Dynamics of Evolutionary Robustness

Alan Piszcz, Terence Soule
Dept. of Computer Science, University of Idaho
Moscow, ID, USA
apiszcz@acm.org, tsoule@cs.uidaho.edu

ABSTRACT

Recently there has been considerable interest in determining whether, and how much, evolutionary pressure for genetic robustness influences evolutionary processes. In this paper, we attempt to show that this evolutionary pressure does have a significant effect in typical genetic programming problems. Specifically we demonstrate that in a standard genetic programming implementation to solve a symbolic regression problem, pressure for genetic robustness forces the population away from high fitness, but less robust, solutions in favor of solutions with lower fitness, but higher genetic robustness.

Categories and Subject Descriptors

I.2.2 [**Artificial Intelligence**]: Automatic Programming, program synthesis.

General Terms

Algorithms, Experimentation, Theory.

Keywords

Dynamics, Elitism, Robustness.

1. INTRODUCTION

Genetic programming (GP) problems typically generate solutions that meet goal criteria for a desired fitness function. Consider two outcomes, the first being optimization which is a popular use for GP, and the second, survival of a population or species as a model. In the first outcome a solution that represents a function or program is the goal and robustness is a secondary concern. In the second outcome, survival is the primary concern with fitness as secondary.

Unless one directs the fitness function to evolve robustness as a feature, we find the majority of research optimizing for a solution. Optimal solutions often trade off robustness for solution time. If growth of the GP solution is permitted it enables the individual to sustain more disruptive events caused by crossover or mutation, thus providing a more robust solution.

Our goal in this research is to determine if highly fit individuals become susceptible and subsequently replaced by less fit and more robust individuals. Four experiments test various behaviors of robust individuals. We show that beyond the generation which contains the optimal fitness individual, the individual may not be optimal to survive evolutionary disruption. Finally we show the dynamic transition of a population moving from a highly fit individual on a narrow peak to a less fit individual on a broader peak for a typical GP symbolic regression problem. The dynamic is discussed further in the background section.

2. BACKGROUND

Consider genetic robustness as a measure of the correlation between genotypic changes and fitness changes. A more robust individual is one in which genotypic changes have a relatively smaller effect on fitness. Several researchers have shown that evolution may favor genetically robust individuals over more fit individuals. Specifically it has been shown that given a landscape consisting of less fit, but broader peaks and of more fit, but narrower peaks under some conditions an evolving population will preferentially converge on the less fit, but more stable peaks rather than the more fit, but narrower peaks.

Figure 1 illustrates this scenario. The broader peaks represent solutions that are more robust; a small genotypic change to an individual on a broad peak is likely to produce an individual with a fitness that is similar to the original individual. In contrast a small genotypic change to an individual on a narrow peak is more likely to shift the individual "off" the peak, producing an individual with a fitness that is very different (and typically worse) than the original individual.

Simulation by GP showing transition phases through evolutionary processes provides an insight to population dynamics. Modeling the complex interactions of real world systems may lead to a better understanding of the properties that affect robustness.

Edlund and Adami model robustness with antagonistic, multiplicative or synergistic as a measure of fitness decay due to epistasis, using this model they find robustness occurs where the phenotype have maximal independence among each other [5]. Wilke and Adami examine neutral mutations and find selective pressure evolves robustness against mutation [18].

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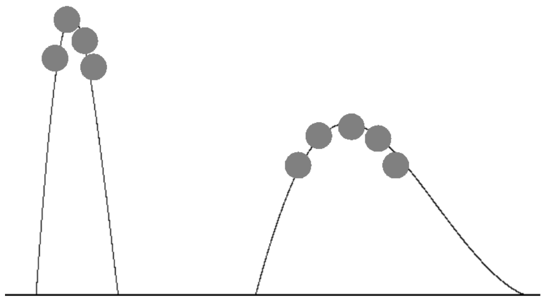

Figure 1: The narrow peak on the left shows the few fit individuals that achieved fitness. The flatter and less fit peak is occupied by more individuals who are more resilient to genotypic changes.

For example, DeVisser et al. define genetic robustness as the 'invariance of phenotypes in the face of heritable permutations (e.g. mutations)' [7]. The research identifies three classes of theories addressing robustness scenarios; adaptive, intrinsic and congruent. Adaptive: mechanisms that buffer a trait from deviation increase the fitness and should be favored by natural selection. Intrinsic: buffering with respect to mutations may be a necessary or likely consequence of adaptation. Congruent: suggests a correlation between genetic and environmental robustness, robustness may evolve as a correlated side effect of evolution for environment robustness. Any trait improving robustness also increases fitness, and can occur when the trait is intrinsically connected to improving its function, or when a trait avoids deleterious variation in an adaptive scenario.

Historically robustness is defined as canalization, which is the developmental buffering and homeostatic processes by which a particular kind of organism forms a relatively constant phenotype although individuals may have a variety of genotypes and environmental conditions may vary [17] [14].

In the evolutionary computation literature, the term resiliency is sometimes used instead of genetic robustness. Streeter states that large trees will be the most resilient in the face of crossover in his examination of code growth [16].

Wilke and Adami show that survival of the flattest influences the growth rate using low and high mutation rates. The individuals cluster near the peak for low mutation rates, alternatively with high mutation rates replication grows faster around the flatter peak where individuals are able to retain fitness values closer to the peak [18].

Krakauer and Plotkin model redundancy by comparing a family of landscapes with varying degrees of steepness. They show a fixed environment will evolve organisms towards maximum redundancy where a genotypic change causes the least change in fitness. This research also shows population sensitivity where small populations prefer shallow landscapes and large populations prefer steep landscapes [11].

Evolving individuals often adopt strategies to increase the genetic robustness. One common example is the code growth phenomenon in GP. Over the course of evolution GP individuals grow progressively by adding additional instructions. These instructions typically have very low functionality, that is they contribute little or nothing to the individuals' fitness. However, they increase the genetic robustness of the individuals by enabling individuals to better withstand deleterious mutation and crossover events. In a small individual with a relatively low proportion of functionally unimportant code, most mutation or crossover events are likely to affect the individual's functionality; it is not very robust. In contrast, in a large individual with a relatively high proportion of functionally unimportant code, most mutation or crossover events are not likely to affect the individual's functionality and if they do, it will typically have a small effect; the individual is robust.

More recently, a novel evolutionary dynamic has been shown based on the evolutionary pressure for both robustness and fitness. It was shown that as individuals, which originally converged on a low fitness, broad peak, increase their inherent genetic robustness, through growth or other mechanisms, they shift from the low fitness, broad peak to high fitness, narrow peaks. The individuals' evolved robustness makes up for the narrowness of the higher fitness peak. This demonstrates that the evolution of genetic robustness is sometimes a necessary precondition to a population of individuals finding a fit, but less robust solution [15]. However, these results were shown for a very simple, illustrative, problem. It is not clear whether a similar phenomenon applies to more typical GP problems. Thus, the goal of this paper is to determine whether similar evolutionary dynamics occur for more typical problems. Specifically, whether high fitness, but less robust individuals are out competed by lower fitness, but more robust individuals.

3. EXPERIMENT GOALS

Previous research demonstrating the shift from lower fitness, broad peaks to higher fitness, narrow peaks used a carefully tailored, toy problem in which the exact fitness landscape was well defined. This is not possible with more typical GP problems, such as symbolic regression. Instead, we seed the population with high fitness, but genetically 'brittle' individuals. These individuals are located on high, but presumably narrow, fitness peaks.

Our hypothesis, is in contrast to a more traditional view that fitness is the driving force where evolutionary computation predicts that the high fitness seed individuals will quickly dominate the population, leading to convergence on those solutions. The hypothesis for this experiment is that high fitness, but genetically brittle individuals (the seeded individuals) will go extinct, replaced by lower fitness, but genetically more robust individuals. Confirming this hypothesis requires demonstration of 1) that high fitness individuals become extinct, 2) when the high fitness individuals go extinct they are replaced by less fit individuals, and 3) when the high fitness individuals go extinct they are replaced by more robust individuals.

We are also interested in the effect of elitism, because elitism will always preserve at least one copy of the highest fitness individual, even if it would otherwise go extinct, due to a lack of robustness. Does preserving one or more copies of this individual improve the evolutionary process, or do these copies only take up space in the population?

3.1 Background on Seed Individuals

We hypothesize that less fit and larger individuals will evolve and replace two initial seeds with perfect fitness. The seed individuals represent the narrow but more fit peak shown earlier. To guarantee that the high fitness individuals are available we insert two high fitness seed individuals into the population at generation 1 with perfect fitness. Typically, once the goal fitness value is achieved for the target or objective function the evolutionary simulation is terminated. In this experiment, the intent is to determine the best fitness individuals at the end of a 'a priori' number of generations, thus we do not end the simulation when the fitness goal is met. We allow evolution to continue for a finite number of generations irrespective of the fitness values.

The transcendental function sine is the objective for the symbolic regression experiment. Two equivalent functions represent highly fit and brittle seeds for the population. Equations 1 and 2 shows the tangent and Taylor series equivalent seed functions for sine. The tangent equivalent seed has a higher probability to undergo deleterious changes due to the complexity of the sequence of terms when compared with the Taylor series. We identify the size of the functions for the tangent equivalent individual has 15 nodes and the Taylor series equivalent is 36 nodes.

The GP environment parameters for the testing the seed are shown in the seed test column of Table 3. The integer constants in the following formulas are created by the GP environment with ephemeral random constants (ERC).

For example, Equation 3 shows a potential outcome of a genetically altered Taylor series. In this case the $+\frac{x^5}{5!}$ is removed, the sign preceding $\frac{x^7}{7!}$ is changed to + and 1 is added to the equation. Similar disruptive events to the tangent equivalent function are more likely to be deleterious due to its structure.

$$sin(x) \approx \frac{2tan\left(\frac{x}{2}\right)}{tan^2\left(\frac{x}{2}\right)+1} \tag{1}$$

$$sin(x) \approx x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \frac{x^{11}}{11!} \tag{2}$$

$$Altered: x - \frac{x^3}{3!} + \frac{x^7}{7!} + \frac{x^9}{9!} - \frac{x^{11}}{11!} + 1 \tag{3}$$

Brittle or non-robust individuals exhibit significant change in fitness with small changes in the genotype. An illustration of brittleness with sine equivalent functions is shown in Figure 2 where equations 1 and 2 have minor alterations.

We create a slight modification to each individual as shown in Equation 4 where the constant value 2 is changed to 3, and Equation 5 where the constant value 3 is changed to 4. We indicate the modified values in bold font. Through the x range -0.5 to 0.5 the mean square error is small, below ≈ -1 and above 1 the mean squared error becomes significant as both altered functions deviate from the target function.

$$sin(x) = \frac{\mathbf{3}tan\left(\frac{x}{\mathbf{3}}\right)}{tan^{\mathbf{3}}\left(\frac{x}{\mathbf{3}}\right)+1} \tag{4}$$

$$sin(x) \approx x - \frac{x^{\mathbf{4}}}{\mathbf{4}!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \frac{x^{11}}{11!} \tag{5}$$

Seed solutions have no introns or non-significant nodes, all nodes of each seed are significant code. This differs from evolved individuals that often have a large percentage of introns, thus increasing the probability that a modification may occur where there is little or no function degradation of the individual.

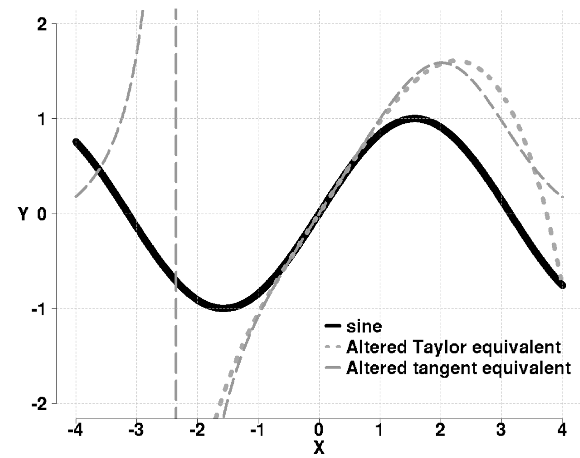

Figure 2: Sine equivalent function, normal, altered Taylor series and tangent equivalent. Slightly modified sine equivalent functions using altered Taylor and tangent Equations 4 and 5.

In Figure 3 we show the insertion of seeds effect evolution by comparing the mean population fitness of 100 trials without seeds against the mean population fitness with seeds. The case with the seeds attains a higher mean fitness due to the initialization with perfect fitness individuals.

3.2 Brief Background of Symbolic Regression

Symbolic regression is the procedure of inducing a symbolic equation, function or program that fits given numerical data. GP is ideal for symbolic regression and most GP applications can be reformulated as a symbolic regression problem. A GP system performing symbolic regression takes a number of numerical input, output relations, called fitness cases, and produces a function or program that is consistent with fitness cases [12].

The term *symbolic* emphasizes that the goal is not finding the optimum parameters but the optimum functions in the form of expressions or symbolic representations.

The goal of symbolic regression is to find a numerically robust solution expression $E(x)$ which minimizes the following mean square error for a given sample set of $\mathcal{A}$ of independent input values x_k and dependent output values $d_k (k = 1, ..., |\mathcal{A}|)$ [13]:

$$MSE(E, \mathcal{A}) = \frac{1}{|\mathcal{A}|} \sum_{k=1}^{|\mathcal{A}|} (E(x_k) - d_k)^2 \tag{6}$$

The symbolic regression output equation provides a method for prediction, analysis and system modeling. Several examples of real world symbolic regression problems include chemical process modeling [8] [9], approximation and rainfall runoff modeling [2], data mining [6], design of experiments [1], and economic decision analysis [10] [4].

4. EXPERIMENTS

We test our hypothesis from section 3 with four experiments. The first experiment confirms robustness is a factor by testing varying depth GP environments, with the

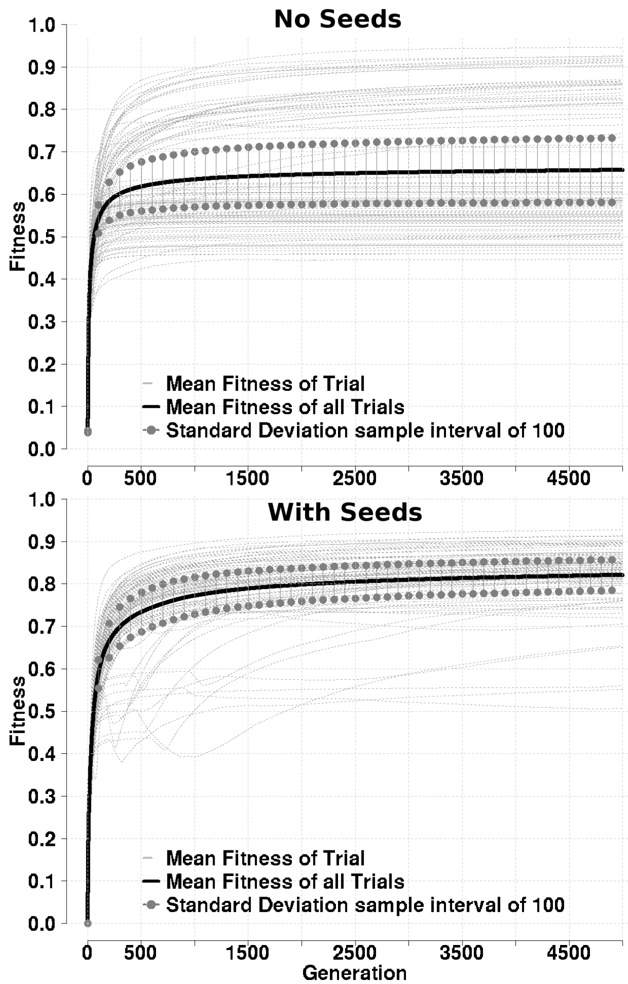

Figure 3: The top plot shows the mean population fitness *without seeds* for 100 runs, and a mean of those 100 runs are shown by the heavy black line. The bottom plot is similar information *with seeds* in the population.

expectation that shallow depth trees are less likely to support robustness. The second experiment confirms that given enough depth most of the seed individuals go extinct between generations 10 and 50. We test the effects elitism pressure in experiment 3 by varying the elitism parameter and confirming it has a significant influence of mean size of the best of run individual. Finally, experiment 4 applies a constant evolutionary pressure of mutation and confirms seed individuals that dominate early generations are replaced by more robust individuals.

4.1 Experiment 1: Robustness

Robustness in our context occurs through redundancy and or replication of the building blocks in an individual. Disruption of the individual by crossover, or mutation are deleterious to non robust individuals. Robustness decreases the probability that a critical subtree or building block will affect the overall fitness of the individual. In testing this hypothesis we analyze the effects of varying depth individuals with a symbolic regression experiment. We test for the frequency of the seed individual in the top 50 rank positions of the population for the symbolic regression problem.

In typical GP, a run terminates when any solution meeting the fitness criteria is satisfied. Recall we disable the run termination criteria to observe the population through generation 5000. This change in termination criteria allows observation of the population and fitness dynamics.

We expect shallow depth GP environments to support less robust individuals. In this experiment we insert a single tangent equivalent seed at the completion of generation 1. If shallow individuals are less robust, we expect the seed to dominate the solution for 4999 generations of the top 50 ranked positions. Since shallow depth trees limit robustness the seeds should dominate shallow environments, robust individuals should increase their ranking as the depth of the environment allows the creation of larger individuals. The problem environment parameters are shown in the experiment 1 column of Table 3. The experiment includes 10 iterations for each trial at depths of 3, 6, 9, and 12. The mean number of size 15 individuals from each trial is shown in column 2 of Table 1. The sweep ratio column in Table 1 shows the number of size 15 individuals divided by the total possible slots ($4999 \times 500 = 24950$).

Table 1: Robustness Demonstration Summary

Depth	Size 15 Ind. Count	Sweep Ratio
3	249736	0.9990
6	190589	0.7625
9	153101	0.6125
12	111753	0.4471

Results: The depth 3 sweep ratio of 0.9990 indicates few non seed individuals evolve to the top 50 ranked positions. As the depth increases, to 6, 9, and 12 we observe the significant decrease in the seed (size 15) individuals holding the top 50 positions of the population. At depth 12, 0.4471 size 15 individuals remain, or ≈55% of the top 50 ranked positions evolve to more robust individuals. We offer the following explanations:

- Larger trees allow individuals to grow introducing replication of building blocks and reduce the probability that a node or subtree disruption will produce a deleterious outcome. This confirms the hypothesis that less fit and more robust programs can achieve higher rates of survival.
- Table 1 demonstrates the brittleness of the tangent individual as the depth increases and more robust and larger individuals replace the seed individual.
- The handcrafted tangent equivalent function seed is brittle. We note that once disrupted, the tangent building blocks only survive a few generations.

4.2 Experiment 2: Robustness or Fitness

Experiment 2 tests the hypothesis that high fitness individuals are replaced by less fit and more robust individuals. Experiment 2 tests the size of the top ranked best fit individual for 100 trials. We identify the seed individuals by their size. To determine the influence of the seeds on descendants we manually inspect subsequent generations for evidence of subtrees from the original seed individuals. The experiment 2 column of Table 3 shows the GP environment parameters.

Results: Recall that the seed tangent equivalent solution is size 15. Initially the tangent individuals dominate the best solutions as shown in selected best of run statistics for the top 10 ranked individuals. However, starting in the region of generation 50 larger individuals of size 50 to 100 begin to appear and replace the tangent individuals. Some of the new, larger individuals have a lower fitness than the tangent individuals (if they had a higher fitness they would be inserted into the top of the ranking). Because they are much larger than the tangent individuals, it is reasonable to assume that they are more robust.

Thus, these results show that a more robust, but less fit solution, here represented by the individuals with a size greater than 50, can replace more fit, but less robust solutions even in a typical GP problem. The individual rank and size of the best 10 individuals from selected trials shown below allow one to visualize the robust or large individuals rank.

We analyzed data from all the trials, two trials are presented due to space considerations. The format of the matrix is row 1 indicates the generation, note Column 1 indicates the top 'n' individual, (row 2 is the rank 1 individual, row 3 is rank 2, etc.). Row 1 (columns 2-9) indicate the generation of the result. Trial 80 shows the tangent individual remaining at rank 1 for the remaining 4999 generations, and rank 2 is the Taylor series, all remaining individuals increase in size and hold the remaining positions. Trial 100 shows both seeds superseded by generation 5 and larger individuals dominating by generation 100.

	Gen->	5	10	50	100	500	1000	2500	5000
	1	15	15	15	15	15	15	15	15
	2	36	36	36	36	36	36	36	36
	3	22	22	22	22	22	22	22	22
	4	43	7	7	163	268	298	339	484
Trial 80	5	37	43	125	157	294	298	377	471
	6	45	36	51	158	298	298	382	469
	7	39	38	51	184	298	301	382	473
	8	39	38	99	177	298	298	383	469
	9	37	40	113	174	298	298	372	469
	10	8	37	113	158	298	298	382	469

	Gen->	5	10	50	100	500	1000	2500	5000
	1	5	5	57	151	429	463	541	579
	2	6	5	41	151	411	455	547	579
	3	6	5	45	151	411	451	547	553
	4	8	5	51	155	405	451	547	553
Trial 100	5	6	5	41	155	411	451	557	599
	6	16	5	29	155	411	451	557	599
	7	23	5	57	155	411	455	557	599
	8	9	5	65	155	411	455	557	599
	9	35	5	71	155	411	451	557	589
	10	15	5	61	151	411	451	557	589

Figure 4 shows the generation range of 1 to 100. The mean individual size of 15 shows the tangent equivalent function dominating the best of run individuals until generation 17. The individual trial plots shown as straight gray lines between size 7 and 40 are the seed individuals before crossover modifies their structure. A significant increase in the mean size of the best of run individual occurs at generation 18. Within the first 18 generations, the random individuals have poor fitness and the best of run is dominated by the seeds. The mean size of the individuals across all 100 trials is approximately 400 nodes at generation 5000.

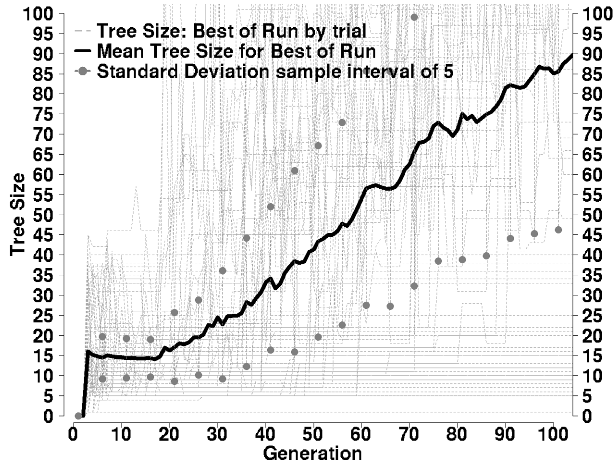

Figure 4: The best of run tree size with a smaller $x - axis$ and $y - axis$ range (0-100), and the mean size of all best of run individuals over all trials with standard deviation.

This result confirms the hypothesis that robust and larger individuals evolve replacing the seed individuals in a typical GP problem. In this experiment the jump in mean size at generation 18 suggests more robust solutions begin to dominate after generation 17. The size 15 seed solutions are replaced by non-seed individuals that grow to 4 times the seed size by generation 60 and in excess of 26 times the seed by generation 5000.

4.3 Experiment 3: Elitism Pressure

Elitism, also referred to as the elitist strategy directly copies the best n individuals from the current generation to the succeeding generation. The elite individuals are not subject to crossover or mutation. Elite individuals go extinct when higher performing individuals displace them in the elite set.

Various selection strategies are possible, frequently it is the best n individuals from a rank order by fitness. The number of elite individuals is typically in the range of 1 to 3, regardless of population size. Elitism often improves search while reducing exploration. DeJong suggests search versus exploration advantages in two types of landscapes: unimodal surfaces improve with an elitist plan, while multi modal surfaces degrade with an elitist plan [3].

Elitism protects the best of run individuals and eliminates the possibility of any future deleterious operations for the generation. These individuals are more likely to dominate best of run ranking for a greater number of generations than without elitism. Without elitism, the probability of selection for crossover reduces the chance of any individual maintaining its structure.

The seeds are compact human developed representations of sine equivalent functions that may be more susceptible to disruption from crossover. We hypothesize elitism will cause the transition from the compact seeds to more robust individuals to be delayed. The experiment 3 column of Table 3 shows the GP environment parameters.

Results: Consider the role of elitism and its selection pressure effect on the ability to develop robust individuals. The final size of the best of run individual is compared with no elitism with five trials that increase the elitism set size. Figure 5 shows an inverse relationship of the elitism set size with best of run individual size over a range of elitism values from 0 to 5. Larger elitism set sizes result in smaller best of run individuals. Table 2 and Figure 5 shows the decrease in individual size with increasing elitism.

The columns of Table 2 denote: E elitism value, $E\%$ elitism as percent of population, $\overline{s}_{BOR(5k)}$ is the mean size of the best of run at generation 5000 for 100 trials, $\overline{s}_{norm}$ is the mean size normalized to elitism of 0, $\overline{f}_{BOR(5k)}$ the Best of Run mean fitness of 100 trials measured at generation 5000, and $\overline{f}_{M(5k)}$ is the population mean fitness of 100 trials at generation 5000.

Table 2: Experiment 3 Summary

E	$E\%$	$\overline{s}_{BOR(5k)}$	$\overline{s}_{norm}$	$\overline{f}_{BOR(5k)}$	$\overline{f}_{M(5k)}$
0	0.000	395.29	1.000	0.860	0.764
1	0.002	281.95	0.713	0.906	0.787
2	0.004	178.14	0.451	0.930	0.819
3	0.006	132.12	0.334	0.951	0.836
4	0.008	87.54	0.221	0.973	0.850
5	0.010	69.59	0.176	0.977	0.858

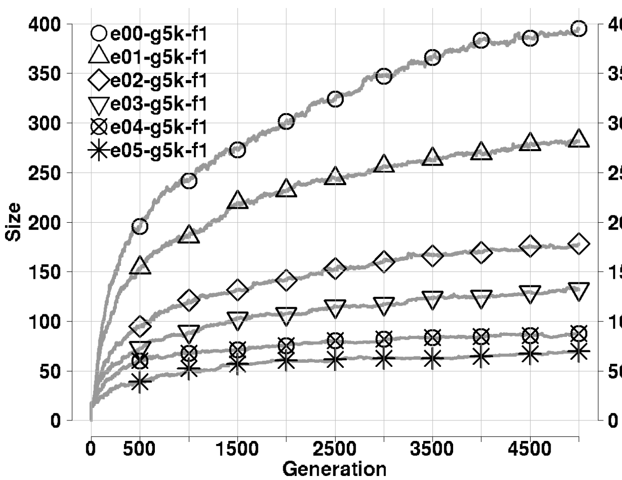

Figure 5: Best of run mean of 100 Trials at Elitism 0, 1, 2, 3, 4, 5. Increasing elitism causes significant changes to the best of run individual size.

Figure 6 plots four values against E on the $x-axis$. The $\circ$ symbol plots the values of $\overline{s}_{norm}$ which depict the decrease in size of the best of run individual relative to its size at generation 5000 with elitism of 0. The $\triangle$ symbol shows the best of run mean fitness for 100 trials. The $\times$ shows the best of population mean fitness for 100 trials.

The $\diamond$ shows Fitness Improvement Ratio F_{IR} given by Equation 7 where f_i is the fitness at a given elitism value, (e.g. 1, 2, ..), f_{e_0} is the fitness with no elitism, and R_{e_i} is the ratio of elitism to the population size at a specific elitism value, (e.g. 0.002 or 0.2% for elitism value of 1). The F_{IR}

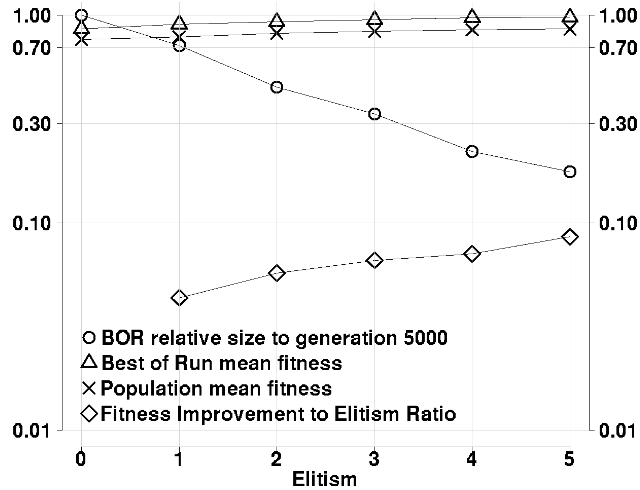

Figure 6: Elitism Effect on Size and Fitness. Increasing the elitism set size increases fitness and decreases robustness.

shows the fitness improvement for a given value of elitism set size. This result confirms the selection pressure from elitism leads to smaller individuals. If it is the case that larger individuals are more robust, than we lose robustness and gain more compact solutions.

$$F_{IR} = \frac{1}{\frac{f_i - f_{e_0}}{R_{e_i}}} \quad (7)$$

4.4 Experiment 4: Disrupting the Seeds

Robustness is a measure of the average fitness and/or phenotype change in response to a genotype change. In our hypothesis, larger individuals are more likely to be robust and improve their fitness when exposed to deleterious phenotype changes. Brittle individuals in the previous experiments use human designed equivalent functions for the transcendental sine function. We test the hypothesis by applying a continuous disruptive pressure through the addition of a mutation phase. Experiment parameters for the mutation phase test are shown in the experiment 4 column of Table 3.

The experiment tests whether the human developed seed individuals maintain a majority of the rank 1 positions, for 100 trials over 5000 generations. If the large individuals are more robust than the human developed sine equivalent functions, then the majority of large individuals will surpass the fitness of the brittle individuals.

Results: Figure 7 shows the best of run individual size histograms for generation 2, 50, 1000 and 5000. The $y-axis$ is the frequency for a given size of the best of run individual. Figure 7 generation 2, shows the majority of the best of run individuals with a size less than 50. By generation 50 shown in Figure 7, we note the size of the best of run individuals range from 50 to 300. In Figure 7 we show generation 1000 where only a few compact individuals remain from size 1 to 10, where the others have grown to a size of 75 to 800. At generation 5000, no rank 1 individuals that are less than size 100 remain. Long term survival shows that robustness is correlated with size.

Table 3: GP Experiment Parameters

Objective: Evolution of a functional equivalent for the transcendental sine function.					
Function set: Addition, division, factorial, power, square root, subtraction, and tangent.					
Parameter	Seed Test	Experiment 1	Experiment 2	Experiment 3	Experiment 4
Population size	500	500	500	500	500
Generations	5000	5000	5000	5000	5000
Maximum depth	17	3, 6, 9, 12	17	17	17
Initialization depth	20% to 60%	20% to 60%	20% to 60%	20% to 60%	20% to 60%
Initialization method	half and half	half and half	half and half	half and half	half and half
Fitness cases	20	20	20	20	20
Function evaluation range	-3.14 to 3.14	-3.14 to 3.14	-3.14 to 3.14	-3.14 to 3.14	-3.14 to 3.14
Internal crossover frequency	0.9	0.9	0.9	0.9	0.9
External crossover frequency	0.1	0.1	0.1	0.1	0.1
ERC	Enabled, 1-20	Enabled, 1-20	Enabled, 1-20	Enabled, 1-20	Enabled, 1-20
Crossover selection	Tournament, Size = 3	Tournament, Size = 3	Tournament, Size = 3	Tournament, Size = 3	Tournament, Size = 3
Crossover frequency	1.0	0.6	0.8888	1.0, 0.998, 0.996, 0.994, 0.992, 0.99	0.8888
Reproduction frequency	0.0	0.01	0.0	0.0, 0.002, 0.004, 0.006, 0.008, 0.01	0.0
Reproduction selection	Best	Best	Best	Best	Best
Mutation parameters					
Frequency	None	0.39	.1112	None	0.1112
Internal node frequency	NA	0.9	0.9	NA	0.9
Leaf node frequency	NA	0.1	0.1	NA	0.1
Replacement tree depth	NA	2	max = 4	NA	max = 4
Depth ramp	NA	NA	0 to 4	NA	0 to 4
Selection method	Best	Best	Best	Best	Best
Subtree creation	NA	half and half	half and half	NA	half and half

Reviewing these results show a clear and significant shift in the size of the individual as mutation and crossover events negatively affect the smaller individuals' fitness. These results indicate growth positively influences adaptation against the deleterious effects of mutation.

5. CONCLUSIONS

These results demonstrate robustness as an evolutionary dynamic, where robust individuals replace high fitness narrow peak or less robust individuals. The evolved individuals achieve resilience to crossover through growth. This increases the probability that non-critical sections of the individual will undergo change due to crossover or mutation. This confirms GP code growth seeks redundant building blocks through replication to improve survivability of crossover events. When early generation individuals evolve as solutions, one may be obtaining a fit solution and not necessarily the most robust solution against evolutionary disruption.

Achieving robustness allows evolution to select to higher fitness individuals who are affected less by perturbation of the environment. This suggests that code growth is a requirement for robustness in complex GP solutions. Limiting the genome size may affect robustness and introduce equilibrium between exploration and robustness. Without the ability to improve through crossover events and individual would effectively be jumping around the optimal point of the narrow peak landscape. Each evolutionary event can potentially reduce the fitness of the individual to make it a candidate for replacement.

The results offer additional areas for future research. In the context of evolution there appears to be two paths possible. In the first path, creation of high fitness and brittle individuals that propagate to future generations and at the same time likely to suppress less fit individuals. The second path is where individuals evolve robustness attributes and avoid extinction.

If growth is necessary to achieve robustness, what is the upper limit? If one can describe the balance between these two modes as it represents a phase change with the seed individuals as shown by Figure 4, perhaps one can determine the minimum number of generations for robustness.

Can the robustness trade space be modeled for even more complex problems in an aim to understand if growth is limited by artificial constraints and thereby constrains the ability of GP to evolve a solution? Do researchers need a balance between robustness and the first solution when considering the more robust solution is likely to take more generations and memory to evolve?

6. REFERENCES

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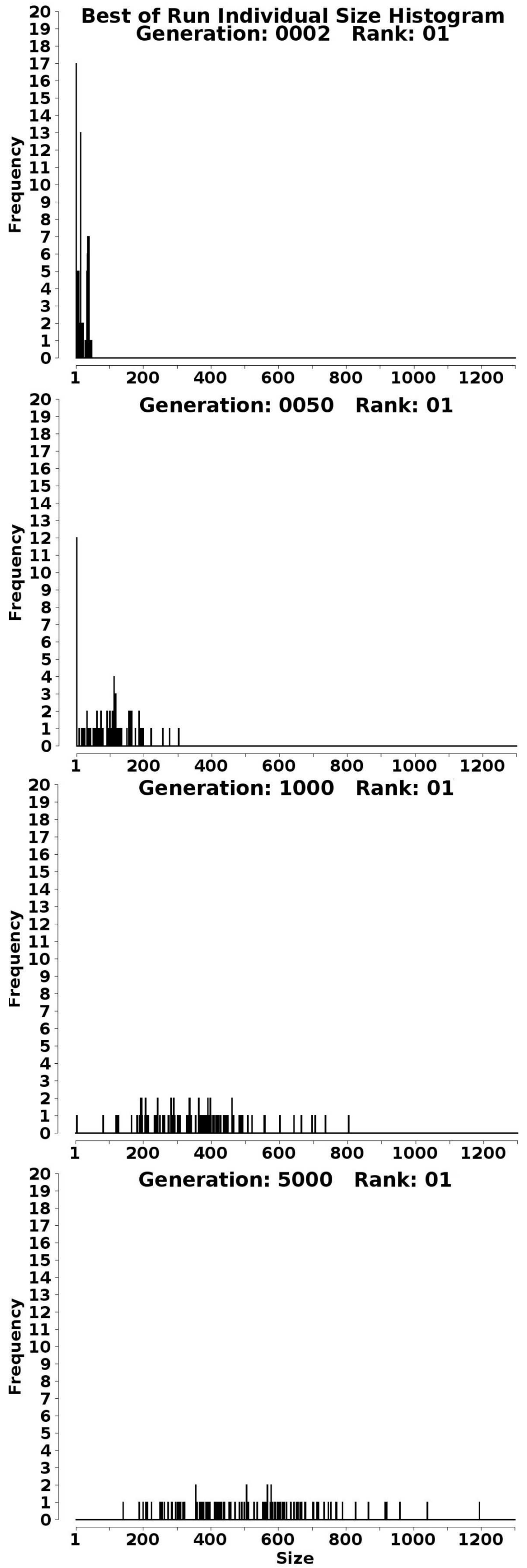

Figure 7: Generation 2, 50, 1000 and 5000, Rank 1 best of run individual size histogram shows the transition from the seed individuals in generation 2 to significantly larger and more robust individuals through generation 5000.

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