

Methodology to Select Solutions from the Pareto-Optimal Set: A Comparative Study

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ABSTRACT

The resolution of a Multi-Objective Optimization Problem (MOOP) does not end when the Pareto-optimal set is found. In real problems, a single solution must be selected. Ideally, this solution must belong to the non-dominated solutions set and must take into account the preferences of a Decision Maker (DM). Therefore, the searching for a single solution (or solutions) in MOOP is done in two steps. First, a Pareto optimal set is found. Multi-Objective Evolutionary Algorithms (MOEA), based on the principle of Pareto optimality, are designed to produce the complete set of non-dominated solutions. Second, a methodology able to select a single solution from the set of non-dominated solutions (or a region of the Pareto frontier), and taking into account the preferences of a Decision Maker (DM), can be applied. In this work, a method, based on a weighted stress function, is proposed. It is able to integrate the user's preferences in order to find the best region of the Pareto frontier accordingly with these preferences. This method was tested on some benchmark test problems, with two and three criteria. This methodology is able to select efficiently the best Pareto-frontier region for the specified relative importance of the criteria.

Categories and Subject Descriptors

I.2.8 [Problem Solving, Control Methods, and Search]: Heuristic methods.

General Terms

Algorithms.

Keywords

Decision making, Optimization, Multi-objective.

1. INTRODUCTION

Real-world optimization problems are characterized by the existence of multiple, often conflicting, objectives. In the absence of information about the relative importance of the various criteria,

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such problems typically admit multiple optimal solutions where any improvement in one objective can only be obtained at the expense of degradation in other objectives [1]. Thus, the final solution of a multi-objective optimization problem results, not only from the optimization process (or search process), but also from the decision process. The integration of the DM preferences on the optimization process must allow the selection, from the set of optimal solutions, the single solution (or a region) that best satisfies these preferences. Three different classes of methods can be defined, taking into account the way how the search and the decision processes are integrated [2, 3]: i) A priori methods: The decision maker must specify her or his preferences, expectations and options before the optimization process takes place. These preferences are defined in terms of an aggregating function which combines individual objective values into a single utility value. ii) A posteriori methods: The Pareto optimal set after been generated, without taking into consideration any DM preferences, is supplied to the DM, which selects the most preferred among the alternatives. iii) Interactive methods: Decision making and optimization occur simultaneously. At each step, partial preference information is supplied by the DM. The optimization algorithm uses this information to generate better alternatives accordingly with the information received from the DM.

The difficulties arising from the process of decision making in a multi-objective environment makes the need, to define a methodology able to combine the DM preferences with the search procedure, necessary. A new methodology, based on a weight Stress Function Approach, to select the best solution (or set of solutions) from a Pareto frontier taking into account the preferences of the DM, is proposed. This methodology is able to define the best region of the Pareto frontier from a set of weights given by a weight vector. The attribution of a relative importance to each criterion helps the optimization algorithm in converging to relevant regions of the Pareto frontier. Thus, in this case the search will be made on a small portion of the Pareto frontier, which facilitates the algorithm convergence for a set of optimal solutions that have particular interest to the DM.

2. METHODOLOGY FOR DECISION MAKING

In this section three different methods for selecting solutions taking into account the preferences of a DM will be described, the aim being to compare the performance of these techniques using some benchmark problems from the literature.

2.1 Reference Point based EMO Approach (RP-EMO)

Deb and co-authors [4] proposed a reference point approach, where the aim was to reach a Pareto frontier region located near a specific pre-defined reference point or points. This is *apriori* method that can be applied using or not a weight vector. The main ideas of this method are the following:

1. Solutions closer to the reference points, in the criteria space, are to be more highlighted.
2. Solutions within a ε -neighborhood to a near-reference-point solution are to be highlighted, in order to maintain a diverse set of solutions near each reference point.

The normalized Euclidian distance (d_{ij}) between each solution of the best non-dominance level and the reference point is calculated, allowing the identification of the solutions located near the reference point:

$$d_{ij} = \sqrt{\sum_{i=1}^N w_i \left(\frac{f_i(x) - \bar{z}_i}{f_i^{\max} - f_i^{\min}} \right)^2} \quad (1)$$

where, $f_{i\max}$ and $f_{i\min}$ are the maximum and the minimum value of the objective function for criterion i , respectively, and \bar{z}_i and w_i are the i -th component of the reference point and weight vector, respectively. This equation takes into account simultaneously the relative importance of the different criteria, quantified through the weight vector, and the distance between the solution and the reference point. The solutions with lower distance to the reference point and, simultaneously, with high weight vector, will be selected preferentially.

This procedure was been implemented through the modification of the NSGA-II algorithm [5,6]. The main change was made on the crowding operator, since in this case the aim is not to obtain a great diversity of solutions along the Pareto frontier but a sub-set of these solutions that minimize the distance to the reference point. The details of these changes have been described by Deb and co-authors [4].

There are two difficulties when this methodology is applied to real problems: First, it is necessary to know beforehand the criteria space, which is not an easy task mainly for problems with many criteria and for problems where the Pareto front is not obtained from simple functions. Second, the definition of the reference points makes necessary that the DM needs to have some knowledge about the location of the best solutions or regions of the Pareto front.

2.2 Weighted Metrics Method (WMM)

The Weighted Metrics [2] is a preference method where the articulation of preference information is used a posteriori. Thus, the solution is chosen from a group of results obtained by the optimization process. For that purpose the Weighted Metrics (Lp) is applied:

$$\text{Minimize } \left(\sum_{i=1}^N w_i |f_i(X) - z_i^*|^p \right)^{1/p} \quad (2)$$

Subject to $x \in S, 1 \leq p < \infty$

where, w_i is the weight vector (taking into account that $w_i > 0$ for $i=1, \dots, N$ and $\sum_{i=1}^N w_i = 1$) and z_i^* is the ideal criteria vector which represents the vector maximizing each one of the objective functions [2].

The Weighted Tchebycheff metrics is obtained from equation 1 making $p=\infty$:

$$\text{Minimize } \max_{i=1, \dots, N} [w_i |f_i(X) - z_i^*|] \quad (3)$$

Subject to $x \in S$

The aim of the Weighted Metrics is to minimize the distance between the solutions (on the criteria space) and the ideal criteria vector. If the ideal criteria vector belongs to the space of admissible solutions, then the best solution will be this ideal vector, since it is the solution that minimizes each one of the individual objective functions. However, this ideal criteria vector only can belong to the admissible region when the criteria are not conflicting.

The method of weighted Tchebycheff metric is often used to generate the Pareto optimal solutions. The main property of this method is the following: for a known ideal objective vector, for each optimal solution X , there is a weighted vector w_i for which X is the solution of the weighted Tchebycheff problem. Thus, all the optimal solutions of a MOOP can be determined through the resolution of the weighted Tchebycheff problem. However, the application of this technique can originate weak Pareto optimal solutions, thus some auxiliary calculations will be needed to identify these weak ones [2].

2.3 Weighted Stress Function Method (WSFM)

The method proposed here (WSFM) integrates the DM preferences after the search process has been made. Therefore, this is a posterior method where the search and the decision process are sequential. The main principle of the WSFM method consists on the idea that the best solution that will satisfy the preferences of the DM must take into account the two following aspects. First, the solution must belong to the Pareto Frontier, *i.e.*, to the set of non-dominated solutions. Second, the selection procedure must take into consideration the ideal objective vector denoted as z^* , *i.e.*, the vector maximizing each one of the objective functions. Since the aim is to maximize the criteria, the individual optimization of each criterion corresponds to the maximum value for this criterion. However, in multi-objective optimization, the relative importance attributed to each criterion will induce a "stress" for searching for solutions that maximize each one of the various criteria.

The method will be explained with Figure 1, where a Pareto frontier of an optimization problem with two criteria, f_1 and f_2 , to be maximized is shown. For each solution belonging to the Pareto frontier, two stresses, γ_{w1} and γ_{w2} , are defined, each one associated to the corresponding criteria, f_1 and f_2 (w_1 and w_2 are the weight attributed to each criterion). These stresses are proportional to the weighted distance between criteria i and the i -th component of ideal objective vector.

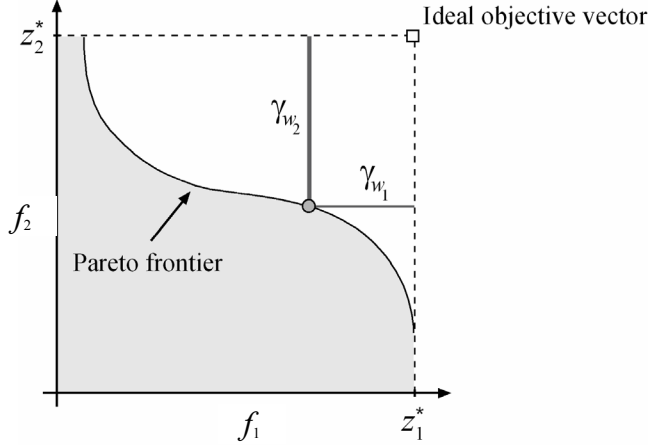


Figure 1- Stresses associated with an optimal solution for an optimization problem with two criteria.

Since both criteria are conflicting the solution of the problem cannot be the ideal objective vector. In fact the stresses associated with the best solution for the problem are not null. The best solution for the problem, in an ideal situation, is the one that have null stresses, but this situation is not possible do to the conflicting relation between criteria. Thus, the best solution will be the one where the differences between the stresses associated to each criterion have the minimum possible value. This means that, for the best solution, the stress γ_{w1} does not constitutes a sufficient stress for searching solutions with best values for f_1 , since an increase in this criterion implies a decrease in f_2 , and consequently an increase on γ_{w2} . The increase of this stress redirects the search for solutions with best values of the second criterion in detriment of losses in the first one. Taking into account these considerations, the weighted stress function associated to the weight w_i is defined as:

$$\gamma_{w_i}(x) = \begin{cases} \frac{w_i}{2} \tan\left(-\frac{\pi}{\psi(w_i)}(x-w_i)\right) + \xi(w_i), & x \leq w_i \\ -\frac{\xi(w_i)}{\tan\left(\frac{\pi}{\varphi(w_i)}(w_i-1)\right)} \tan\left(-\frac{\pi}{\varphi(w_i)}(x-w_i)\right) + \xi(w_i), & x > w_i \end{cases} \quad (4)$$

where,

$$\varphi(w_i) = \frac{3}{4}(1-w_i)^2 + 2(1-w_i) + \delta_1 \quad (5)$$

$$\psi(w_i) = \varphi(w_i) + 4w_i - 2 \quad (6)$$

$$\xi(w_i) = -\frac{1}{\tan\left(-\frac{\pi}{2+2\delta_2}\right)} \tan\left(\frac{\pi}{1+\delta_2}\left(w_i - \frac{1}{2}\right)\right) + 1 \quad (7)$$

This definition considers that the criteria are normalized, i.e., $f_i \in [0, 1]$, consequently, the ideal vector has the following coordinates (1,1).

Figure 2 represents the stress function (γ_{w_i}) and the geometric interpretation of functions φ , ψ and ξ . For a specific location of w_i , $\xi(w_i)$ is equal to $\gamma_{w_i}(f_i)$ for f_i equal to w_i (since f_i and w_i are normalized, ranging in the interval $[0,1]$). Simultaneously, the distance between asymptote A1 and w_i is $\psi(w_i)/2$ and the distance between asymptote A2 and w_i is $\varphi(w_i)/2$. When w_i and f_i , simultaneously, approaches 0 or 1 functions $\psi(w_i)$ and $\varphi(w_i)$, respectively, cannot be null in order to avoid that, in these cases (i.e., for w_i equal to 0 or 1), $\gamma_{w_i}(f_i)$ assumes an infinite value. To enable that the weight range in the interval $[0,1]$, a small value (δ_i) must be added to $\psi(w_i)$ and $\varphi(w_i)$ functions, without changing considerably the shape of the $\gamma_{w_i}(f_i)$ function. Finally, function $\xi(w_i)$ must be constant in all w_i domain, except for w_i values near the interval limits (i.e., for values of w_i near to 0 and 1). This is accomplished using the parameter δ_2 in equation 7. Various values of δ_1 and δ_2 were tested and the best values are $\delta_1=0.002$ and $\delta_2=0.008$.

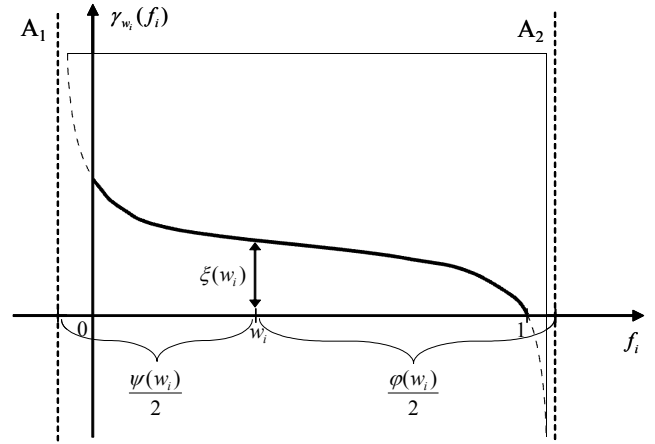


Figure 2- Stress function for a specific weight.

Thus, for a set of weights w_i , the resolution of MOOP with N criteria consists in solving the following problem with a single objective:

$$\text{Minimize } T(X) = \sum_{i < j} \left| \gamma_{w_i}(f_i(X)) - \gamma_{w_j}(f_j(X)) \right| \quad (9)$$

Subject to $X \in S$

2.4 Methodology for DM

In the majority of the optimization problems only small regions of the Pareto frontier have practical importance [7, 8]. These sub-sets of the Pareto frontier are defined by the integration of the DM preferences on the problem under study. The definition of an "ability" function, combining the non-dominance concept and the relative importance of each criterion, will allow the selection of the solutions satisfying both features. For that purpose, a new fitness function (denoted as DF) will be defined, depending of both on the rank value - $Rank_s(X)$ and on the $T(X)$ value defined by equation 9.

$$DF(X) = Rank_s(X) + \frac{T(X)}{T(X) + 1} \quad (10)$$

This function specifies with detail the fitness of each solution taking into consideration two objectives: the non-dominance level and the importance of the solution considering the preferences of the DM.

These two objectives are improved when DF decreases. This model for calculating the fitness can be used to define a new decision rank, since the first term of equation 10 corresponds to the standard rank of the solution. Thus, the values of DF contain information concerning the non-dominance level. The extension of the sub-set of the Pareto frontier obtained by the application of this methodology is controlled by a dispersion parameter (ε) that ranges in the interval $[0,1]$. For small values of this parameter the methodology produces a single solution and for high values the entire Pareto frontier will be obtained. In order to implement the methodology proposed the Non-Dominated Sorting Genetic Algorithm (NSGA-II) proposed by Deb et al. [5,6] was been modified. During N_1 generations (search generations) the NSGA-II was applied without modifications in order to find a good approximation to the Pareto frontier. Then, during N_2 generations (decision generations) the new methodology (equation 10) was applied in order to the populations converges to the region defined by the weights given by the DM.

3. RESULTS AND DISCUSSION

3.1 Case Studies

Three different test problems will be used to test the validity of the algorithm:

ZDT1 [5] (Minimize f_1 and f_2 , $L=30$, $x \in [0,1]$):

$$\begin{aligned} f_1(x) &= x_1 \\ f_2(x) &= g(x) \left[1 - \sqrt{x_1/g(x)} \right] \\ g(x) &= 1 + 9 \left(\sum_{i=2}^L x_i \right) / (L-1) \end{aligned} \quad (11)$$

ZDT2 [5] (Minimize f_1 and f_2 , $L=30$, $x \in [0,1]$):

$$\begin{aligned} f_1(x) &= x_1 \\ f_2(x) &= g(x) \left[1 - (x_1/g(x))^2 \right] \\ g(x) &= 1 + 9 \left(\sum_{i=2}^L x_i \right) / (L-1) \end{aligned} \quad (12)$$

DTLZ2 [9] (Minimize f_1, f_2 and f_3 , $L=12$, $x \in [0,1]$):

$$\begin{aligned} f_1(x) &= (1 + g(x)) \cdot \cos\left(x_1 \frac{\pi}{2}\right) \cdot \cos\left(x_2 \frac{\pi}{2}\right) \\ f_2(x) &= (1 + g(x)) \cdot \cos\left(x_1 \frac{\pi}{2}\right) \cdot \sin\left(x_2 \frac{\pi}{2}\right) \\ f_3(x) &= (1 + g(x)) \cdot \sin\left(x_1 \frac{\pi}{2}\right) \\ g(x) &= \sum_{i=3}^L (x_i - 0.5)^2 \end{aligned} \quad (13)$$

First, the influence on the algorithm performance, of the dispersion parameter (ε) and of the weight vector, will be evaluated, and then the methodology proposed (WSFM) will be compared with the RP-EMO and WMM approaches.

There are metrics available able to compare entire Pareto frontiers, but since the aim here is to evaluate the location of small portions of

the Pareto frontier that take into consideration the DM preferences. Thus, in this work the comparison will be made visually.

The Pareto frontiers were obtained with the NSGA-II algorithm with real codification using the SBX operator with an index of 10 and polynomial mutation with an index of 20. The population size was fixed at 100 for ZDT1 and ZDT2 problems and at 200 for DTLZ2 problem, the number of search generations (N_1) was 300 for ZDT1 and ZDT2 problems and 500 for DTLZ2 and the number of decision generations (N_2) was 30 generations for all problems. Due to the stochastic nature of the optimization algorithm, five different runs were carried out for each case using different seed values.

3.2 Study of the Influence of the Dispersion Parameter (ε)

This parameter controls the dispersion of the solutions along the Pareto frontier when the proposed methodology is applied. The ideal situation will be the one where only a single solution is found and, simultaneously, the DM preferences are considered. However, a balance between the dispersion and the concentration of the solutions must be done, mainly when the DM does not have a good knowledge of the process.

Figure 3 shows the influence of the dispersion parameter, for ZDT2 test problem, on the performance of the decision methodology proposed. As expected, for low values of ε (0.005 and 0.2) the solutions converge for small regions of the Pareto frontier. The parameter allows to control the extension of the Pareto set obtained, *i.e.*, to control the balance between the preferences of DM and the non-dominance relation. If the DM has a good knowledge of the problem it can use low values for this parameter. In this case the number of alternative solutions is reduced ($\varepsilon=0.005$). If the DM wants to obtain more alternatives a reasonable option is to use ε equal to 0.2. For a dispersion parameter higher than 0.6 the entire Pareto frontier is obtained. The same type of conclusion can be demonstrated from an equal study made for the remaining test problems. Figure 4 shows these results for DTLZ2 test problem with three criteria.

3.3 Study of the Influence of the Weight Vector

Figure 5 shows the results obtained for ZDT1 test problem using different weight vectors and ε equal to 0.1. These results show that an increase of the importance of the first criterion (f_1) produces solutions with lower values for this criterion. When a weight of 80% is attributed to f_1 (and, as a consequence, 20% to f_2) the main aim is to minimize f_1 and not to minimize f_2 . The main task, in this case, is the searching of solutions with lower value for f_1 , and the task of minimizing f_2 is less important. The reverse situation occurs when the importance of the second criterion is higher than the importance attributed to the first one. In this case, the minimization of f_2 will be the main task. The same type of behavior was observed for the remaining test problems.

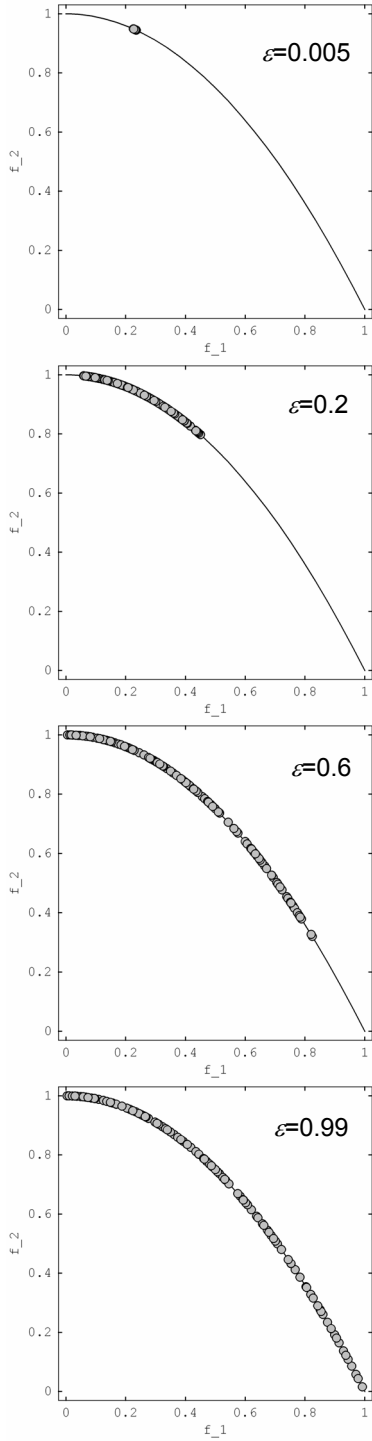


Figure 3- Results for ZDT2 problem for various values of the dispersion parameter - weight vector: (0.8, 0.2).

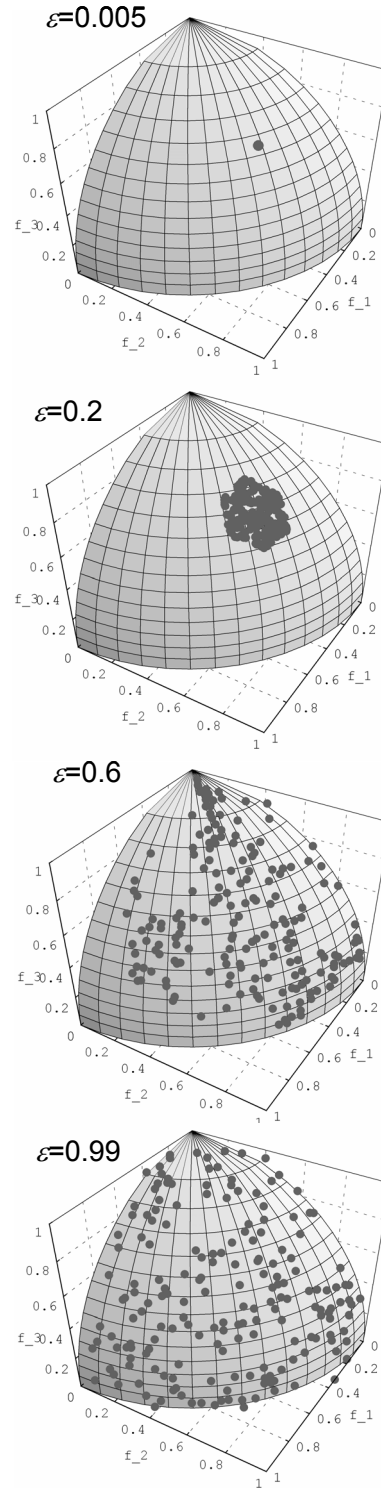


Figure 4- Results for DTLZ2 problem for various values of the dispersion parameter - weight vector: (0.5, 0.3, 0.2).

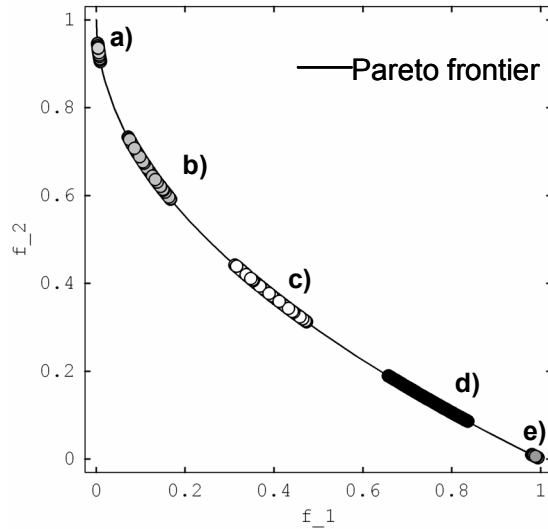


Figure 5- Results for ZDT1 problem for various weight vectors:
a) (0.98,0.02), b) (0.8,0.2), c) (0.5,0.5), d) (0.2,0.8) and e)
(0.02,0.98).

3.4 Comparison with the RP-EMO Approach

Figure 6 shows the results obtained for the ZDT1 using: A) the WSFM and B) the RP-EMO approach, for different weights values and a single reference point located at the origin (*i.e.*, the ideal objective vector). Differences between the two methods are attained mainly for the set of weights i) and iii). For the first case (i) the results obtained by RP-EMO approach are located around the point (0.2, 0.55), while for the WSFM they are located around the point (0.1, 0.7). Taking into account that the aim is, mainly, to minimize criteria f_i (since the weight attributed to this criteria is higher), the region obtained by the WSFM seems to satisfy better the DM preferences. The same type of justification can be given for the set of weights iii), but now the higher importance is attributed to criteria 2.

Figure 7 presents the results obtained for test problem ZDT2 under the same conditions. In this case the main differences are attained for the set of weights represented by i) and ii). The solutions obtained for case i) seems to not attribute any importance to criterion 2 ($f_2 \approx 0$). Simultaneously, the solutions obtained by WSFM approach (A) have, also, low values of f_2 , but without “forgetting” some of the importance given to criterion 2. In the case of weight vector ii), equal importance is attributed to both criteria. However, the RP-EMO approach produces solutions with higher values for f_1 (*i.e.*, solutions that confer more importance to criterion 1). Again, the WSFM approach seems to produce solutions more balanced, taking into account that the DM is searching for solutions where both criteria have the same importance.

Finally, Figure 8 presents identical results for the DTZL2 problem, using WSFM (i) and RP-EMO (ii) approaches. The weight vector used was (0.07, 0.8, 0.13) and the reference point was located at (0, 0, 0). Taking into account that the importance attributed to criteria 1 and 3 is identical, the WSFM approach produces solutions where this equilibrium (between criteria 1 and 3) is achieved and, simultaneously, the higher importance for criterion 2 is considered. The results obtained by the RP-EMO approach does not take into

consideration the equilibrium between criteria 1 and 3, the solutions obtained preserve the importance of criterion 3.

A global comparison between WSFM and RP-EMO approaches seems to indicate that WSFM approach is able to take into consideration in a more equilibrated way the DM preferences. The WSFM approach is able to perform well for convex and concave problems and for problems with more than two criteria.

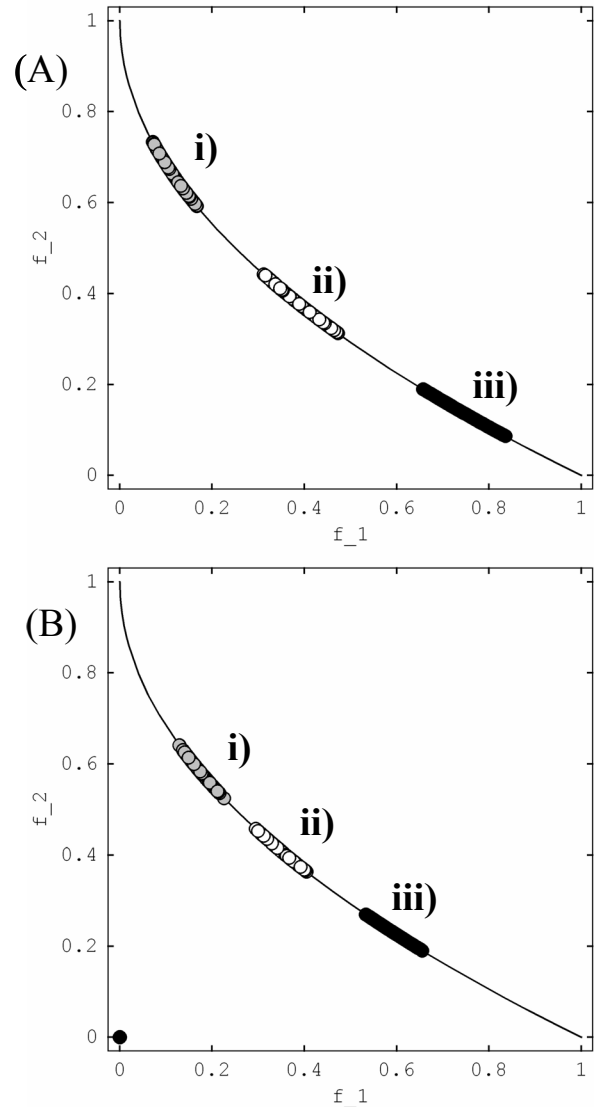


Figure 6- Comparison between the WSFM (A) and the RP-EMO (B) approaches for ZDT1 problem- weight vectors: i) (0.8,0.2), ii) (0.5,0.5) and iii) (0.2,0.8).

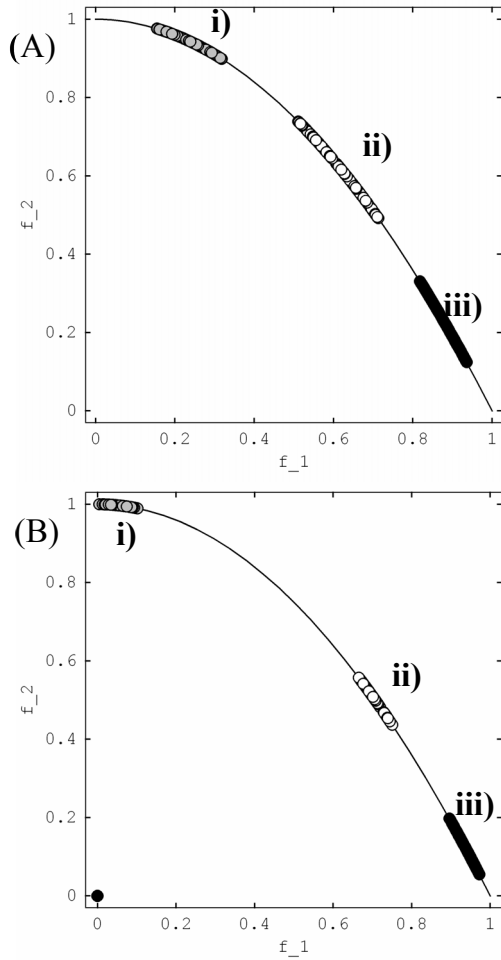


Figure 7- Comparison between the WSFM (A) and RP-EMO (B) approaches for ZDT2 problem- weight vectors: i) (0.8,0.2), ii) (0.5,0.5) and iii) (0.2,0.8).

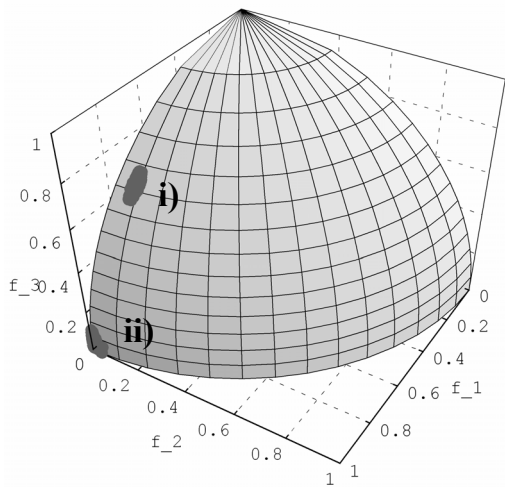


Figure 8- Comparison between the i) WSFM and ii) RP-EMO approach for DTLZ2 problem - weight vector for both: (0.07,0.8,0.13).

3.5 Comparison with the WMM

Figures 9, 10 and 11 show the results obtained using the WMM approach for test problems ZDT1, ZDT2 and DTLZ2, respectively. The results for the test problems with two criteria (ZDT1 and ZDT2) produced by WSFM and WMM approaches are very similar. However, the comparison between these approaches for the DTLZ2 test problem (Figure 11) evidences some differences. As the RP-EMO approach, the WMM approach gave more importance to criterion 3 than to criterion 1, which does not consider the equilibrium between these two criteria as defined by the weight preferences vector defined by the DM.

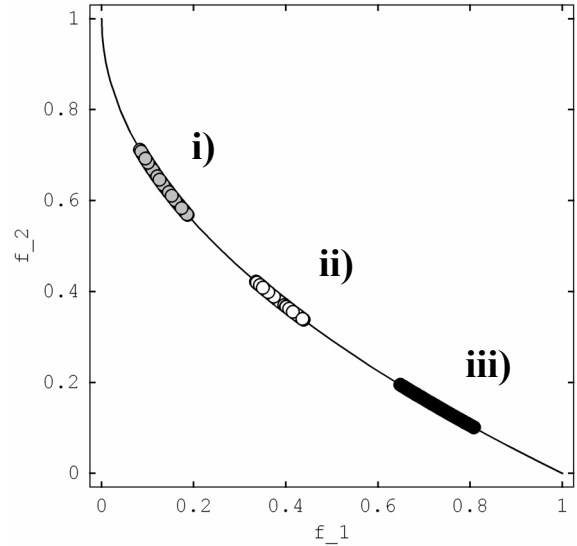


Figure 9- Results for ZDT1 problem using the WMM approach- weight vectors: i) (0.8,0.2), ii) (0.5,0.5) and iii) (0.2,0.8).

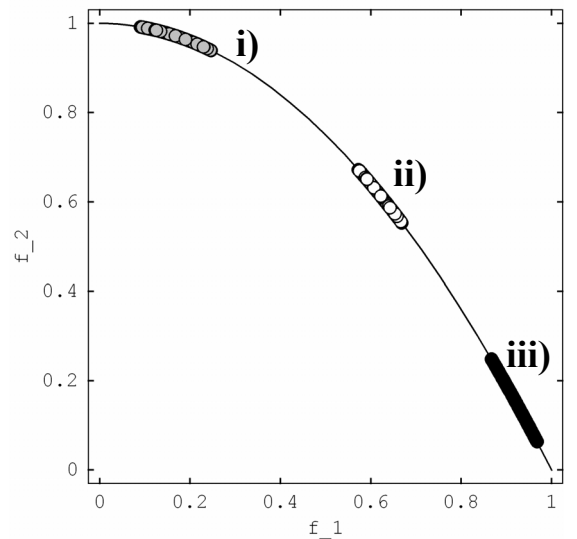


Figure 10- Results for ZDT2 problem using the WMM approach - weight vectors: i) (0.8,0.2), ii) (0.5,0.5) and iii) (0.2,0.8).

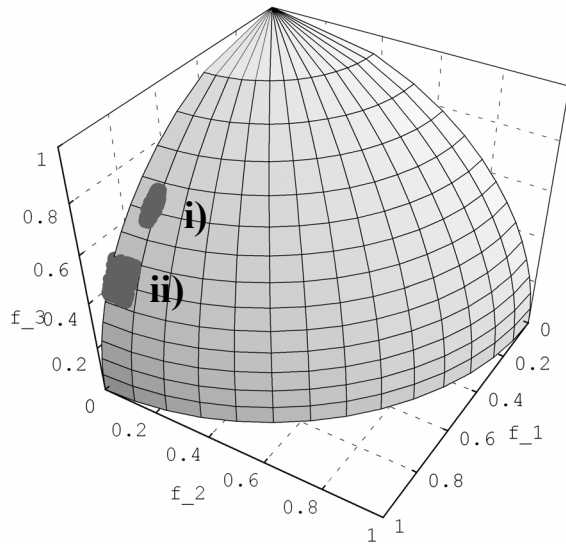


Figure 11- Comparison between the i) WSFM and ii) WMM approaches for DTLZ2 problem - weight vector for both: (0.07,0.8,0.13).

4. CONCLUSIONS

The WSFM approach allows the selection of the best solutions taking into account the DM preferences. This methodology produces good results for problems with two and three criteria and for convex and non-convex Pareto frontiers. The method proposed was not suitable for discontinuous fronts. However, a modification of this method was tested with good results for this type of problems.

The implementation of this method allows the possibility of the DM to conduct the search process for specific regions of the Pareto frontier. Also, the dispersion parameter allows one to control the size of the final Pareto frontier region obtained.

The assessment with other methods available shown that the WSFM approach is a good alternative for selecting solutions taking into account the DM preferences.

5. ACKNOWLEDGMENTS

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