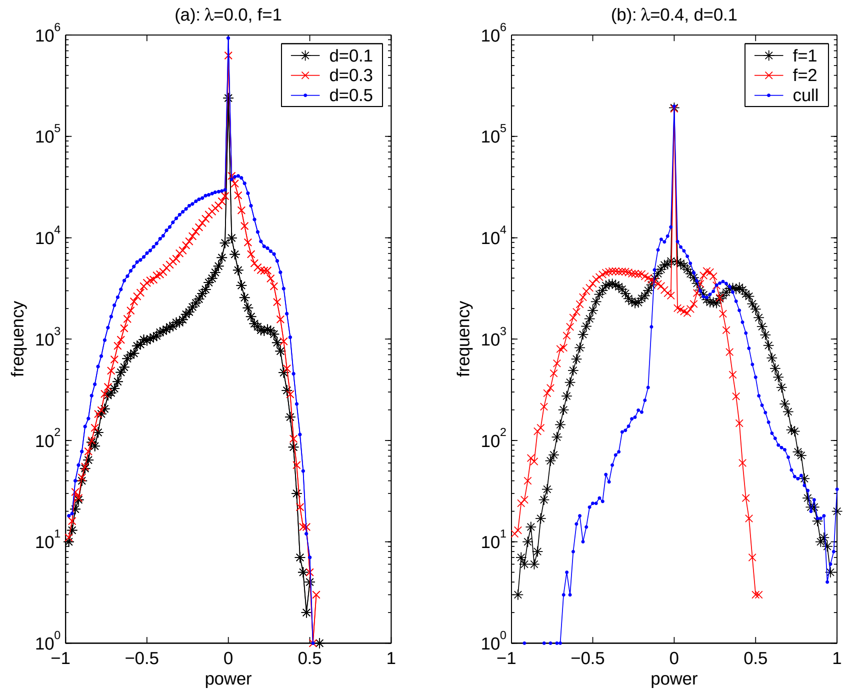


**Figure 6: Power distribution – frequency *vs* power for typical trials. (a) The regular lattice ($\lambda = 0$) and $f = 1$ and three different density values. (b) A small–world network ($\lambda = 0.4$) with fixed density, $d = 0.1$ and three different scenarios: $f = 1$, $f = 2$, and cull model (threshold = 4 when $f = 1$).**

# 5. CONCLUSION

In this paper, we have investigated self-organizing social hierarchies using an agent-based model. The model consisted of a homogenous virtual world (represented by alternative small-world networks) inhabited by artificial agents. Interactions between agents – fights to gain control of a resource / occupy sites in the network – represent self-reinforcing dominance mechanisms, which amplify differences between agents leading to a heterogeneous population.

Our simulation results show that by altering the underlying interaction links, the critical population density at which the society jumps from a homogenous society to a hierarchical society also changes. As the level of "randomness" increases, the critical density generally decreases. However, the magnitude of this threshold value depends on the relative strengths of the reward mechanisms. We have also examined a scenario where perturbations were introduced into the stratified system. Here, an agent clone, randomly selected from the local neighbourhood, replaced the nominated oppressed agents. Similar patterns in diversity were observed in the dynamic hierarchies, however, the critical population density at which the transitions occurred was significantly higher.

The multi-agent system examined here is an abstraction and simplification of a natural system. However, it does provide a framework for investigating the hierarchies observed in social organizations as well as in technological networks. Understanding the kinds of behaviour observed in such complex systems may pave for the way for the engineering of large-scale, self-organizing autonomous agent applications.

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