Hybrid Search for Cardinality Constrained Portfolio Optimization

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ABSTRACT
In this paper, we describe how a genetic algorithm approach added to a simulated annealing (SA) process offers a better alternative to find the mean variance frontier in the portfolio selection process. The nonlinear mixed integer quadratic programming model is considerably more difficult to solve than the original model; but some computational experiments have shown that hybrid heuristics offer a good alternative for these types of problems.

Categories and Subject Descriptors
I. Computing Methodologies I.6. Simulation and modeling. I.6.5. Model Development

General Terms
Management.

Keywords
Portfolio selection, Markowitz model, Mixed integer programming.

1. INTRODUCTION
The problem of optimally selecting a portfolio among n assets was formulated by Markowitz in 1952 as a constrained quadratic minimization problem [7]. The main assumption on Markowitz's model is that the aim of the investor is to design a portfolio which minimizes risk while achieving a predetermined expected return. An investor should be compensated with an increase in the portfolio's return if she accepts an increased level of risk.

The basic mean-variance model has been studied extensively but is often too simplistic to represent the complexity of real-world portfolio selection problems in an adequate fashion. In order to enrich the model, we introduced more realistic constraints that create a mixed integer and combinatorial problem so the decision of which assets to include or not is a crucial one; which turns out to be an NP hard problem.

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2. THE MEAN VARIANCE PROBLEM
2.1 An Extended Portfolio Selection Problem
The model considers that each asset is characterized by a return varying randomly with time. The variance of its return measures the risk of each asset. If each component x_i of the vector $X=\{x_1, x_2, .., x_n\}$ represents the proportion of an investor's wealth allocated to asset i and r_i is the individual asset return contained in a vector $R=\{r_1, r_2, .., r_n\}$, then the total return of the portfolio is given by the scalar product of X by R. The set of optimal solutions of the Markowitz model, parameterized over all possible values of the expected return constitutes the mean-variance frontier of the portfolio. Markowitz [7] assumes that the aim of the investor is to design a portfolio which minimizes risk while achieving an expected return.

Some research started with meta-heuristics techniques in local search for the portfolio selection problem using evolutionary algorithms [4]. Some attempts have also been made using a variety of metaheuristics to the portfolio selection problem without considering commercial restrictions [1]. Mansini and Speranza have also studied several portfolio selection problems with cardinality constraints and some others have considered minimum costs on transactions using heuristics techniques [3] [5]. A technique that incorporate cardinality constraints and uses simulated annealing and evolutionary strategies was also tested on portfolio selection [6]; however they do not incorporate additional constraints. One of the more complex representations was established by Crama and Schyns [2]. They added cardinality constraints, trading constraints, floor and ceiling constraints and turnover constraints to the original model and used simulated annealing to find the efficient frontier. In this paper we used the complex model of portfolio selection that follows.

Min. $\sum_{i=1}^{n}\sum_{j=1}^{n}\sigma_{ij}x_i x_j$ Objective Function (2.1)

s.t. $\sum_{i=1}^{n} r_i x_i = R_{esp}$ Expected return constraint (2.2)

$\sum_{i=1}^{n} x_i = 1$ Budget constraints (2.3)

$\underline{x_i} \le x_i \le \overline{x_i}$ Floor and Ceiling constraints (2.4)

$\max\left(x_i - x_i^{(0)}, 0\right) \le \overline{B_i}$ Turnover(Purchase) Constraints (2.5)

$\max \left(x_i^{(0)} - x_i, 0\right) \le \overline{S_i}$ Turnover (Sale) Constraints (2.6)

$x_i = x_i^{(0)}$ or $x_i \ge \left(x_i^{(0)} + \underline{B_i}\right)$ Trading constraints

or $x_i \le \left(x_i^{(0)} - \underline{S_i}\right)$ (2.7)

$\left|\{i \in \{1,...,n\} : x_i \ne 0\}\right| \le N$ Number of assets (2.8)

$\forall i \in \{1,...,n\}$

2.2 Hybrid Evolutionary Approach

The main idea of SA is to start with some arbitrary solution and have it modified by allowing random changes whenever they result in an improvement. Additionally, to overcome local optima changes that come with a "no good" solution are accepted with decreasing probability. However, unlike the standard version of SA, where just one solution is considered, this approach uses a whole "population" of individuals which are permanently ranked according to evolutionary principles. The best of the population's members are selected whereas the worst are eliminated and replaced either by a clone of one of the best individuals or some new solution which is gifted with properties of the best solutions.

The algorithm starts with a random initialization of a solution each representing a portfolio. This includes selecting k of the N available assets and assigning them random positive weights w_i such that their sum adds up to 1; i.e., w_i is the weight of asset i in a portfolio during iteration t. The subsequent iterations consist of three stages: evaluation and ranking of the portfolios; modification of the portfolio structure and replacement of the poorest portfolios in the population. The algorithm finishes when a fixed number of iterations is reached and the best solution, which corresponds to the elitist portfolio, is reported. The process is repeated for each required value of the return in order to create the mean-variance frontier.

3. RESULTS AND CONCLUSIONS

The results show that once the additional constrains were added, the required time to find the best portfolio is increased. A model with a lot of restrictions causes noise when looking for the efficient frontier. This noise is also a problem for decision takers because a little change in the required returns offers dramatic changes in the associated risks. This is one of the problems reported in literature for complex portfolio selection models.

The experiments showed that the return calculated using the simulated annealing process alone is increased by up to 1.7% greater than the new approach. However, when the analysis of the variance was done, we found that the variance obtained from the return proposed by the hybrid approach was almost 41% smaller than the previous simulated annealing approach. the combination of evolutionary strategies added to the simulated annealing process reflects a smoother curve and it is very close to the quadratic curve (see Fig. 1).

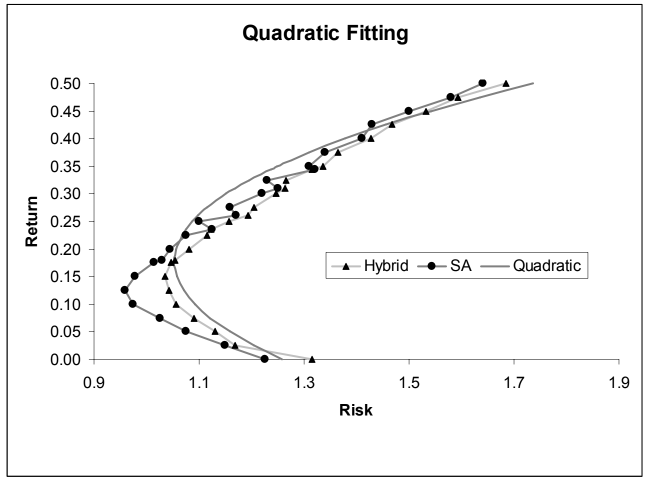

Figure 1. Quadratic fitting compared to Hybrid and Simulated annealing frontiers

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