Analysis of Greedy Heuristics and Weight-Coded EAs for Multidimensional Knapsack Problems and Multi-Unit Combinatorial Auctions

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Categories and Subject Descriptors: I.2.8 [Artificial Intelligence]: Problem Solving and Search

General Terms: Algorithms

Keywords: multidimensional knapsack problem

1. INTRODUCTION

The multidimensional knapsack problem (MDKP) assumes one knapsack being packed with a number of items x_j such that the total profit $\sum p_j$ of the selected items is maximized. Each item has m different properties r_{ij} $(i = 1, \ldots, m; j = 1, \ldots, n)$ consuming capacity c_i of the knapsack which must not be exceeded. Recently [1], it was noticed that the IP formulation of the MDKP is equal to the one of the winner determination problem (WDP) in the context of multi-unit combinatorial auctions (CA). In CAs bidding is allowed on bundles of goods, which allows bidders to express synergies between goods they want to obtain.

This paper compares the performance of greedy heuristics and Raidl's weight-coded EA [3] for instances of WDPs and MDKPs. Greedy heuristics reach high-quality solutions very fast, while the EA can further improve the solutions at the expense of much higher computational effort.

2. RESULTS

The greedy heuristics, we analyze, first sort the items decreasingly according to some criteria. Then, they construct a solution by adding the items one after another to the knapsack as long as no restrictions are violated. From the literature, we chose the following four criteria for sorting:

1. *Normalized Bid Price (NBP):* $\frac{p_j}{\sqrt{\sum_{i=1}^{m} r_{ij}}}$
2. *Scaled NBP (SNBP):* $\frac{p_j}{\sum_{i=1}^{m} \frac{r_{ij}}{c_i}}$
3. *Relaxed LP Solution (RLPS): solutions of the relaxed LP* ($x_j \in [0, 1]$)
4. *Shadow Surplus (SS):* $\frac{p_j}{\sum_{i=1}^{m} a_i r_{ij}}$, *where* a_i *are the solutions of the dual LP.*

The greedy heuristics and the weight-coded EA were tested on MDKP instances from the OR library[1], multi-unit instances from the CA test suite CATS [2], and instances generated with the method presented by Leyton-Brown et al. [2].

Table 1: Performance of primal greedy heuristics.

	m	n	α	NBP gap	SNBP gap	RLPS gap	SS gap
1	5	100	.25	8.706	6.525	1.478	1.557
2			.50	4.163	2.764	0.835	1.175
3			.75	2.386	1.695	0.877	0.981
4	5	500	.25	6.945	3.101	0.383	0.326
5			.50	3.587	2.108	0.212	0.179
6			.75	2.221	1.089	0.129	0.116
7	14	2500	.00	7.350	12.209	6.482	6.722
8	595	1500	.05	6.328	16.131	4.110	4.565

Table 1 shows the high performance of the four greedy heuristics (MDKP: lines 1-6; MUCA: lines 7,8), where *gap* is in percent to the optimum. We confirmed the hypothesis that for all considered MDKP instances relaxed capacity restrictions (higher α) cause lower gaps (ANOVA, 5% significance level, $M = 3.74$ vs. 1.86 vs. 1.07, $F = 8.749$, $p < 0.001$). This result contradicts work by Raidl and Gottlieb [4], who concluded that the trend does only hold for gaps to the relaxed LP solution, since it becomes tighter.

In a next step we tested Raidl's weight-coded approach which uses the SS heuristic as bias for focussing the search [3]. The average gap of 1.86% of the SS heuristics to the relaxed optimum was outperformed by a gap of 0.62%. Yet, compared to the few ms runtime of the SS heuristics, with 154 seconds per run, the EA was much slower.

3. REFERENCES

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GECCO'07, July 7–11, 2007, London, England, United Kingdom.
ACM 978-1-59593-697-4/07/0007.