# PSO with Randomized Low-Discrepancy Sequences 

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#### Abstract

We initialize a global-best particle swarm with a Halton sequence, comparing it with uniform initialization on a range of benchmark function optimization problems. We see substantial improvements in performance, particularly with high complexity problems /small populations. Halton initialization yields equivalent performance to uniform initialization with substantially smaller populations.


## Categories and Subject Descriptors

I.2.8 [Computing Methodologies]: Problem Solving, Control Methods and Search - heuristic methods.
General Terms: Experimentation

## Keywords

Randomized Halton Sequence, Particle Swarm Optimization.

## 1. INTRODUCTION

Several studies in evolutionary computation (EC) have shown that different pseudo-random generators can have significant effects on performance [1-3]. However they still assume that uniform random initialization is appropriate. In Monte Carlo simulations, randomized low discrepancy sequences $[4,5]$ have been found more effective. We trialed a randomized Halton sequence [6] for particle swarm initialization (PSO, [7]).

## 2. EXPERIMENTS AND RESULTS

We implemented the glbest algorithm [7], with the initial swarm values generated in two ways: uniform pseudo-random (U-PSO) with the random number generator of Press et al [8], and a randomized Halton (SH-PSO) sequence. Otherwise, the two algorithms are identical, both using Press' generator for later stages. The benchmark functions used were Spherical ( $\mathbf{f}_{1}$ ), Quadratic ( $\mathbf{f}_{2}$ ), Ackley ( $\mathbf{f}_{3}$ ), Griewank ( $\mathbf{f}_{4}$ ), Rastrigin ( $\mathbf{f}_{5}$ ), and Rosenbrock ( $\mathbf{f}_{6}$ ) [2].

We used a fixed budget, 20000 function evaluations, and swarm sizes 50,100 , and 200 . We varied problem dimensionality over $10,15,20,25,30,35$, and 40 . For each combination, we ran 100 runs and recorded mean best performance.. Due to space limits we only show the results (Table 1) with swarm size 50, and dimensionalities $10,20,30$, and 40 .

## 3. CONCLUSIONS AND FUTURE WORK

Halton sampling gives better results than uniform, particularly on problems of high dimensionality, with limited swarm size. Halton gives better exploration, equivalent to an increase in population. We plan to investigate this further, through diversity and local fitness landscape studies, in subsequent work.

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Table 1. Average Best Solution Values Number of particles $=\mathbf{5 0}$ ( $\mathbf{( 0 0 0}$ generations)

|  | Dim. | SH-PSO | U-PSO |
| :--- | :--- | :--- | :--- |
| $\mathbf{f}_{1}$ | 10 | 0 | 0 |
|  | 20 | $0.21 \pm 0.31$ | $0.24 \pm 0.36$ |
|  | 30 | $250 \pm 214$ | $316 \pm 224$ |
|  | 40 | $(5.0 \pm 2.2)^{*} 10^{3}$ | $(7.6 \pm 3.6)^{*} 10^{3}$ |
| $\mathbf{f}_{2}$ | 10 | $0.003 \pm 0.007$ | $0.003 \pm 0.005$ |
|  | 20 | $(7.2 \pm 3.9)^{*} 10^{3}$ | $(8.0 \pm 4.8)^{*} 10^{3}$ |
|  | 30 | $(1.4 \pm 0.4)^{*} 10^{5}$ | $(1.7 \pm 0.6)^{*} 10^{5}$ |
|  | 40 | $(6.8 \pm 2.2)^{*} 10^{5}$ | $(7.0 \pm 2.2)^{*} 10^{5}$ |
| $\mathbf{f}_{3}$ | 10 | $(1.1 \pm 1.8)^{*} 10^{-6}$ | $(1.0 \pm 1.3)^{*} 10^{-6}$ |
|  | 20 | $0.33 \pm 0.39$ | $0.29 \pm 0.33$ |
|  | 30 | $5.5 \pm 1.3$ | $5.8 \pm 1.3$ |
|  | 40 | $12.4 \pm 1.6$ | $13.7 \pm 2.0$ |
|  | 20 | $0.079 \pm 0.037$ | $0.080 \pm 0.043$ |
|  | 30 | $1.56 \pm 0.48$ | $0.23 \pm 0.19$ |
| $\mathbf{f}_{5}$ | 40 | $12.2 \pm 4.9 \backslash$ | $1.71 \pm 0.50$ |
|  | 10 | $3.3 \pm 2.1$ | $3.5 \pm 2.4$ |
|  | 20 | $33.2 \pm 9.5$ | $34.5 \pm 10.9$ |
|  | 30 | $131 \pm 34$ | $137 \pm 37$ |
|  | 40 | $336 \pm 78$ | $397 \pm 73$ |
| $\mathbf{f}_{6}$ | 10 | $8.2 \pm 7.9$ | $10.23 \pm 9.8$ |
|  | 20 | $32.4 \pm 18.0$ | $38.4 \pm 22.3$ |
|  | 30 | $194 \pm 159$ | $333 \pm 401$ |
|  | 40 | $(8.3 \pm 1.6)^{*} 10^{3}$ | $(10.8 \pm 9.5)^{*} 10$ |
|  |  | 3 |  |

