# Ant Colony Optimization Technique for Equilibrium Assignment in Congested Transportation Networks

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## ABSTRACT
This paper deals with transport user equilibrium. A modified version of the ant colony system is proposed where the ant colony heuristic is adapted in order to take into account all aspects characterizing the transport problem: multiple ODs (Origin-Destination pairs), link congestion, non-separable cost link functions, elasticity of demand, multi classes in demand.

## Categories and Subject Descriptors
I.2.8 [**Problem Solving, Control Methods, and Search**]: *Heuristic methods*

## General Terms
Algorithms, Performance, Experimentation.

## Keywords
Ant colony system, User equilibrium assignment, Transportation networks, Non separable cost.

## 1. TRAFFIC ASSIGMENT PROBLEM
The best known approaches to traffic assignment problem are Deterministic User Equilibrium (DUE) and Stochastic User Equilibrium (SUE).

Given a directed graph $G(V,E)$, with $V$ being the nodes and $E$ the links, we can identify a subset of nodes in $V$ and call them centroids. Let $\boldsymbol{d}$ be a vector with components representing the average number of trips going from centroid origin $o$ to centroid destination $d$ within a give time period. Each origin-destination (OD) flow generates on the network path flows $F_i$, with $i \in I_{od}$, where $I_{od}$ is the subset of all admissible paths connecting the pair of centroids $o$ and $d$. For a given link $i \in E$, the sum of all path flows crossing this link is called the *link flow:*

$$f_i = \sum_k a_{ik} F_k \,, \qquad (1)$$

where $a_{ik}$ is 1 if the link $i$ is used by the path $k$ and 0 otherwise.

A model of a transportation system describes the behavior of traffic demand $\boldsymbol{d}$ and its relationship with link flows. By introducing a cost $c_i(\boldsymbol{f})$ for traveling on a certain link $i$, depending on the observed traffic $\boldsymbol{f}$, one can express traffic demand and the way it is distributed on links as a function of the vector of costs $\mathbf{c}$, in particular the relationship between $\boldsymbol{F}$, $\boldsymbol{f}$ and $\boldsymbol{d}$ becomes $F = P(c(f))d(c(f))$ and $f = AP(c(f))d(c(f))$; where $P$ is a matrix with each element corresponding to the fraction of traffic demand routed on a certain path and $C=A^T c$ represents the vector of path choices costs (which has been considered the sum of the composing link costs). These equations, as shown in figure 1, describe the circular dependence at the base of the equilibrium problem.

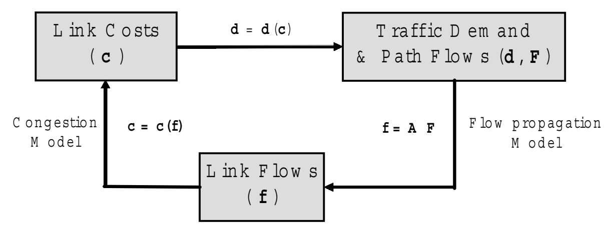


**Figure 1. Equilibrium relationship between traffic demand, flows and costs.**

A method to solve DUE is the Frank-Wolfe algorithm. Daganzo [2] proposed a model for optimization for symmetric SUE. A plain complete review of SUE and DUE as well models is presented by Cascetta in [1].

## 2. ANT SYSTEM FOR THE TRAFFIC ASSIGMENT PROBLEM
We think about an Ant System [3] for the user equilibrium assignment problem in the following way. An agent/ant decides to travel on a path using the information left before by other ants then it distributes a new quantity of pheromone in function of the "goodness" of the path. In order to effectively explore the space of solutions there must be a relationship between pheromone and flow, because ants deposit pheromone not flow. For each link, for instance, a quantity of flow proportional to the pheromone present on it could be associated. Then, after pheromone distribution, the new proportional flow assignment implies a variation of costs that leads the ants following to have a different evaluation of paths. All this can be viewed in the same manner as the circular relationships of equilibrium between traffic demand, flows and costs explained previously, where pheromone substitutes traffic demand (Figure 2).


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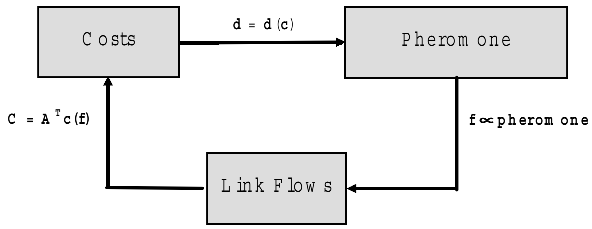


**Figure 2: Equilibrium relationships between pheromone distribution, flows and costs.**

At every iteration step each ant deposits a quantity of pheromone $\Delta\tau^k_{ij}(t)=1/C^k(t)$ on each arc belonging to the tour made by ant $k$ at iteration $t$, and $C^k(t)$ is its cost. For every *od*-couple there is an ant colony, with its own nest (centroid O) and a food source (centroid D). Every ant of the same colony distributes pheromone of the same type, so that the ants in that colony can recognize and follow only paths that lead to the same food source. Every ant colony is independent and its ants have to route a quantity of flow equal to the corresponding flow demand from an origin O to a destination D.

At time $t$ on link $(i,j)$ there is a quantity of flow equal to:

$$f_{ij}(t)=\sum_{c=1}^{N_{OD}} d_c \frac{\tau^c_{ij}(t)}{\tau^c_{FS}(t)} \tag{2}$$

where $d_c$ is the flow demand of colony $c$, $N_{OD}$ is the number of colonies, $\tau^c_{ij}(t)$ is the quantity of pheromone of colony $c$ on arc$(i,j)$ and $\tau^c_{FS}(t)$ is the sum of pheromone quantities present on the arcs of the forwarding star of node $i$ of colony $c$. An ant must follow paths leading to the food source (i.e., its destination) only of its colony, and so it has to pay attention only to the information (pheromone) left by the other ants of the colony. The ant-decision table $A^c_i=[a^c_{ij}(t)]_{|Ni|}$ of node $i$ and colony $c$ is obtained by the composition of the local pheromone trail values with a heuristic weight of the minimum path as follows:

$$a^c_{ij}(t)=\frac{[\tau^c_{ij}(t)+w_{ij}(t)]^\alpha}{\sum_{l\in N_i}[\tau^c_{il}(t)+w_{il}]^\alpha}, \forall j\in N_i \tag{3}$$

where $\tau^c_{ij}(t)$ is the amount of pheromone trail of colony $c$ on link $(i,j)$ at time $t$ and $w_{ij}(t)$ is a weight value of arc $(i,j)$. The value of $\tau^c_{ij}(t)$ used in Equation (3) may be the exactly amount of pheromone released by ants or the perceived one extracted according to a certain distribution (e.g. the normal distribution).

The probability at time $t$ an ant $k$ chooses to go from node $i$ to node $j\in N_k$ when constructing its trip is given by:

$$p^k_{ij}(t)=\frac{a_{ij}(t)}{\sum_{l\in N^k_i} a_{il}(t)} \tag{4}$$

where $N^k_i\subseteq N_k$ is the set of nodes connected to node $i$ that ant $k$ has not yet visited.

After all the ants have completed their trips, pheromone evaporation on all arcs is triggered, and, after that, each ant $k$ deposits a quantity of pheromone $\Delta\tau^k_{ij}(t)$ on each used link: the shorter the tour made, the greater the amount of pheromone deposited. In practice, the addition of new pheromone by ants and pheromone evaporation are implemented by the following rule applied to all the arcs:

$$\tau_{ij}(t)\leftarrow(1-\rho)\tau_{ij}(t)+\Delta\tau_{ij}(t)$$
$$\Delta\tau_{ij}(t)=\sum_{k=1}^{m}\Delta\tau^k_{ij}(t) \tag{5}$$

where $m$ is the number of ants at each iteration (maintained constant), and $\rho\in(0,1]$ is the pheromone trail decay coefficient. The initial amount of pheromone $\tau_{ij}(0)$ is set to the same small positive constant value $\tau_0$ on all arcs in order to randomly explore the network at the first iteration of the algorithm.

## 3. EXPERIMENTAL VALIDATION

Tests have been conducted to verify the actual properties of the proposed ACS in different scenarios and deterministic user equilibrium. Four networks with different structures and OD demand are tested. In Table 1 the main features, number of links, number of nodes and OD centroids of the networks are reported.

**Table 1: Networks features and validation results**

| | Links | Nodes | OD | Computing Time[s] | Iterations (<10%) |
|---|---|---|---|---|---|
| Trial | 12 | 6 | 4 | 0.2 | 9 |
| Non Separable Costs | 28 | 12 | 8 | 0.2 | 10 |
| Maggi (Milan) | 373 | 189 | 1283 | 8.3 | 15 |
| Extra-urban (Naples) | 1363 | 994 | 1483 | 1.8 | 8 |

Convergence (90%) is obtained easy for lower values of $\rho$ and with a limited number of iterations for all networks. Comparisons with Frank-Wolfe (FW) DUE solution show a good similarity, but not same results.

## 4. CONCLUSIONS

The modified version of ACS is suitable for application in almost all real cases to solve the UE assignment problem due to its versatility without assuming simplifying hypotheses. The solution found by ACS does not depend on the shape of the objective function and therefore also the particular cases of non-separable cost link functions or multi-class demand can be tackled easy and successfully.

Applications to real networks show a computation time that is short enough also in complex networks and it can be improved through a parallel programming that is easy enough to apply thanks to the similar nature of ACS which is intrinsically parallel.

Future research concerns different probability functions used by ants when choosing best path and how they build their decision table, that is how they perceived path costs.

## 5. REFERENCES


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