A Comparison of Evolutionary Computing Techniques Used to Model Bi-Directional Reflectance Distribution Functions

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ABSTRACT

Bi-Directional Reflectance Distribution Functions, or BRDFs, are used in many fields including computer animation modeling, military defense (radar, lidar, etc.), and others. This paper explores a variety of approaches to modeling BRDFs using different evolutionary computing (EC) techniques. We concentrate on genetic programming (GP) and in hybrid GP approaches, obtaining very close correspondence between models and data.

The problem of obtaining parameters that make particular BRDF models fit to laboratory-measured reflectance data is a classic symbolic regression problem. The goal of this approach is to discover the equations that model laboratory-measured data according to several criteria of fitness. These criteria involve closeness of fit, simplicity or complexity of the model (parsimony), form of the result, and speed of discovery. As expected, free form, unconstrained GP gave the best results in terms of minimizing measurement errors. However, it also yielded the most complex model forms. Certain constrained approaches proved to be far superior in terms of speed of discovery. Furthermore, application of mild parsimony pressure resulted in not only simpler expressions, but also improved results by yielding small differences between the models and the corresponding laboratory measurements.

Categories and Subject Descriptors

I.2.8 [**Artificial Intelligence**]: Problem Solving, Control Methods, and Search – *heuristic methods, dynamic programming.*

General Terms

Algorithms

Keywords

Genetic programming, evolutionary computation, hybrid genetic programming, symbolic regression, Bi-directional reflectance distribution function, BRDF, parsimony, Phong model

1. INTRODUCTION

The bi-directional reflectance distribution function (BRDF) measures the radiance reflected from a surface as a function of the incident and the viewing angles. When measured from a single direction it is equal to the radiance reflected from the surface divided by the incident radiance.

To model the reflected radiance from an object we should ideally know the characteristics of the small surface patch reflecting the incident energy. In addition, we should know the geometry, illumination strength and frequency distribution, polarization orientation of sources and detectors, source locations, velocity of sources, objects, and detectors, surface texture, and other factors. For this paper we are interested only in modeling the patch reflectance measured under laboratory conditions where these other variables are held constant. Laboratory measurements were made for fixed illumination and detector locations with specific sample materials (see Figure 1). Technically this is a mono-directional reflectance distribution function (MRDF) rather than a BRDF, but the latter term is more widely used. After these reflectance measurements were made awe needed to model the BRDF as a two-parameter set of equations. In most of the examples used here, only a single angle was varied at a time and both the illumination source and the detector were in a plane normal to the surface.

A Huntsville-based defense contractor, sought to model BRDF measurements for a variety of surfaces and approached the US Army Space and Missile Defense Command's Advanced Research Center (ARC) for assistance. A widely used, empirically-derived model, the Phong model, was considered adequate to model the reflectance data. The principal difficulty resided in finding the right set of model parameters that would yield the measured data with sufficient accuracy.

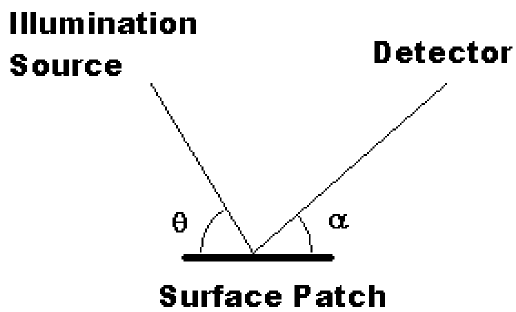

Figure 1. Relation between incidence angle (θ) and reflection angle (α)

The Phong model has several limitations, mainly the fact that it was developed for different wavelengths—the visible spectrum—and under different conditions. Another difficulty with it is that it is not based on any fundamental physical parameters of the surface under study. This allows for the possibility of using other models, with different mathematical forms, to replicate the laboratory-measured data. Such models were also developed taking ease-of-integration into consideration.

In this paper we will illustrate the relative importance of a variety of EC techniques to help with the modeling of a real-world data problem. We will show how it is possible to use parsimony pressure to force GP to develop models fitting specific mathematical forms and how this can expedite convergence.

1.1 Description of Data Sets

A collection of thirteen sample data sets were presented to the COLSA Corporation authors at the ARC, each data set comprising just a few measurements (typically 17 or so) for initial evaluation. Each data set represented the reflectance for a different surface at angles varying from 0 to 90 degrees.

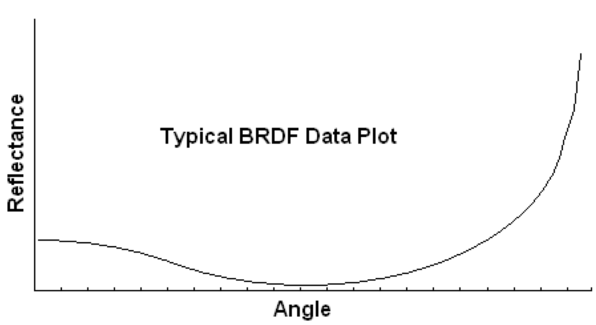

Figure 2. Typical shape of the BRDF data measured in the laboratory for different surfaces.

Figure 2 provides a typical shape of the BRDF function in the data sets as measured in the laboratory for a reflecting surface. Since the data itself is not available for publication at this forum, we cannot provide specific details of the BRDF measurements acquired for the different surfaces under study. Only the general shape of the measurements is given in Figure 2.

2. FORMULATION AS A PROBLEM IN EVOLUTIONARY COMPUTING

The discovery of equations to model discrete data points using GP is a classic symbolic regression problem. In this paper, we limit discussion to the simple case of a single independent variable since the data provided to us involved only the variation of a single angle at a time. We had to discover equations that best fit the measurements, but were also a function of just a single variable.

Other factors also entered into the task requirements: speed of discovery, simplicity or complexity of equations (so-called parsimony), form of equations, and the ability to easily integrate the resulting equations (also related to parsimony).

2.1 Error Function

The quality of the fit was measured as the sum of the absolute values of the differences between the measured and modeled data values as shown in Figure 3. Absolute error was chosen as the error measurement for this paper.

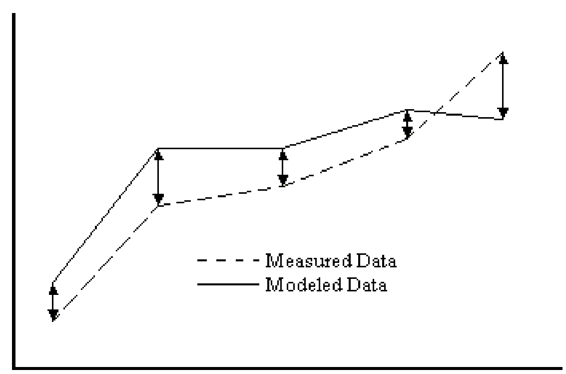

Figure 3. Fitness function is calculated as the total sum of the errors at each point. Lower fitness values indicate a better match between model and measurements.

3. DIFFERENT MODELS

In the next sections we will show how we used different EC techniques to find parameters for a variety of models to fit the measured reflectance data and to develop models of different mathematical form. We will discuss our work with the Phong model, a requirement of the customer. However, we will also discuss how the GP results were made to fit a trigonometric, a power, and an exponential model. In addition, we will review the results of using parsimony and using hybrid models that combined classical GP with the previous model forms.

3.1 Fixed Phong Model

The Phong model equation was used as the basis for comparing all results. Although we modeled multiple forms of the Phong equation, only the main form is reported herein for brevity:

$$\mathbf{R(\theta) = a * \cos(\theta) + b * \cos^{c}(\theta - \beta)}$$

We used classic evolutionary strategy (ES) to evolve good solutions for the parameters **a**, **b**, **c**, and **β**. (Here by ES, we simply mean a real-valued genetic algorithm.) Since this equation has a smooth fitness landscape, we are somewhat confident that we quickly found essentially the optimal values for matching the data. Thus the Phong result serves effectively as a basis for comparing the other modeling techniques.

3.2 Genetic Programming Model

The second approach was to use classic GP to evolve an equation that models the measured data. A variety of operator sets were made available and used. For simplicity, we limited the operator set to the four basic arithmetic operators, plus logarithm, exponentiation, and trigonometric functions. Exponentiation was based on real numbers.

The customer had requested an ability to *easily* integrate the resulting discovered equation (symbolically). Thus a degree of parsimony pressure was applied to force the results to be simpler.

3.3 Other Constrained Models

After modeling with GP, we also wanted to discover some constrained forms of solution where the evolved solution comprised trigonometric, exponential, or polynomial terms, or various combinations thereof. Again this is basically an ES type formulation where the parameters are discovered to fit a fixed specified equation form. Combinations of the following forms were then used:

Trigonometric: $\mathbf{k_1 * \sin(k_2 * x + k_3 * y + k_4 * z + ... + k_5)}$

Power: $\mathbf{k_1 + k_2 * x + k_3 * y + ... + k_8 * x\hat{}2/2! + k_9 * x * y/2! + ... + k_{15} * x\hat{}3/3! + ...}$

Exponential: $\mathbf{k_1 * 2.0\hat{}(k_2 * x + k_3 * y + k_4 * z + ... + k_5)}$

However we only have a single independent variable, **θ**, so the equations simplify to:

Trigonometric: $\mathbf{k_1 * \sin(k_2 * \theta + k_3)}$

Power: $\mathbf{k_1 + k_2 * \theta + k_3 * \theta\hat{}2/2! + k_4 * \theta\hat{}3/3! + ...}$

Exponential: $\mathbf{k_1 * 2.0\hat{}(k_2 * \theta + k_3)}$

where the $\mathbf{k_n}$ values are the parameters to be discovered.

Although not as good as GP, these models evolved very fast to values nearly as good as GP.

3.4 Hybrid Models

The tool suite used allows for combining the various above models into a hybrid. In particular, a number of genes in each chromosome were allocated for the trigonometric, polynomial, and exponential form while the remaining genes were used to encode a classic GP solution. The fitness value used in the error comparison was simply the sum of the constrained and GP evaluations.

3.5 Parsimony

Both the GP and hybrid models allow application of a degree of parsimony that applies a fitness penalty to overly complex results during the selection process. Here complexity can be a variety of measures such as expression depth, size of evolved constants, operator type, non-uniqueness, and several others. Furthermore, each of these criteria may be applied in different strengths.

Parsimony was applied only during the selection process so that elsewhere the resulting GP-discovered solutions would be displayed with their inherent fitness values. Only during the selection process was the fitness penalty applied.

After evolving for a while without parsimony, parsimony factors were calculated on the result and these became the base settings. Then an overall constant multiplier was applied to each of these parsimony factors. This constant multiplier is referred to as the parsimony pressure. The higher the parsimony pressure, the simpler one could expect the resulting equation to be, although most likely less fit than the equation evolved at lower parsimony pressure.

4. RESULTS

Table 1 provides a comparison of the results obtained when using the parsimony, Phong model, and genetic programming techniques. The comparison indicates, as one might expect, that the unconstrained GP model achieved the best result when compared to the Phong equation for the same data set. This was true in every case and by a wide margin. The unconstrained form, however, produced very complex models that would be different every time the evolution was allowed. In addition, it tended to "over fit", i.e., to adjust even to the measurement errors. This was in part due to the small number of data points available in the laboratory measurements.

The evolved constrained forms combined up to four trigonometric terms, three polynomial terms, and one exponential term, but most typically we used fewer terms. The results were not only easy to evaluate, and to integrate symbolically.

The hybrid results proved to be somewhat of a surprise, being similar in quality to GP while being quite a bit simpler. This is especially true with higher parsimony pressure. We found the hybrid approach was also highly desirable for a different non-BRDF data analysis application.

Finally, one of the bigger surprises was that for this problem a very mild application of parsimony pressure produced superior match to the measured data, i.e., a lower fitness error value. One might have expected the fitness value to suffer with *any* parsimony pressure. At higher-pressure parsimony settings, the model forms became simpler, but at the expected cost of being less fit.

Table 1. Comparison of techniques. Ratio of Phong model and hybrid results to the best GP results

Data Set	Phong/GP	Hybrid/GP
0	3.70	24.27
1	3.74	11.26
2	4.95	15.70
3	2.48	10.97
4	6.92	1.15
5	6.53	1.15
6	6.65	2.54
7	8.91	27.45
8	2.68	33.01
9	4.70	19.80
10	3.28	1.98
11	2.36	1.62
12	3.49	0.99
Average	4.65	11.69

Note that in Table 1, for data set 12, the hybrid result was actually superior to the GP result, presumably because this set of measurements actually fit this model better and the GP runs were too short to have discovered this form.

Table 2 presents parsimony results that were quite interesting. With the mildest parsimony setting tried, it was a unexpected to see a four percent improvement (0.96 ratio) when it was expected that the simplicity would result in a cost in fitness value.

At the heaviest parsimony pressure tried, the fitness value suffered by about 78 percent, but was nevertheless an improvement over the Phong model. The evolved expressions, however, were far simpler than the equations derived without any parsimony pressure.

5. ACKNOWLEDGMENTS

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Table 2. Results with and without mild parsimony pressure.

Data Set	With	Without	Ratio
0	0.005320	0.007182	0.7407
1	0.004567	0.005378	0.8492
2	0.002796	0.003897	0.7175
3	0.008709	0.010284	0.8468
4	0.003821	0.004135	0.9241
5	0.004696	0.004249	1.1052
6	0.002484	0.002423	1.0252
7	0.009657	0.009074	1.0642
8	0.004882	0.004642	1.0517
9	0.004395	0.005888	0.7464
10	0.014761	0.010562	1.3976
11	0.014320	0.017165	0.8343
12	0.020476	0.017204	1.1902
			Ave = 0.961007

6. SUMMARY

EC is well suited to find the parameters necessary to fit a particular model to measured data. In particular, ES was very useful in determining the parameters to be used in the Phong model to reproduce laboratory measured BRDF data for different surface materials. In addition, GP used in conjunction with parsimony pressure could provide alternative models for the BRDF laboratory data that had any desired mathematical form. This research suggests that BRDF models based on surface material physical characteristics could be developed and adjusted with relative ease to fit measured reflectance data.