Configuring an Evolutionary Tool for the Inventory and Transportation Problem

Anna I Esparcia-Alcazar*
Lidia Lluch-Revert
Ken C Sharman
Instituto Tecnologico de Informatica
Edif. 8G Acc. B,
Ciudad Politecnica de la Innovacion
46022 Valencia, Spain
{anna, lillure, ken}@iti.upv.es

Manuel Cardos
Departamento de Organizacion de Empresas
Universidad Politecnica de Valencia
Camino de Vera s/n
46022 Valencia, Spain
mcardos@doe.upv.es

J.J. Merelo
Departamento de Arquitectura y Tecnologia de Computadores
Universidad de Granada
Periodista Daniel Saucedo Aranda s/n.
18071 Granada, Spain
jmerelo@geneura.ugr.es

ABSTRACT

EVITA, standing for **Ev**olutionary **I**nventory and **T**ransportation **A**lgorithm, aims to be a commercial tool to address the problem of minimising both the transport and inventory costs of a retail chain that is supplied from a central warehouse. In this paper we study different issues involved in finding the appropriate settings for EVITA, so that it can be employed by a non-expert user over wide range of problems.

The aim is not to define a new algorithm for resolution of the ITP, but to determine whether it is possible to find a set of input parameters that can provide good results on a wide range of problem configurations, hence eliminating the need for user adjustment once the tool is employed in a commercial setting.

We focus on the influence of three parameters: the population size, the tournament size and the mutation probability. After extensive experimentation and statistical analysis we are able to find a good configuration for the three factors.

Categories and Subject Descriptors: F.2.2 [Theory of Computation]: Nonnumerical Algorithms and Problems

General Terms: Algorithms.

Keywords: Inventory and Transportation Problem, evolutionary algorithms.

1. INTRODUCTION

The Inventory and Transportation Problem was introduced by Cardós and García-Sabater in [5] and can be defined as the minimisation of the inventory and transportation costs of a retail chain served from a central warehouse, subject to the operational constraints taking place at the shop level. As a consequence[1] several *delivery frequencies* are allowed for each shop and the inventory costs can be calculated accordingly, assuming a periodic stock review policy for the retail chain shops. For a given delivery frequency, expressed in terms of number of days a week, there can also be a number of *delivery patterns*, i.e. the specific days of the week in which the shop is served. Once these are established, the transportation costs can be calculated by solving the vehicle routing problem (VRP) for each day of the week.

This research is based on a work made for DRUNI SA, a major regional cosmetics retailer in Spain.

In [10] Esparcia-Alcázar *et al.* presented the first work on a hybrid evolutionary tool that could optimise simultaneously both inventory and transport costs. Our objective in this paper is making such a tool amenable to commercial use by finding an appropriate configuration that eliminates the need for user adjustment.

The rest of the paper is laid out as follows. In Section 2 the problem is explained and the state of the art outlined. Section 3 describes the characteristics of the EVITA tool. Section 4 is devoted to the experiments carried out, the analysis of which is done in Section 5. Finally, conclusions and further work are given in Section 6.

2. PROBLEM DESCRIPTION AND RELATED LITERATURE

The Inventory and Transportation Problem (ITP) arises when a company owns both a chain of shops and the warehouse that supplies them. The aim is to minimise total inventory and transportation costs subject to the operational constraints at the shop level, basically:

1. A periodic review stock policy applies for the shop items. So, a target cycle service level has been established for every item category, which is usually lower for the slow-moving ones. As a consequence, stock out is allowed.

*Corresponding author.

[1] This will be explained later

GECCO'07, July 7–11, 2007, London, England, United Kingdom.

2. Every shop has a limited stock capacity.

3. The retail chain tries to fulfil a backorder in few days, so there is a lower bound for delivery frequency depending on the target client service level.

4. Most of the expected stock reduction between replenishments is due to very high moving items. This reduction cannot be too high in order to avoid two problems: (a) the unappealing empty-shelves aspect of the shop just before the replenishments; and (b) replenishment orders too large to be placed on the shelves by the shop personnel in a short time compatible with their primary selling activity.

5. Conversely, the expected stock reduction of very high moving items must be high enough to perform an efficient allocation of the replenishment order.

6. Sales are not uniformly distributed along the horizon. Hence, in order to match deliveries to sales, only a given number of delivery patterns are allowed for every feasible frequency.

Because a pattern assumes a given frequency, the problem is limited to obtaining the optimal patterns (one per shop) and set of routes (one per day). The optimum is defined as that combination of patterns and routes that minimises the total cost, which is calculated as the sum of the individual inventory costs per shop (inventory cost) plus the sum of the transportation costs for all days of the week (transport cost). These two objectives are in general contradictory: the higher the frequency of delivery the lower the inventory cost, but conversely, a higher frequency involves higher transportation costs.

Additionally we will consider that: (a) the shops have no time windows, i.e. the deliveries can take place at any time; (b) the fleet is considered homogeneous (i.e. only one type of vehicle is used); (c) the number of vehicles is unlimited (as they are subcontracted); (d) no single shop can consume more than the capacity of any one vehicle, i.e. a shop is served by one and only one vehicle; (e) the transport cost only includes the cost per km as a consequence of the transport being subcontracted; and (f) the load is containerised, so the number of units is an integer. The problem data can be found in the Appendix.

Both inventory and transportation management have received lots of attention in the logistics literature; however, this is not the case for the joint problem. Previous research on it can be classified depending on whether or not the vehicle routing problem (VRP) is included. For example, Constable and Whybark [8] consider a single product controlled by an order-point system and a cost per unit transportation cost. Burns et al. [3] develop an approximate analytical method for minimising inventory and transportation costs under known demand, based on an estimation of the travel distance. Benjamin [2] analyses production, inventory and transportation costs in a capacitated network, yet once again transportation costs are assumed to be proportional to the units moved. Speranza and Ukovich [17] focus on a multi-product production system on a single link. Ganeshan [13] presents a (s, Q)-type inventory policy for a network with multiple suppliers by replenishing a central depot, which in turn distributes to a large number of retailers; the author considers transportation costs, but only as a function of the shipment size. Qu et al. [15] deal with an inbound material-collection problem so that decisions for both inventory and transportation are made simultaneously; however, vehicle capacity is assumed to be unlimited so that it is solved as a traveling salesman problem (TSP). Finally, Zhao et al. [19] present a modified economic ordering quantity for a single supplier-retailer system in which production, inventory and transportation costs are all considered.

The inventory routing problem (IRP) belongs to the category of models including the VRP. This problem arises when the vendor delivers a single product and implements a Vendor Inventory Management policy with its clients, so that the vendor decides the delivery (time and quantity) in order to prevent the clients from running out of stock while minimising transportation and inventory holding costs. This is the case of Campbell and Savelsbergh [4]. Unfortunately retail chains do not fall into this category, as thousands of items are involved.

Also related to the VRP is the periodic vehicle routing problem (PVRP), which appears when customers have established a predetermined delivery frequency and a combination of admissible delivery days within the planning horizon. The objective is to minimise the total duration of the routes, while the restrictions usually involve a limited capacity of the delivery vehicles and a maximum duration of each itinerary. See for instance, [9] and [18].

The Inventory and Transportation Problem (ITP) can be viewed as a generalisation of the IRP to the multiproduct case. Additionally, the ITP can also be viewed as a generalisation of the PVRP, as it includes inventory costs[2] and a set of delivery frequencies instead of a unique delivery frequency for each shop.

3. EVITA: A TWO-LEVEL TOOL FOR THE ITP.

3.1 Introducing the tool

As explained above, the ITP consists of finding:

- The *optimal frequency*, f, or number of days a week to serve each shop
- The *optimal pattern*, p, for that frequency. A pattern p represents a set of days in which the shop is served.
- The *optimal routes* for each day of the working week, by solving the VRP for the shops allocated to that day by the corresponding pattern

Previous approaches to problems in this field have presented a number of drawbacks: high computational expense, a limited exploration of the search space and, more importantly, that they are not practical tools to use from a business point of view. To circumvent these problems, [10] proposed a hybrid evolutionary algorithm that calculated patterns for the shops while the routes where calculated employing a simple algorithm.

The EVITA tool that we present here is based on that idea. In this paper we investigate different configurations of parameters for the evolutionary algorithm (top level algorithm). Additionally, we have substituted the routing al-

[2]The PVRP can be seen as an ITP in which the inventory costs are zero.

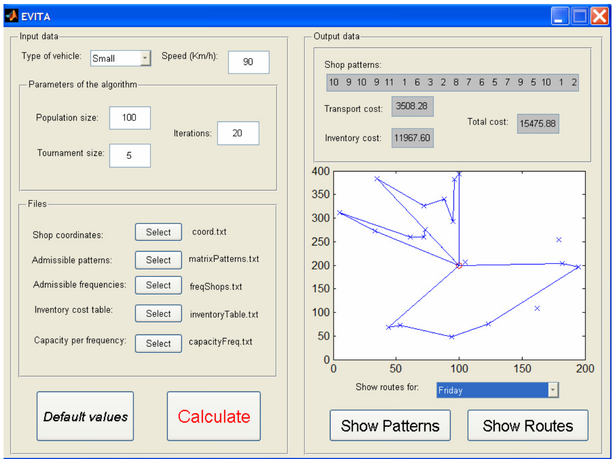

Figure 1: Graphical user interface of the EVITA tool

gorithm used in [10] (lower level algorithm) with one that obtains better results.

We perform a factorial design of experiments with different combinations of parameters and test them extensively on eight problem instances that have been generated by selecting difficult routing problems extracted from the literature and adding our own values of inventory costs. Statistical tests are then used to compare the different options.

3.2 The evolutionary algorithm

In our representation the chromosomes are vectors of length equal to the number of shops, whose components p_i, or genes, are integers representing a particular delivery pattern,

$$p_i \in [1, P]$$

where P is the total number of admissible patterns (see Table 6) and $i \in [1 \dots nShops]$.

The fitness of such a chromosome is calculated as the sum of the associated inventory and transportation costs,

$$f = InventoryCost + TransportCost \qquad (1)$$

To calculate the inventory costs, given the patterns for each shop, we know the associated delivery frequency and with this we can look up the inventory cost per shop, see Table 7. To obtain the transport cost we use the CWLS algorithm, as explained in Section 3.3.

The pseudo-code for the evaluation function is given in Table 1.

3.3 The routing algorithm

There exists a vast literature on both the Vehicle Routing Problem, VRP, and the periodic VRP (PVRP) (see for instance [18]). It is not the objective of this work to propose a new algorithm for calculation of routes, nor to employ the best such algorithm found so far. On the contrary, our objective is to find an algorithm that can provide good results in a commercially acceptable timescale.

In [10] the daisy algorithm was employed, which is a simple and easy to implement algorithm, but presenting many disadvantages. For instance, it is not appropriate when not all the shops are interconnected; further, it generates non-overlapping "petals". In [11] the same authors studied the

Procedure Evaluate
input:
  Chromosome $\{p_1, \dots, p_{nShops}\}$,
  problem data tables
  $costPerKm$
output: Fitness
  [Calculate inventory cost]
    InventoryCost $= 0$
    for $i = 1$ **to** $nShops$
      Look up frequency f_i for pattern p_i
      Look up cost c_i for shop i and frequency f_i
      $InventoryCost+ = c_i$
  [Calculate transportation cost]
    $day = Monday$;
    $totalDistance = 0$
    repeat
      Identify shops to be served on day
      Run CWLS algorithm to get $dayDistance$
      $totalDistance+ = dayDistance$
      $day++$
    until $day = lastWorkingDay$;
    $TransportCost = totalDistance * costPerKm$
  [Calculate fitness]
    $TotalCost = InventoryCost + TransportCost$
  return $TotalCost$
end procedure;

Table 1: Evaluation function.

influence of the choice of the routing algorithm in the total costs, coming to the conclusion that a more efficient algorithm provided better results, not only as per the transport cost (which would be obvious) but also in the inventory cost of the obtained solutions.

We performed a study of several simple routing algorithms, comparing both the results obtained and the executions times, in order to choose the most appropriate one to incorporate in the EVITA tool. We chose 50 different configurations of the VRP that are publicly available [20][21] and tried three different algorithms on them: daisy, GRASP with different numbers of iterations [12] and Clarke and Wright's algorithm [7]. This is based on the concept of *saving*, which is the diminution in the traveled length achieved when combining two routes. We employed the parallel version of the algorithm, which works with all routes simultaneously.

Due to the fact that the solutions generated by C&W algorithm are not guaranteed to be locally optimal with respect to simple neighbourhood definitions, it is almost always beneficial to apply a local search to attempt to improve each constructed solution. For this we designed a simple (and fast) local search method. We also enhanced the daisy algorithm in the same manner.

We did not include in this short study other algorithms of known efficiency, such as tabu search, as we considered that the potential improvement in the results would not be compensated by the additional complexity they would introduce in the EVITA tool.

The statistical analysis performed[3] shows that Clarke and

[3]ANOVA followed by Tukey's HSD multiple comparisons

Encoding	The gene i represents the pattern for shop i. The chromosome length is equal to the number of shops ($nShops$).
Selection	Tournament in 2 steps. To select each parent, we take $tSize$ individuals chosen randomly and select the best.
Evolutionary operators	1 and 2 point crossover and n point mutation. The mutation operator changes simultaneously n shops, where $n = U(1, shops2Mut)$.
Termination criterion	Terminate when the total number of fitness evaluations (including the evaluation of the initial population) $numEvals$ equals or exceeds a maximum value, $MaxEvals$ $numEvals \geq MaxEvals$ The algorithm runs in steady-state, so one iteration is equivalent to performing a tournament, i.e. selecting 2 parents and producing 2 children. Hence, $numEvals = popSize + iterations * 2$.
Parameters under study	Population size, $popSize$ Tournament size, $tSize$ Mutation probability, Pm
Fixed parameters	Maximum number of shops to mutate, $shops2Mut = round(0.05 * nShops)$ Maximum fitness evaluations, $MaxEvals = 5000$.

Table 2: Configuration of the base evolutionary algorithm employed in the experiments.

Wright with local search (from now on we will refer to it as CWLS) and GRASP obtain the best results, while not being significantly different from the optimum reported in the literature. The conclusion seems to be that any of these algorithms would be appropriate to use in conjunction with the evolutionary algorithm. However, the execution times for GRASP are about ten times bigger that those of CWLS. Hence, we will select the latter as the routing algorithm to incorporate in the EVITA tool.

The pseudo-code for the CWLS algorithm is given in Table 3.

4. EXPERIMENTS WITH THE EVOLUTIONARY ALGORITHM

Our aim is to find an optimal configuration of parameters for the evolutionary algorithm that the user can employ successfully on a wide class of problems [6][16]. The base algorithm is shown in Table 2; with this we performed a factorial design of experiments (see [14]) with the following levels:

- Population size, $popSize \in \{100, 200, 500\}$
- Mutation probability, $pM \in \{0.33, 0.5, 0.66\}$
- Tournament size, $tSize \in \{2, 5, 7\}$

We tested these different configurations with 5 runs for each one of eight different geographic layouts taken from Augerat *et al.* [1], which are shown in Table 4. In the original problem the n indices represented the number of shops plus one and the k indices the maximum number of vehicles allowed. In our case we will ignore the latter data, as we are not considering restrictions in the number of vehicles.

The problems from set A correspond to shops scattered more or less uniformly on the map; the problems from set B correspond to shops that are grouped in clusters, see [20][21]. We chose four problems from each set, with different levels of number of shops and eccentricity. The latter represents the distance between the warehouse and the geographical centre of the distribution of shops. For instance, an instance with low eccentricity would have the warehouse centered in the middle of the shops while in another with high eccentricity most shops would be located on one side of the warehouse.

It must be noted that we are only using the spatial location and not other restrictions given in the bibliography, such as the number of vehicles (as mentioned above) or the loads demanded by the shops. For the latter we used our own values; we also added the inventory costs, given in Table 7. The table, containing data for up to 80 shops, is the same across all different geographical instances used.

We performed 5 runs per configuration and problem, i.e. a total of $5 \times 3 \times 3 \times 3 \times 8 = 1080$ runs, with a termination criterion in all cases of 5000 fitness evaluations. The maximum run time was of 120 minutes in the computers employed[4].

5. ANALYSIS

In order to be able compare results between instances, we normalised the fitness values by defining the *relative percentage deviation*, RPD, given by the following expression:

$$RPD = \frac{fitness - fitness_{min}}{fitness_{min}} \times 100$$

where $fitness$ is the fitness value obtained by an algorithm configuration on a given instance. The RPD is, therefore, the average percentage increase over the lower bound for each instance, $fitness_{min}$. In our case, the lower bound is the best result obtained for that instance across all parameter configurations.

With the RPD results of the 1080 runs we ran an n-way ANOVA test, whose results are shown in Table 5. The analysis of these is done as follows.

We begin by selecting the factor or interaction that has the greatest F-ratio and, by checking its different levels in a means plot we can tell which level best suits the response

procedure. This was implemented using the `anova` and `multcompare` functions in MATLAB

[4]PCs with Intel Celeron processor, between 1 and 3GHz, between 256 and 512 MB RAM.

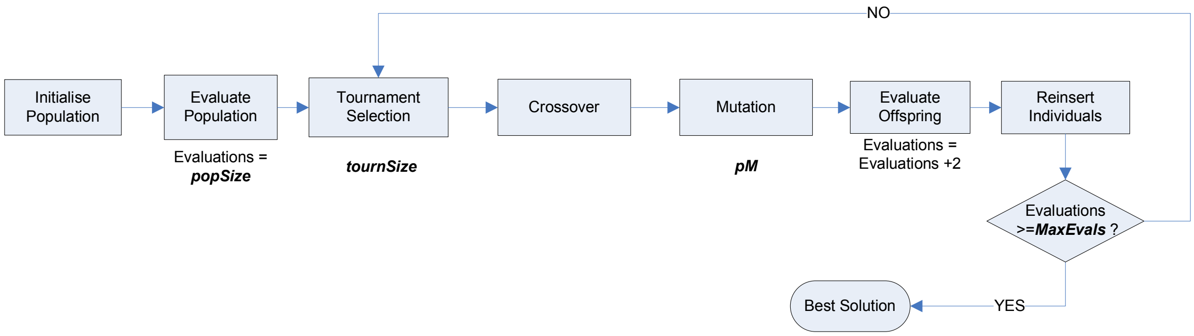

Figure 2: Flow chart of the evolutionary algorithm employed

Algorithm CWLS
    [Initialisation]
        Build $nShops$ `routes` as follows
        $r_i = (0, i, 0)$
    [Calculate savings]
        Calculate $s_{i,j}$ for each pair of shops i, j
        $s_{i,j} = cost_{i0} + cost_{0j} - cost_{ij}$
    [Best union]
        repeat
            $s_{i*j*} = \max s_{i,j}$
            Let r_{i*} be the route containing i
            Let r_{j*} be the route containing j
            if
                $i*$ is the last shop in r_{i*}
                and $j*$ is the first shop in r_{j*}
                and the combination is feasible
            then combine r_{i*} y r_{j*}
            delete s_{i*j*};
        until there are no more savings to consider;
    [Local search]
        Improve each route r_i separately
        Improve considering exchanges between routes
    return `routes`
end algorithm;

Table 3: Clarke & Wright's algorithm with local search (CWLS).

ID	Instance	Distribution	*nShops*	Eccentricity
A32	A-n32-k5.vrp	uniform	31	47.4
A33	A-n33-k5.vrp	uniform	32	20.2
A69	A-n69-k9.vrp	uniform	68	15.3
A80	A-n80-k10.vrp	uniform	79	63.4
B35	B-n35-k5.vrp	clusters	34	60.5
B45	B-n45-k5.vrp	clusters	44	16.6
B67	B-n67-k10.vrp	clusters	66	19.9
B68	B-n68-k9.vrp	clusters	67	49.2

Table 4: Problem instances used in the experiments and their characteristics, taken from the bibliography.

Source	Sum Sq.	d.f.	Mean Sq.	F	Prob>F (p-value)
Pm	0.892	2	0.4459	4.93	0.0074
tournSize	15.068	2	7.534	83.32	0
popSize	25.971	2	12.9853	143.61	0
Pm*tournSize	0.349	4	0.0873	0.97	0.4257
Pm*popSize	0.713	4	0.1783	1.97	0.0966
tournSize*popSize	18.27	4	4.5676	50.51	0
Error	95.936	1061	0.0904		
Total	157.199	1079			

Table 5: ANOVA Table for the algorithm configurations used.

variable and fix the factor at this desired level. Then we pick the second highest F-ratio and apply the same procedure. We finish when all simple factors are fixed to some level.

The biggest F-value that results in statically significant differences in the response variable corresponds to the factor *popSize*. We can study the average performance of all combinations grouped by the levels of this factor. This is normally done with a means plot along with a multiple comparison test, which in our case is the least significant difference. The result is shown in Figure 3, left, where it can be observed that a high population size value yields significantly worse results. We will therefore fix the *popSize* value at 100 individuals.

Next, it can be observed that the factor *popSize* has a significant interaction with the tournament size. From the interaction plot, shown in Figure 4, it can be determined which value of the *tSize* factor is best for the *popSize* value already fixed. In this case the value selected is a tournament size of 2. Interestingly, it can be noticed that this value is the worst for the remaining values of *popSize*.

The next factor in order of importance is the mutation probability, Pm. The means plot for this factor is shown in Figure 3, right. In it we find that, on average, the combinations with small mutation probabilities are around a 0.52% average increase over the lower bound whereas the combinations with high mutation rates are about 0.44%. Therefore, we choose $Pm = 2/3$ because this produces the minimum RPD for the *popSize* and *tSize* factors fixed. Their interactions can be observed in Figures 4, left, and 5.

6. CONCLUSIONS AND FUTURE WORK

We have explored what input parameters are more con-

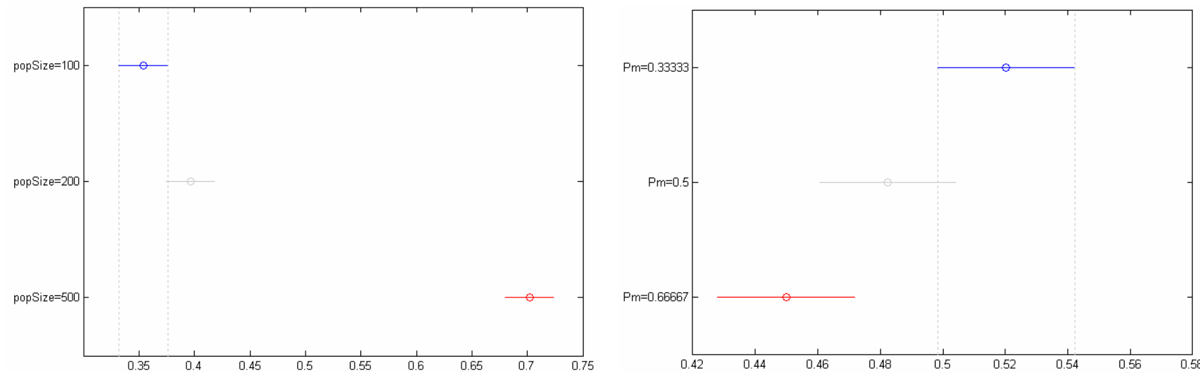

Figure 3: Multiple comparison plots for the population size and the mutation probability

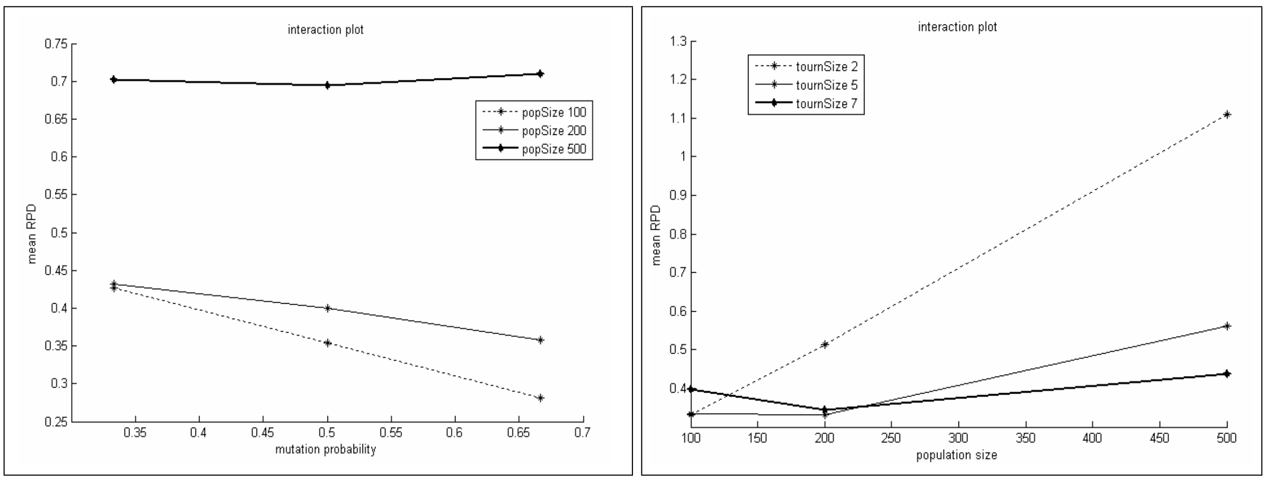

Figure 4: Interaction plots for pM vs $popSize$ (left) and $popSize$ vs $tSize$ (right)

venient to employ within the EVITA tool so that it can be used by a non-expert user over a wide range of problem instances. The aim has been to optimise the hybrid evolutionary tool by studying different combinations of the input parameters of the evolutionary algorithm, concluding that a good setting in terms of quality of solutions and computational speed is obtained using $popSize = 100$, $Pm = 2/3$ and a $tSize = 2$, plus a fast routing algorithm such as CWLS.

Future work will involve:

- Regarding the problem: incorporation of more restrictions (limited number of vehicles, time windows in shops); using non-Euclidean distances, considering cases where distances are different one way and back; incorporation of other transport cost structures; fleet size optimisation
- As per the higher level algorithm: paralelisation of the evolutionary algorithm; exploration of other paradigms (ant colonies, particle swarms, GRASP, etc.)
- With respect to the lower level algorithm: implementation of an additional step of off-line optimisation of the obtained routes, to apply once the evolution is finished; incorporation of new routing algorithms, studying the balance between the computational cost and the improvement obtained; improvement of the local search algorithm.

Acknowledgement

This work was part of projects ANaLog (supported by the Instituto de la Pequeña y Mediana Industria Valenciana, IMPIVA, Ref. IMCOCA/2006/22, and partially funded by the European Regional Development Fund) and NADEweb (Spanish Ministerio de Educación y Ciencia - TIC2003-09481).

7. REFERENCES

[1] Augerat, P. Belenguer, J.M., Benavent, E. Corbern, A. Naddef,D. and Rinaldi, G.: Computational Results with a Branch and Cut Code for the Capacitated Vehicle Routing Problem, Res. Rep. 949-M, Univ. Joseph Fourier, Grenoble, France (1995).

[2] Benjamin, J.: An Analysis of Inventory and Transportation Costs in a Constrained Network, *Transportation Science*, 23(3): 177-183 (1989)

[3] Burns, L.D., Hall, R.W., Blumenfeld, D.E. and Daganzo, C.F.: Distribution Strategies that Minimize

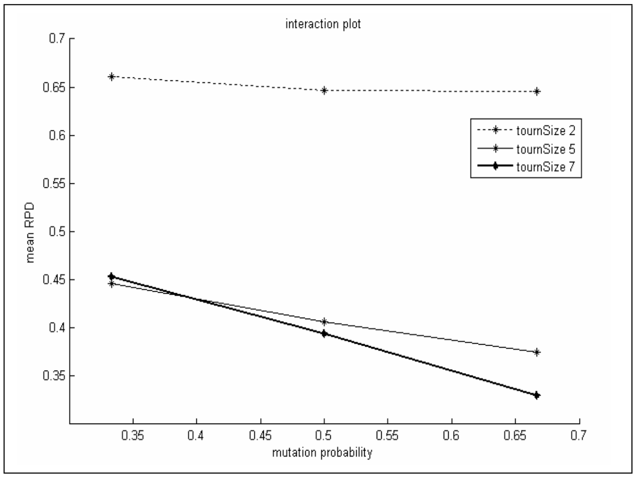

Figure 5: Interaction plot for Pm vs $tSize$

Transportation and Inventory Costs, *Operations Research*, 33(3):469-490 (1985)

[4] Campbell A.M. and Savelsbergh, M.W.P. : A Decomposition Approach for the Inventory-Routing Problem, *Transportation Science*, 38(4):488-502 (2004)

[5] Cardós, M. and García-Sabater, J.P.: Designing a consumer products retail chain inventory replenishment policy with the consideration of transportation costs. *Intl. Jour. of Prod. Econ.*, 104(2):525535 (2006). doi:10.1016/j.ijpe.2004.12.022

[6] Castillo, P.A., J.J. Merelo, A. Prieto, I. Rojas and G. Romero: Statistical analysis of the parameters of a neuro-genetic algorithm. *IEEE Transactions on Neural Networks*, 13(6), November 2002.

[7] Clarke, G. and Wright, W.: Scheduling of vehicles from a central depot to a number of delivery points. *Operations Research*, 12:568-581 (1964)

[8] Constable, G.K. and Whybark, D.C.: The interaction of transportation and inventory decisions, *Decision Sciences*, 9(4):688-699 (1978)

[9] Cordeau, J.-F., Gendreau M. and Laporte, G.: A Tabu Search Heuristic for Periodic and Multi-Depot Vehicle Routing Problems, *Networks*, 30(2):105-119 (1997).

[10] Esparcia-Alcázar, A.I., Lluch-Revert, L., Cardós, M., Sharman, K. and Andrés-Romano, C.: Design of a Retail Chain Stocking Up Policy with a Hybrid Evolutionary Algorithm. In J. Gottlieb and G.R. Raidl (Eds.): *EvoCOP 2006*, Lecture Notes on Computer Science LNCS 3906, pp. 49-60. Springer (2006)

[11] Esparcia-Alcázar, A.I., Lluch-Revert, L., Cardós, M., Sharman, K. and Andrés-Romano, C.: A comparison of routing algorithms in a hybrid evolutionary tool for the inventory and transportation problem. In Piero Bonissone, Gary Yen, Lipo Wang and Simon Lucas, (Eds.): *Proceedings of the IEEE Congress on Evolutionary Computation, CEC 2006* pages 5605-5611, Vancouver, Canada, July 2006. IEEE, Omnipress. ISBN: 0-7803-9489-5.

[12] Feo, T.A. and Resende M.G.C. : A probabilistic heuristic for a computationally difficult set covering problem. *Operations Research Letters*, 8:67-71 (1989)

[13] Ganeshan, R.: Managing supply chain inventories: A multiple retailer, one warehouse, multiple supplier model. *Intl. Jour. of Prod. Econ.*, 59(1-3):341-354 (1999)

[14] Montgomery, D.C.: *Design and Analysis of Experiments*. John Wiley and Sons (2005)

[15] Qu, W.W., Bookbinder, J.H. and Iyogun, P.: An integrated inventory-transportation system with modified periodic policy for multiple products. *European Journal of Operational Research*, 115(2): 254-269 (1999)

[16] Rojas, I., J. Gonzalez, H. Pomares, J. J. Merelo, P. A. Castillo and G. Romero: Statistical analysis of the main parameters involved in the design of a genetic algorithm. *IEEE Transactions On Systems Man and Cybernetics Part C-Applications and Reviews*, 32(1): 31-37, February 2002.

[17] Speranza, M.G. and Ukovich, W.: Minimizing Transportation and Inventory Costs for Several Products on a Single Link, *Operations Research*, 42(5):879-894 (1994)

[18] Toth, P. and Vigo, D.: The Vehicle Routing Problem. *SIAM monography on Discrete Mathematics and Applications*. Society for Industrial and Applied Mathematics, Philadelphia, PA, USA (2001)

[19] Zhao, Q.H., Wang, S.Y., Lai, K.K. and Xia, G.P.: Model and algorithm of an inventory problem with the consideration of transportation cost. *Computers and Industrial Engineering*, 46(2): 398-397 (2004)

[20] http://branchandcut.org/VRP/data/#A

[21] http://branchandcut.org/VRP/data/#B

Appendix: Problem data

The following tables contain the admissible patters, the inventory cost data per shop and other data related to the problem.

Pattern	Freq	Mon	Tues	Wed	Thu	Fri
1	2	✓	-	-	✓	-
2	2	✓	-	-	-	✓
3	2	-	✓	-	✓	-
4	2	-	✓	-	-	✓
5	2	-	-	✓	-	✓
6	3	✓	-	✓	-	✓
7	3	-	✓	-	✓	✓
8	3	-	✓	✓	-	✓
9	4	✓	-	✓	✓	✓
10	4	✓	✓	✓	-	✓
11	5	✓	✓	✓	✓	✓

Table 6: Admissible patterns per frequency. The checkmark represents that the shop is served on that day, the minus that it is not. As a consequence of the business logic, we will only consider 11 patterns out of the 31 that are possible. .

	Inventory cost (€)					Delivery size (roll containers)					Admissible frequencies						Inventory cost (€)					Delivery size (roll containers)					Admissible frequencies				
Shop	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5	Shop	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
1	436	363	351	336	325	9	5	4	2	2	-	-	-	✓	✓	41	373	310	292	285	283	6	3	2	2	1	-	✓	✓	✓	✓
2	435	362	350	335	325	9	5	4	2	2	-	-	-	✓	✓	42	371	308	290	283	282	6	3	2	2	1	-	✓	✓	✓	✓
3	433	361	349	334	324	9	5	4	2	2	-	-	-	✓	✓	43	369	307	289	282	280	6	3	2	2	1	-	✓	✓	✓	✓
4	432	360	348	334	323	9	5	4	2	2	-	-	-	✓	✓	44	368	305	287	280	279	6	3	2	2	1	-	✓	✓	✓	✓
5	431	359	347	333	322	9	5	4	2	2	-	-	-	✓	✓	45	366	303	286	279	277	6	3	2	2	1	-	✓	✓	✓	✓
6	430	359	346	332	321	9	5	4	2	2	-	-	-	✓	✓	46	364	302	284	278	276	6	3	2	2	1	-	✓	✓	✓	✓
7	429	358	346	331	321	9	5	4	2	2	-	-	-	✓	✓	47	362	300	283	276	275	6	3	2	2	1	-	✓	✓	✓	✓
8	428	357	345	330	320	9	5	4	2	2	-	-	-	✓	✓	48	360	299	281	275	273	6	3	2	2	1	-	✓	✓	✓	✓
9	427	356	344	329	319	9	5	4	2	2	-	-	-	✓	✓	49	358	297	280	273	272	6	3	2	2	1	-	✓	✓	✓	✓
10	426	355	343	329	318	9	5	4	2	2	-	-	-	✓	✓	50	356	296	279	272	270	6	3	2	2	1	-	✓	✓	✓	✓
11	425	354	342	328	317	9	5	4	2	2	-	-	-	✓	✓	51	355	294	277	270	269	6	3	2	2	1	-	✓	✓	✓	✓
12	424	353	341	327	317	9	5	4	2	2	-	-	-	✓	✓	52	353	293	276	269	267	6	3	2	2	1	-	✓	✓	✓	✓
13	423	352	340	326	316	9	5	4	2	2	-	-	-	✓	✓	53	351	291	274	268	266	6	3	2	2	1	-	✓	✓	✓	✓
14	422	351	340	325	315	9	5	4	2	2	-	-	-	✓	✓	54	349	290	273	266	265	6	3	2	2	1	-	✓	✓	✓	✓
15	421	351	339	324	314	9	5	4	2	2	-	-	-	✓	✓	55	347	288	271	265	263	6	3	2	2	1	-	✓	✓	✓	✓
16	420	350	338	324	313	9	5	4	2	2	-	-	-	✓	✓	56	345	286	270	263	262	6	3	2	2	1	-	✓	✓	✓	✓
17	419	349	337	323	313	9	5	4	2	2	-	-	-	✓	✓	57	343	285	268	262	260	6	3	2	1	1	-	✓	✓	✓	✓
18	417	348	336	322	312	9	5	4	2	2	-	-	-	✓	✓	58	341	283	267	260	259	6	3	2	1	1	-	✓	✓	✓	✓
19	416	347	335	321	311	8	5	4	2	2	-	-	-	✓	✓	59	340	282	265	259	258	6	3	2	1	1	-	✓	✓	✓	✓
20	415	346	334	320	310	8	5	4	2	2	-	-	-	✓	✓	60	338	280	264	258	256	6	3	2	1	1	-	✓	✓	✓	✓
21	414	345	334	320	309	7	4	3	2	1	-	-	-	✓	✓	61	336	279	263	256	255	5	3	2	1	1	-	✓	✓	✓	✓
22	411	342	329	316	308	7	4	3	2	1	-	-	-	✓	✓	62	336	278	262	256	255	5	3	2	1	1	-	✓	✓	✓	✓
23	409	340	324	313	308	7	4	3	2	1	-	-	-	✓	✓	63	335	278	262	256	256	5	3	2	1	1	-	✓	✓	✓	✓
24	406	337	320	310	307	7	4	3	2	1	-	-	-	✓	✓	64	335	278	262	256	256	5	2	2	1	1	-	✓	✓	✓	✓
25	403	334	315	307	306	7	4	3	2	1	-	✓	✓	✓	✓	65	335	278	262	256	257	5	2	2	1	1	-	✓	✓	✓	-
26	401	333	314	306	304	7	4	3	2	1	-	✓	✓	✓	✓	66	334	277	261	255	256	5	2	2	1	1	-	✓	✓	✓	-
27	399	331	312	305	303	7	4	3	2	1	-	✓	✓	✓	✓	67	333	276	261	255	255	5	2	2	1	1	-	✓	✓	✓	-
28	397	330	311	303	301	7	4	2	2	1	-	✓	✓	✓	✓	68	333	276	260	254	255	5	2	2	1	1	-	✓	✓	✓	-
29	396	328	309	302	300	7	3	2	2	1	-	✓	✓	✓	✓	69	332	275	259	253	254	4	2	1	1	1	-	✓	✓	✓	-
30	394	327	308	300	299	7	3	2	2	1	-	✓	✓	✓	✓	70	331	274	259	253	253	4	2	1	1	1	-	✓	✓	✓	-
31	392	325	306	299	297	7	3	2	2	1	-	✓	✓	✓	✓	71	330	274	258	252	253	4	2	1	1	1	-	✓	✓	✓	-
32	390	324	305	297	296	7	3	2	2	1	-	✓	✓	✓	✓	72	329	273	257	252	252	4	2	1	1	1	-	✓	✓	✓	-
33	388	322	303	296	294	7	3	2	2	1	-	✓	✓	✓	✓	73	328	272	257	251	252	4	2	1	1	1	-	✓	✓	✓	-
34	386	321	302	295	293	7	3	2	2	1	-	✓	✓	✓	✓	74	328	271	256	250	251	4	2	1	1	1	-	✓	✓	✓	-
35	384	319	300	293	291	7	3	2	2	1	-	✓	✓	✓	✓	75	327	271	255	250	250	4	2	1	1	1	-	✓	✓	✓	-
36	383	317	299	292	290	7	3	2	2	1	-	✓	✓	✓	✓	76	326	270	255	249	250	3	2	1	1	1	✓	✓	✓	-	-
37	381	316	298	290	289	7	3	2	2	1	-	✓	✓	✓	✓	77	325	269	254	248	249	3	2	1	1	1	✓	✓	✓	-	-
38	379	314	296	289	287	7	3	2	2	1	-	✓	✓	✓	✓	78	324	269	254	248	248	3	2	1	1	1	✓	✓	✓	-	-
39	377	313	295	287	286	7	3	2	2	1	-	✓	✓	✓	✓	79	324	268	253	247	248	3	2	1	1	1	✓	✓	✓	-	-
40	375	311	293	286	284	7	3	2	2	1	-	✓	✓	✓	✓	80	323	267	252	247	247	3	2	1	1	1	✓	✓	✓	-	-

Table 7: Inventory cost (in euro), size of the deliveries per shop depending on the delivery frequency and admissible frequencies per shop.

Capacity	12 roll containers
Transportation cost	0.6 €/Km
Average speed	60 km/h
Unloading time	15 min
Maximum working time	8h

Table 8: Vehicle data