A Multi-objective Evolutionary Algorithm with Weighted-Sum Niching for Convergence on Knee Regions

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ABSTRACT

A knee region on the Pareto-optimal front of a multi-objective optimization problem consists of solutions with the maximum marginal rates of return, i.e. solutions for which an improvement on one objective is accompanied by a severe degradation in another. The trade-off characteristic renders such solutions of particular interest in practical applications. This paper presents a multi-objective evolutionary algorithm focused on the knee regions. The algorithm facilitates better decision making in contexts where high marginal rates of return are desirable for Decision Makers. The proposed approach computes a transformation of the original objectives based on weighted-sum functions. The transformed functions identify niches which correspond to knee regions in the objective space. The extent and density of coverage of the knee regions are controllable by the niche strength and pool size parameters. Although based on weighted-sums, the algorithm is capable of finding solutions in the non-convex regions of the Pareto-front.

Categories and Subject Descriptors

I.2.M **Computing Methodologies**

General Terms

Algorithms

Keywords

Multi-objective optimization, Genetic algorithms

1. INTRODUCTION

Multi-objective optimization entails conflicting and often incomparable and non-commensurable objectives. Where objectives cannot all be optimized in concert the notion of trade-off becomes central. As several solutions corresponding to differing trade-off in the objective space are equally optimal, the element of human preference becomes a critical factor in deciding upon the *best* solution.

A posteriori multi-objective evolutionary algorithms (MOEA) [1] relegate preference-based selection to the post-optimal stage. These algorithms are concerned with obtaining a comprehensive set of *Pareto-optimal* solutions, which are characterized by varying trade-offs in the objective space. Equipped with the trade-off data, the DM selects a solution according to preference. *A posteriori* algorithms enjoy higher popularity than *a priori* or interactive algorithms but introduce other problems. Navigating through the candidate solutions post-optimal selection could be a sufficiently challenging problem of its own [2].

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In some practical applications only a subset of the Pareto-optimal front constitutes relevant alternatives for the DM. The significance of "knee" regions on the Pareto-optimal front, in particular, has been highlighted in [3-6]. The presence of a knee region is scale-invariant and solutions on the knee regions are interesting as they involve steep trade-off between objectives.

The focus on knee regions is not new [3-6]. In this paper a MOEA which focuses on the knee regions of the Pareto-optimal front is proposed. The algorithm biases the search towards knee regions by employing an objective-function transformation to identify niches corresponding to potential knee regions as the secondary selection criterion. The primary selection criterion is Pareto-domination, to encourage convergence to the true Pareto-front. The uniqueness of the approach proposed here is that the extent and density of coverage of the knee-regions on the Pareto-front could be controlled using two simple parameters.

2. THE PROPOSED APPROACH

2.1 Focusing on knee-region: biased selection

In the proposed algorithm, the secondary selection criterion is invoked once competing individuals are non-dominated. It is accomplished by computing **T**, a transformation of the objective functions **F** as follows. First a matrix weight set **W** of size PxM is generated, where P is the pool size and M is the number of objectives. The matrix **W** obeys the following:

$$\sum_{m=1}^{M} w_{pm} = 1 \quad w_{pm} \in R; w_{pm} > 0 \tag{1}$$

W is a transformation matrix applied on the objective functions as follows:

$$\mathbf{V^k} = \mathbf{W}\, x\, \mathbf{F^k} \tag{2}$$

where $\mathbf{V^k}$ is a Px1 vector and **F** the column vector containing the objective function values for individual k in the set X .

Once the vector $\mathbf{V^k}$ has been computed for all competing individuals in X they are assembled in a matrix, which is then sorted along the rows. For each individual, the best Q rank figures out of the available P are selected and assembled as rows of a matrix **T** of size $(N+1)xQ$, where N+1 is the number of competing individuals and Q is the predetermined niche size. The matrix **T** is the objective function transform. Given **T,** the algorithm in Figure 3 computes the worst individual to discard.

for each individual $k \in [1, N]$
    sign $= 0$
    for each objective q
        $\Psi(k,q) = \mathrm{T}(k,q) - \mathrm{T}(N+1,q)$
        sign $= \mathrm{sgn}(\Psi(k,q)) +$ sign

    if (abs(sign) $= Q$)
        $\Delta_k = \sum_q (\Psi(k,q))$
    else
        $\Delta_k = 0$

  sort (Δ_k)
  if (max(Δ_k) > 0)
      replace parent corresponding to max(Δ_k) with child$_j$
  end

Figure 1 Biased selection focusing on knee regions

By extracting the best rank figures, each row of **T** represents the weighted-sums where each corresponding individual perform best, the niche weighted-sums

3. SIMULATION RESULTS

Simulations with test problems DO2DK and DEB2DK [6] are conducted. DO2DK is a minimization problem with parameters **K** and **s** which could set to introduce knees and skew in the true Pareto-optimal front. The simulations employed DO2DK with values of K=1, s=0 and K=4 and s =1. DEB2DK contains non-convex region in the Pareto-optimal front. For each test function, the algorithm was run several times. Figs. 2 to 3 illustrate the individuals obtained against the true Pareto-front (traced line). It was observed that the algorithm proposed converged to the knee regions of the true Pareto-front.

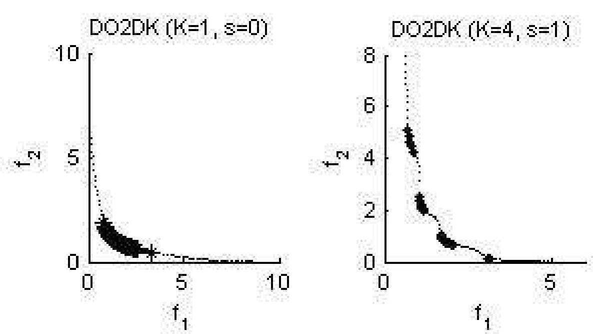

Figure 2 Population obtained for test problem DO2DK with 1 knee and 4 knees and skew

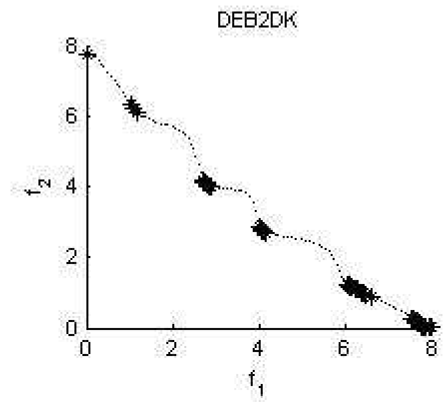

Figure 3 Population obtained for test problem DEB2DK

The effect of varying Q for a given P was also examined. Results showed that the extent of the coverage and the focus on the knee region could be controlled by setting P and Q.

4. CONCLUSION

The paper presented a multi-objective evolutionary algorithm capable of focusing on the knee regions of the Pareto-front. Simulations on difficult test functions show the ability of the algorithm to converge to the knee-regions of the true Pareto front. The number of weighted-sums employed in objective transformation depends on the number of knees expected, making the algorithm scalable with respect to the number of objectives. The extent of coverage of the knee regions may be controlled via the parameters P and Q. Adaptive measures to the niching strategy could be employed to refine the controllability and is a subject of future works.

5. REFERENCES

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