Comparison of Multi-Objective Evolutionary Algorithms in Optimizing Combinations of Reinsurance Contracts *

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ABSTRACT

Our paper concerns optimal combinations of different types of reinsurance contracts. We introduce a novel approach based on the Mean-Variance-Criterion to solve this task. Two state-of-the-art MOEAs are used to perform an optimization of yet unresolved problem instances. In addition to that, we focus on finding a dense set of solutions to derive analogies to theoretic results of easier problem instances.

Categories and Subject Descriptors

I.2.1 [**Artificial Intelligence**]: Applications and Expert Systems; I.2.8 [**Artificial Intelligence**]: Problem Solving, Heuristic methods

General Terms

Algorithms, Design, Experimentation, Performance

Keywords

Multi-Objective Evolutionary Algorithm, Optimal Reinsurance, Mean-Variance-Criterion, Value-at-Risk

1. INTRODUCTION

In many branches of the insurance business the insurance company is not willing (or not able) to hold the entire risk on its own. There are several types of treaties which an insurance company can use to cede an amount of its claims to another insurance company, mainly a so-called reinsurance company. Some focus purely on the severity of claim sizes, others deal more with the deviation of frequency. Thus, it seems quite natural to combine some types of reinsurance. A substantial amount of research has been performed to find individually optimal structures for each of these types of reinsurance treaties. However only few results (e.g. [3]) exist when the goal is to find optimal combinations of reinsurance treaties because of computational complexity or non-convexity of the objective functions. We therefore propose a multi-objective approach to minimize the expenses that come with contracting reinsurance protections and at the same time to minimize the retained risk after reinsurance.

*See [5] for the full version of this paper.

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2. REINSURANCE BUSINESS

In this section we briefly introduce the necessary methodology in insurance mathematics. For a comprehensive introduction we refer to [2].

We consider three important types of reinsurance in this contribution. The simplest form is the *quota share* (QS), where a fixed relative amount $a \in [0;1]$ of the claims and the premium income is ceded. In the remaining two types claims are cut at a positive line called priority. The *excess of loss* (XL) reinsurance cuts each individual claim at the priority R. The *stop loss* (SL) reinsurance cuts the sum of all claims in a year of business at the priority L.

Reinsurance pricing is done via the expected value of the amount of claims ceded to the reinsurance company $\overline{S}$, i.e. $(1+\lambda)E(\overline{S})$. The factor λ describes the risk loading and depends on the reinsurance type.

The risk of the net sum of claims $\underline{S}$ is measured using the variance and alternatively, using the common risk measure *Value at Risk* (VaR_α). The latter is the α-quantile of the distribution of $\underline{S}$.

3. REINSURANCE OPTIMIZATION

We based our work on a modified *Mean-Variance-Criterion* similar to the one used in [6]. As a first objective function we choose the minimization of the cost of $\overline{S}$. The second objective function is the minimization of the risk of $\underline{S}$. The distribution of the claims is based on real world data.

Our heuristic approach has several advantages compared to the standard approach using the Lagrange method of

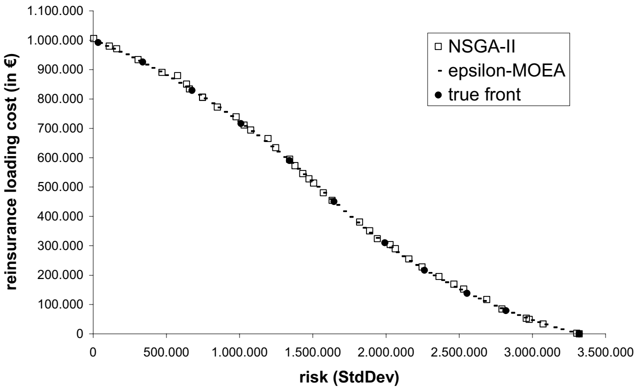

Figure 1: Optimal fronts of the combination of QS and XL.

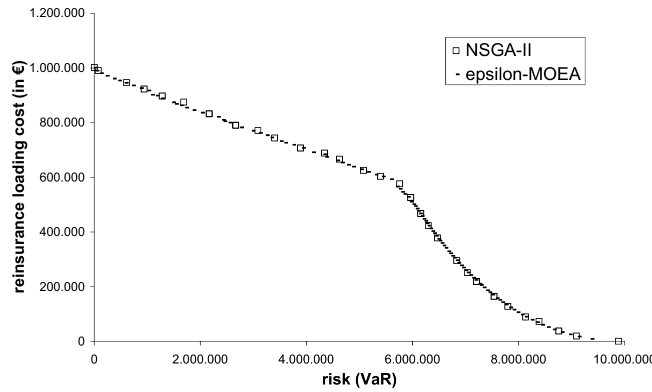

Figure 2: Optimal fronts of the combination of QS, XL and SL.

multipliers. We do not need a transformation to a single objective function. Our model incorporates discrete step sizes which is more realistic than a continuous framework. Centeno [3] proves that even in simple problem instances the feasible search space is not convex in general. Verlaak [6] states that the computation of optimal reinsurance contracts in the case of a combination of XL and SL is an open problem. In fact this problem of computational complexity is caused by the need to compute the distribution of the sums $\underline{S}$ and $\overline{S}$ numerically for each combination of R and L.

4. MOEA APPROACH

We pursue two main goals: At first, we intend to solve reinsurance optimization problems for cases which pose open problems to researchers. Secondly, our aim is to find a converged and diverse but dense front of Pareto-optimal solutions in a single run to choose the desired relationship between expense and risk afterwards. A high density of the front is then used to deduce information on the structure of the reinsurance variables of an optimal combination.

We compare two popular and state-of-the-art MOEAs in this paper: the NSGA-II [1] and the ε-MOEA [4]. For the chosen parameters we refer to our full paper [5].

5. SAMPLE RESULTS

One of the problems discussed in [6] is a combination of QS and XL under the risk measure variance. In Fig. 1 optimal fronts derived by the two MOEAs are compared to the true front computed with the method from [6]. As one can easily see, the MOEA fronts have converged well to the true front. The performance results of this problem show the capability of these MOEAs to converge to the true Pareto front of problems which cannot be computed directly. An example of such a problem is the combination of QS, XL, and SL (cf. [6]). To make the problem structure even harder we choose the risk measure VaR. We present the best Pareto fronts of this problem after a typical run of the two MOEAs in Fig. 2. For more detailed interpretations of the behaviour of the reinsurance variables a, R and L, we refer to [5].

We use the metrics spacing and coverage to measure the performance of both MOEAs in both problems. Spacing values of the NSGA-II are about 0.028 while ε-MOEA yields 0.006. The NSGA-II dominates 0% resp. 4% of the individuals of the ε-MOEA front in the mean, while the latter dominates about 26% of the NSGA-II individuals. Additionally, the ε-MOEA benefits from its growing archive that provides a denser front than the NSGA-II.

6. OUTLOOK

The high adaptability of MOEAs gives us the opportunity to change individual problem specifications very easily. Accordingly, we can deal with problems that are more relevant to real world applications. Such specifications are the reinsurance types which are allowed in the combination, the risk measure, the method for cost calculation and the underlying distribution.

7. REFERENCES

[1] S. Agrawal, K. Deb, T. Meyarivan, and A. Pratap. A fast elitist non-dominated sorting genetic algorithm for multi-objective optimisation: NSGA-II. *Technical Report 2000001, Indian Institute of Technology, (KanGAL)*, 2000.

[2] R. L. Carter. *Reinsurance*. Kluwer Publishing (in association with The Mercantile and General Reinsurance Company Limited), Brentford, Middlesex, 1995.

[3] M. de Lourdes Caracas Centeno. *Some Theoretical Aspects of Combinations of Quota-share and Non-Proportional Reinsurance Treaties*. The British Library Document Supply Centre, May 1985.

[4] K. Deb, M. Mohan, and S. Mishra. A fast multi-objective evolutionary algorithm for finding well-spread pareto-optimal solutions. *Technical Report 2003002, Indian Institute of Technology, (KanGAL)*, 2003.

[5] I. Oesterreicher, A. Mitschele, F. Schlottmann, and D. Seese. *Comparison of Multi-Objective Evolutionary Algorithms in Optimizing Combinations of Reinsurance Contracts*. University of Karlsruhe, Institute AIFB, www.aifb.uni-karlsruhe.de/CoM/publications/GECCO2006.pdf.

[6] R. Verlaak and J. Beirlant. An optimal combination of several reinsurance protections on an heterogeneous insurance portfolio. In *IME 2002 Conference Proceedings*, 2002.