Guided Hyperplane Evolutionary Algorithm

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ABSTRACT

A new evolutionary technique for multicriteria optimization called Guiding Hyper-plane Evolutionary Algorithm (GHEA) is proposed. The originality of the approach consists in the fact that the fitness assignment is realized by using a guiding hyperplane and a new non Pareto optimality concept. Numerical experiments illustrate the performance of GHEA compared with the popular NSGA-II and SPEA2.

Categories and Subject Descriptors

I.2.8 [**Artificial Intelligence**]: Problem Solving, Control Methods, and Search—*Heuristic methods*

General Terms

Algorithms

Keywords

evolutionary multiobjective optimization, guiding hyperplane

1. INTRODUCTION

A new evolutionary multiobjective optimization technique called Guiding Hyperplane Evolutionary Algorithm (GHEA) is proposed. GHEA introduces a new fitness assignment procedure. Based on a guiding hyperplanes concept GHEA provides an intuitive form of guidance towards the optimal frontier. Individuals in the current population are assigned two fitness values that are computed with respect to the guiding hyperplane. Each one of them is used during a different type of selection process. A *predomination* relation for individuals is introduced. Selection for survival is based on this relation.

2. GHEA BASIC IDEAS

Two issues arise regarding population dynamics within evolutionary algorithms: the first refers to preserving population diversity as an important factor for the success of the

search process and the second one refers to the convergence towards the problem solutions. For complex problems such as multicriteria optimization problems, the balance between search diversification and search intensification is crucial.

The main goal in developing GHEA is to produce efficient population dynamics with respect to both diversity and intensification aspects.

2.1 Guiding hyperplane

Consider a real multicriteria optimization problem involving n objective functions defined in the Euclidean space R^s. In the objective space the Pareto front is a hyper-surface of n order. The intuitive idea is to use a hyperplane to guide the search in the objective space. This guiding hyperplane is updated each generation in order to lead the search towards the most promising region possible for the current population.

For constructing the Guiding Hyperplane (GH) the following criteria are envisaged:

(i) GH does not intersect the region covered by individuals in the current population i.e. the convex combination (CC) of the points in the current population;

(ii) No hyperplane points are dominated by any individual of the current population.

Such a hyperplane would guide the population towards promising areas of the search space. The population dynamics - guided by the hyperplane - eventually leads to convergence to the Pareto front.

The construction of a hyperplane parallel to the principal direction of the current population and which does not intersect the convex combination of the current population members is suggested.

The guiding hyper-plane which is built for each generation is also called the *target* of the current generation.

The mechanism of replacing parents with offspring is designed to accelerate population convergence, i.e. to favor the survival of the best solutions already found.

2.2 Fitness assignment

Individuals in the current population are evaluated by means of their relation to the guiding hyperplane. Each generation the evolution of the current population towards the corresponding target hyperplane is realized by focusing on the production of offspring closer to the target and not necessarily on the convergence to the Pareto front.

The GH constructed along the principal direction can also be used to estimate the diversity of the current population.

Projecting the individuals of the present population on the target hyperplane provides an image of the population's distribution. In order to maintain a good distribution during the search process GHEA is endowed with a supplementary mechanism that favors the search in the areas having relatively few individuals.

With respect to the GH two measures are associated to each individual in the current population:

(i) the distance to the guiding hyperplane - used in the selection for recombination;

(ii) the agglomeration degree of the individuals projection in the guiding hyperplane - used in the selection for mutation process.

Both measures are intended to set the balance between exploration and exploitation, which is necessary in ensuring a good population dynamics.

3. CONSTRUCTING THE GUIDING HYPERPLANE

Let $D \subset R^s$ be the search space. Consider n objective functions $f_i : D \rightarrow R, i = 1, ..., n$ and denote $F(x) = (f_1(x), f_2(x), ..., f_n(x))^T$, $F : D \rightarrow R^n$. All vectors are considered as column vectors. Consider the following unconstrained minimization problem:

$$\begin{cases} \text{minimize} & F(x) = (f_1(x), f_2(x), ..., f_n(x))^T, \\ & x \in D \end{cases} \quad (1)$$

The current population may be viewed as a cluster of n-dimensional points in the objective space. The current cluster population in the objective (n-dimensional) space can be described using the principal components analysis [4].

Let $A = (\alpha^2, \alpha^2, ..., \alpha^r)$ be the current population of r individuals forming a cluster in R^n.

Let m be the mean vector (the centroid) of the set A. The most important directions along which cluster A is spread out are given by the eigenvectors of the scatter matrix S defined as:

$$S = \sum_{i=1}^{d} (\alpha^i - m)(\alpha^i - m)^T.$$

Eigenvectors of the matrix S are called the principal directions of the cluster A [4].

The eigenvector u corresponding to the largest eigenvalue indicates the most important direction along which the cluster is spread out.

Denote by x^0 the vector

$$x_i^0 = \min\{\alpha_i^k | k \in \{1, 2, ..., d\}\}, i, \in \{1, 2, ..., n\}.$$

Therefore each component x_i^0 is the minimum with respect to all components α_i^k of the vectors $\alpha_k \in A$. x_0 represents an 'ideal' vector formed with the best components of the vectors in A.

The hyperplane GH parallel to direction u and passing though x^0 represents the guiding hyperplane of the set A and is described by the equation:

$$u^T(x - x^0) = 0.$$

GH is used to evaluate individuals with the purpose to guide the search of population A towards what could be called the most promising region provided by it.

Figure 1 illustrates the construction of GH for a population of individuals evaluated using problem ZDT1 [5]. GH is represented by a line passing through x^0 and parallel to the principal direction of the cluster formed by the individuals in the current population.

4. GHEA FITNESS ASSIGNMENT

A vectorial fitness function is considered. Each component addresses a specific purpose. Consider individual c in the current population $P(t)$. We write $eval(c)$ as the three dimensional vector

$$eval(c) = (\delta(c), \gamma(c), \omega(c, Q)),$$

where $\delta(c)$ and $\gamma(c)$ are computed with respect to the guiding hyperplane GH, $\omega(c, Q)$ is computed using a proposed predomination relation and Q represents a population of individuals.

$\delta(c)$ *distance to GH* .

$\delta(c)$ measures the distance between individual c and GH and it is used during the selection for recombination process. Thus individuals that are 'closer' to GH have higher chances to be selected. $\delta(c)$ represents the distance between c and its projection c' on GH which is computed using

$$c_i' = c_i + u^T(x^0 - c_i)u.$$

$\gamma(c)$ - *agglomeration of population.*

$\gamma(c)$ measures the minimum distance between the protection of individual c on GH and the projections on GH of all the other individuals in the current population. It is used during the selection for mutation process: individuals in crowded areas have higher chances to be selected for mutation thus ensuring population diversity.

Predomination Relation.

A new relation on $D \subset R^s$ called *predomination* is proposed.

Definition 1. Let a and b be two feasible solutions for Problem 1. We say that a *predominates* b if there exists a least one objective f_i such that

$$f_i(a) < f_i(b).$$

Remark 1. An individual a predominates individual b if a is not dominated by b, or if either a dominates b or a is indifferent to b. It is possible that a predominates b and b predominates a.

Using the predomination relation we define $\omega(c, Q)$ as the number of individuals in the the population Q that are predominated by c, i.e.

$$\omega(c, Q) = |\{q \in Q | c \text{ predominates } q\}|.$$

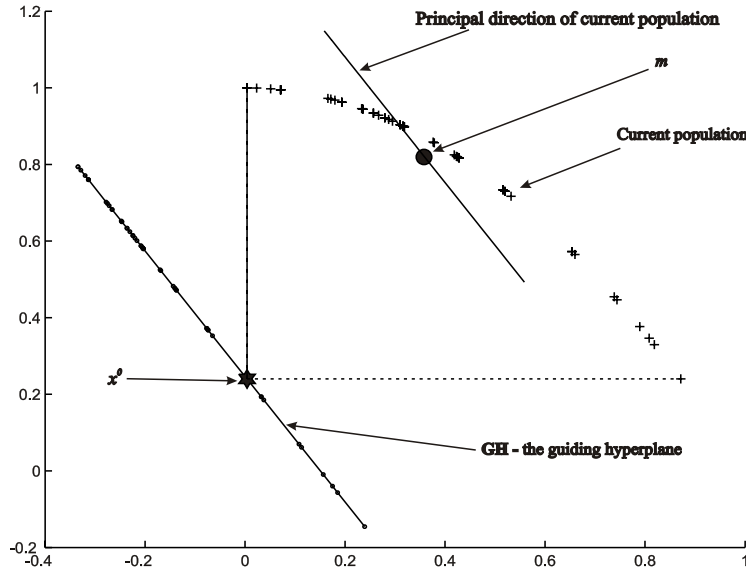$\omega(c, Q)$ is used in the selection for survival process.

Figure 1: Illustration of a guiding hyperplane: "+" represent individuals in the current population, the filled circle represents m, the centroid of the cluster formed by current population, the "⋆" represents x^0 and GH is represented by the line containing the projections of individuals on it.

5. COMPUTING NEXT GENERATION

Offspring obtained by crossover form an intermediary population $P_1(t)$, while descendants obtained by mutation are included into a second intermediary population $P_2(t)$.

A larger population $\Gamma(t)$ is obtained, comprising all parents and descendants

$$\Gamma(t) = P(t) \cup P_1(t) \cup P_2(t).$$

Each individual c in $\Gamma(t)$ is assigned a rank equal to $\omega(c, \Gamma(t))$. Best individuals (having the highest rank) replace the parent population constituting the next current population $P(t+1)$.

6. OUTLINE OF GHEA

GHEA uses a guiding hyperplane to conduct the search of the population toward the Pareto frontier of a MOP. The guiding hyperplane GH is constructed parallel to the principal direction of the cluster formed by the objective values of the population and contains the 'ideal' vector having as components the minima of all components of individuals in the current population. Each generation the hyperplane is updated.

Individuals are assigned three fitness values - each of them for different selection purposes. The distance δ to GH is used for the selection for recombination, γ - providing the agglomeration degree of an individual is used during the selection for mutation process and ω is used during the selection for survival.

GHEA may be generalized using a different mutation mechanism. According to this mechanism the mutation operator acts in two ways: uniform mutation of a single gene or uniform mutation of all genes, depending on a randomly generated parameter that decides which type of mutation will be applied further. Guided Hyperplane Evolutionary algorithm is outlined as follows.

Algorithm 1 Guiding Hyperplane Evolutionary Algorithm

Parameters setting;
randomly initialize population $P(0)$;
t:=0;
while not termination condition **do**
    Compute the target hyperplane GH for the current population $P(t)$;
    Evaluate the population $P(t)$ with respect to GH;
    $P_1(t) :=$ Crossover $(P(t))$;
    $P_2(t) :=$ Mutation $(P(t))$;
    Construct population $\Gamma(t) = P(t) \cup P_1(t) \cup P_2(t)$;
    Evaluate population $\Gamma(t)$ by using performance function ω;
    Set $P(t+1)$ by replacing current population $P(t)$ with the best distinct individuals of the population $\Gamma(t)$;
    t:=t+1;
end while

Output of GHEA is the set of nondominated individuals in the final population.

7. NUMERICAL EXPERIMENTS

Numerical experiments illustrating the efficiency of GHEA are presented. The ZDT1-ZDT4 [5] and DTLZ1-DTLZ3 [3] suite of test problems are considered. Results obtained by using GHEA are compared with those obtained in similar conditions by the popular SPEA2 [7] and NSGA II [2]. Ten runs are considered for each problem and each algorithm.

The considered set of test functions share a particular feature: the set of decision variables is given a structure through which two subsets are defined. Thus each variable is written $X = (X_1, X_2)$, where $X_1 = x_1$ and $X_2 = (x_2, x_3, ..., x_m)$. We shall treat the two subsets differently when applying variation operators. Thus the probability of

Table 1: Parameter settings for the experiments

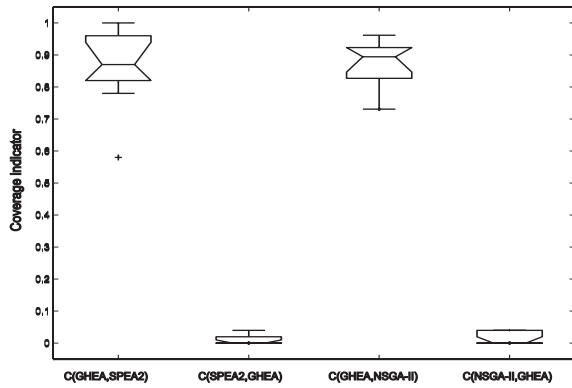| Parameter	GHEA	SPEA2	NSGA-II
Population size	50	50	52
Maximum no. of gen.	100	100	100

Figure 5: Problem ZDT1: values for coverage metric

selecting a gene also depends on the subset that includes it. More specific X_1 will be given a higher probability for mutation.

The parameter settings for GHEA and SPEA2 and NSGA-II are presented in Table 1 for all the test problems. The PISA implementation of SPEA2 [1] and the one of NSGA-II from the Deb's KANGAL repository were used. A population of 50 individuals is considered for only 100 generations. The specific parameters for SPEA2 and NSGA-II provided with the implementations were not modified.

Results are compared using two performance metrics.

Coverage indicator.

The coverage indicator $I_{C(A,B)}$ introduced in [6] gives for a pair (A,B) of approximation sets the fraction of solutions in B that are weakly dominated by one or more solutions in A.

Boxplots for the results obtained over the ten runs for the six problems considered are presented in figures 5-10. Results indicate GHEA approximation sets to be better than the others with respect to this indicator. Descriptive statistics regarding coverage metric values are presented in tables 2 and 3.

Statistical evidence that the results obtained by GHEA are significantly better than those obtained by SPEA2 and NSGA-II in terms of coverage indicator is provided by a
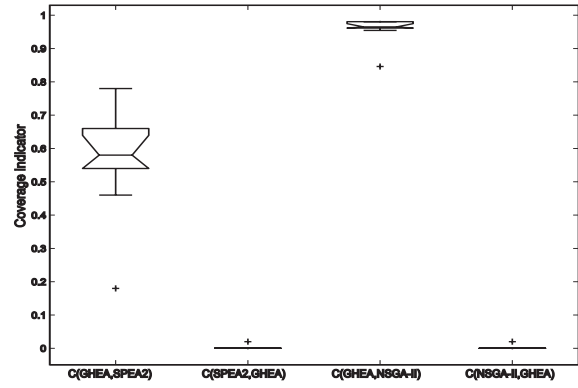

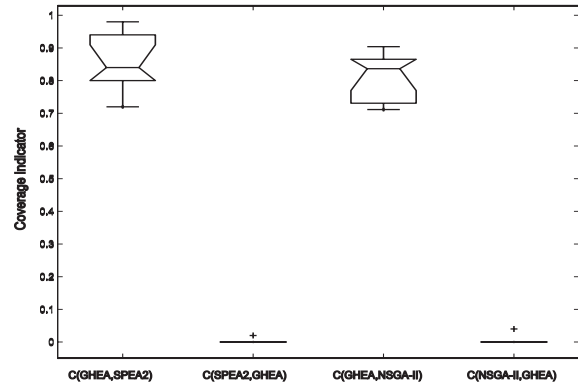**Figure 6: Problem ZDT2: values for coverage metric**

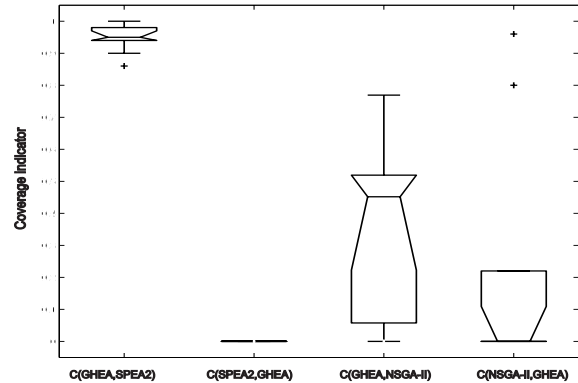**Figure 7: Problem ZDT3: values for coverage metric**

Figure 8: Problem DTLZ1: values for coverage metric

Table 2: Coverage indicator: descriptive statistics

	C(GHEA, SPEA2)		C(SPEA2,GHEA)	
	Mean	St.dev.	Mean	St.dev.
ZDT1	**0.86**	0.119	0.01	0.016
ZDT2	**0.57**	0.166	0.002	0.006
ZDT3	**0.856**	0.083	0.002	0.006
ZDT4	**0.524**	0.299	0.000	0.000
DTLZ1	**0.948**	0.043	0	0
DTLZ2	**0.542**	0.089	0	0
DTLZ3	**0.932**	0.044	0	0

Table 3: Coverage indicator: descriptive statistics

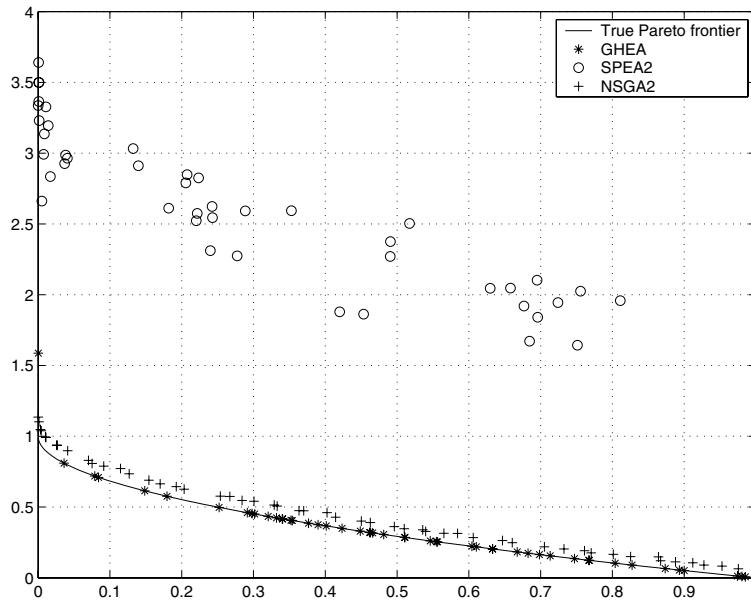|	C(GHEA, NSGA-II)		C(NSGA-II,GHEA)	
	Mean	St.dev.	Mean	St.dev.
ZDT1	**0.873**	0.075	0.012	0.019
ZDT2	**0.957**	0.040	0.002	0.006
ZDT3	**0.813**	0.072	0.008	0.016
ZDT4	**0.505**	0.251	0.067	0.203
DTLZ1	**0.388**	0.281	0.198	0.367
DTLZ2	**0.051**	0.033	0.014	0.044
DTLZ3	**0.153**	0.095	0	0

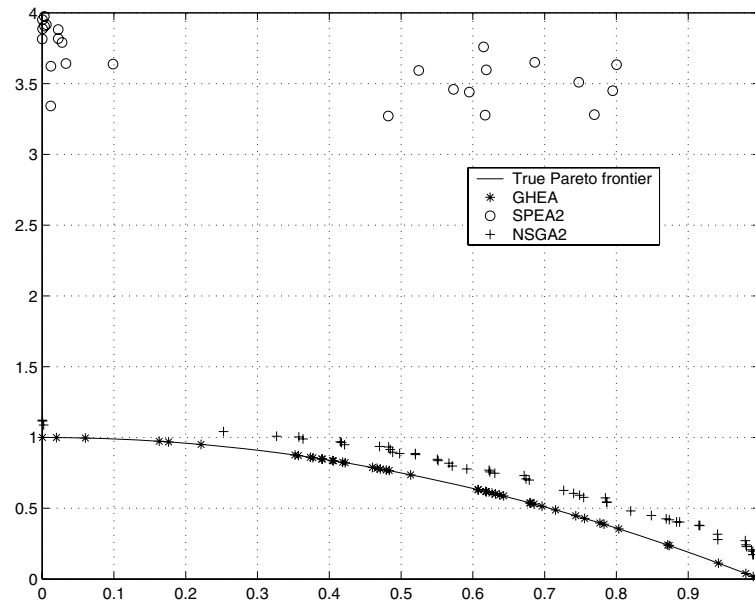**Figure 2: Problem ZDT1: approximation sets provided by GHEA, NSGA-II and SPEA2**

Figure 3: Problem ZDT2: approximation sets provided by GHEA, NSGA-II and SPEA2

Table 5: Hypervolume indicator: descriptive statistics

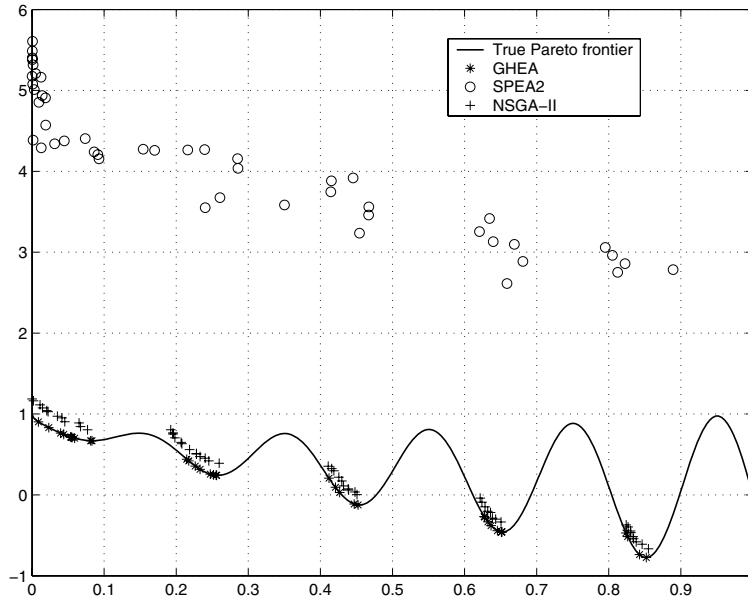|	GHEA		NSGAII		SPEA2	
	Mean	St.dev.	Mean	St.dev.	Mean	St.dev.
ZDT1	**7.59**	0.038	7.56	0.017	5.53	0.13
ZDT2	**6.29**	0.029	6.17	0.042	3.68	0.147
ZDT3	**12.86**	2.11	9.58	0.01	6.79	0.21
ZDT4	**140.34**	0.4157	138.40	1.1474	123.24	7.8943
DTLZ1	1.2495e8	2.46e5	**1.2499e8**	184.18	1.2435e8	2.49e5
DTLZ2	9.49	0.25	**10.18**	0.08	9.53	0.14
DTLZ3	4.094e9	9.81e4	**4.095e9**	1.35e5	3.917e8	2.894e7

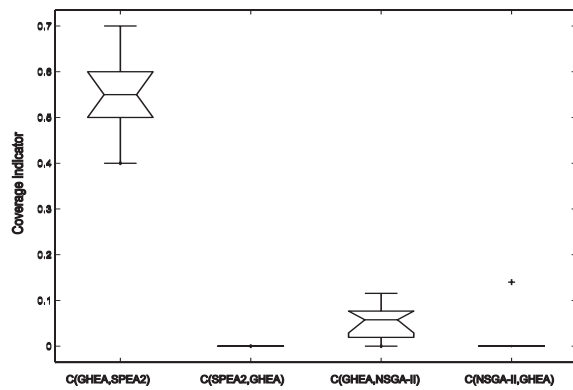**Figure 4: Problem ZDT3: approximation sets provided by GHEA, NSGA-II and SPEA2**

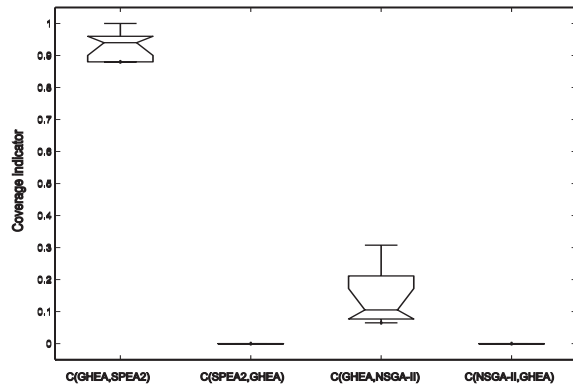**Figure 9: Problem DTLZ2: values for coverage metric**

Figure 10: Problem DTLZ3: values for coverage metric

Table 4: Coverage indicator: t-values and p-values for the Welch Two Sample t-test

	GHEA- SPEA2		GHEA-NSGA-II	
	t-value	p-value	t-value	p-value
ZDT1	**22.31**	9.73e-10	**34.74**	3.38e-12
ZDT2	**10.77**	9.37e-7	**74.01**	1.12e-14
ZDT3	**32.17**	5.45e-11	**34.19**	5.73e-12
ZDT4	**5.54**	1.7e-4	**4.2889**	2.41e-4
DTLZ1	**69.05**	7.07e-14	1.30	0.10
DTLZ2	**19.11**	6.67e-9	**2.14**	0.02
DTLZ3	**66.34**	1.014e-13	**5.07**	0.0003

Welch Two Sample t-test with 0.95 significance. The corresponding t statistics and p-values are presented in table 4. All t-values greater than 2.10 indicate that the alternative hypothesis can be accepted. Results indicate significant differences for all cases except problem DTLZ2.

The Hypervolume indicator.

The hypervolume indicator $I_H(A)$ computes the size of the space dominated by the given approximation set A. It requires a reference point that we have computed by choosing the maximum objective values from all the results obtained for each problem. Descriptive statistics regarding the values obtained are presented in table 5.

Shapiro-Wilk W test indicates that not all the results follow a normal distribution therefore the non parametric test Wilcoxon rank-sum test is used to analyze them. W-values and p-values obtained are presented in table 6. Results indicate significant differences with respect to the hypervolume indicator except two cases: DTLZ2 for GHEA vs SPEA2 and ZDT1 for GHEA vs. NSGA-II. These results are consistent with the corresponding descriptive statistics indicating that GHEA to be a competitive technique.

Table 6: Hypervolume indicator: W and p-values for the Wilcoxon rank sum-test

	(GHEA, SPEA2)		(GHEA,NSGA-II)	
	W-value	p-value	W-value	p-value
ZDT1	100	1.083e-05	82	0.01469
ZDT2	100	1.083e-05	98	4.33e-05
ZDT3	100	1.083e-05	100	1.083e-05
ZDT4	100	1.083e-05	99	3.29e-04
DTLZ1	100	1.083e-05	0	8.745e-05
DTLZ2	50	1	0	1.083e-05
DTLZ3	100	0.1e-4	10.5	0.003

Figure 12: Problem DTLZ1: Best attainment surface for NSGA-II after 100 generations
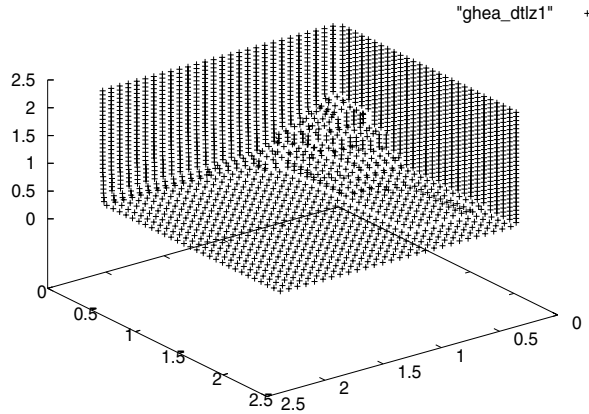

Figure 11: Problem DTLZ1: Best attainment surface for GHEA after 100 generations

Attainment surfaces.

Figures 11 and 12 present best attainment surfaces from the ten runs from GHEA and NSGA-II respectively approximation sets.

Results from SPEA2 are not drawn for problems DTLZ1 and DTLZ3 because there are very few solutions located in the bounding area of the graphics.

Best attainment surfaces obtained for problem DTLZ2 are presented in figures 13 and 15.

Discussion of results.

Three indicators for comparing the results obtained by GHEA, NSGA-II and SPEA2 have been used: coverage indicator, hypervolume indicator and for DTLZ1-DTLZ3 best attainment surfaces have been drawn.

According to coverage indicator more individuals resulting from GHEA dominate individuals resulting from NSGA-II and SPEA2, except for problem DTLZ1 where the difference in means of values of coverage for the results obtained with NSGA-II and GHEA is not significant.

Wilcoxon sum rank test used for the hypervolume indicator I_H values show GHEA to achieve better values than NSGA-II and SPEA2 for problems ZDT2 and ZDT3. For problem ZDT1 results from GHEA are better than those from SPEA2 but not different from NSGA-II.

For problems DTLZ1-3 the best values for I_H are obtained by NSGA-II and then by GHEA. For DTLZ2 problem the Wilcoxon test indicates that the difference between results of SPEA2 and GHEA is not significant.
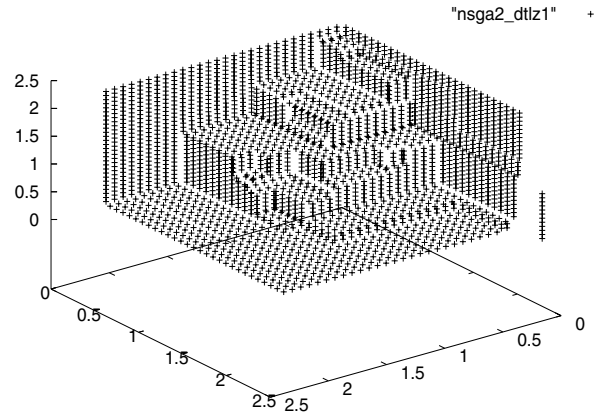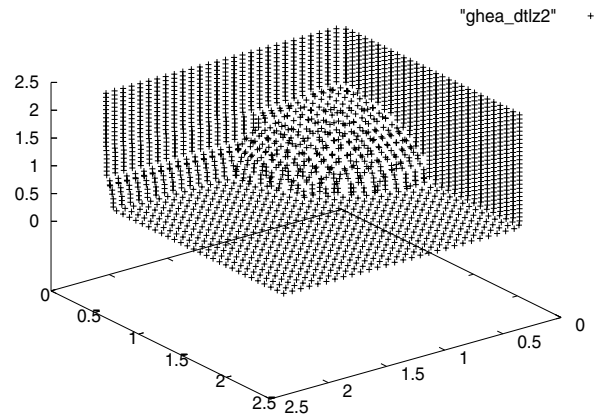

Figure 13: Problem DTLZ2: Best attainment surface for GHEA after 100 generations

Because the values of the hypervolume indicator can be affected by the reference point, in order to show that results obtained by GHEA are competitive best attainment surfaces are drawn for problems DTLZ1and DTLZ2 . In the case of DTLZ1 GHEA presents the best surface and for DTLZ2 surfaces drawn for GHEA and NSGA-II are visibly similar.

8. CONCLUSIONS

A new approach for evolutionary optimization called Guiding Hyperplane Evolutionary Algorithm (GHEA) is proposed. The main principle of GHEA is based on computing an adaptive guiding hyperplane that controls the population dynamics with the purpose of leading the search toward the Pareto frontier.

As far as the main features of a good approximation of the Pareto front are concerned, i.e. diversity and convergence, two fitness functions are used to evaluate each solution with regard to the current target hyperplane: one measuring the distance to the hyperplane and the other one the crowdedness of the projections of current solutions into the hyperplane.
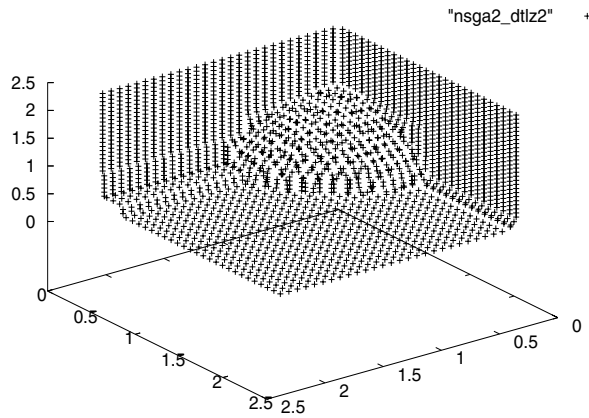
"nsga2_dtlz2"   +

Figure 14: Problem DTLZ2: Best attainment surface for NSGA-II after 100 generations
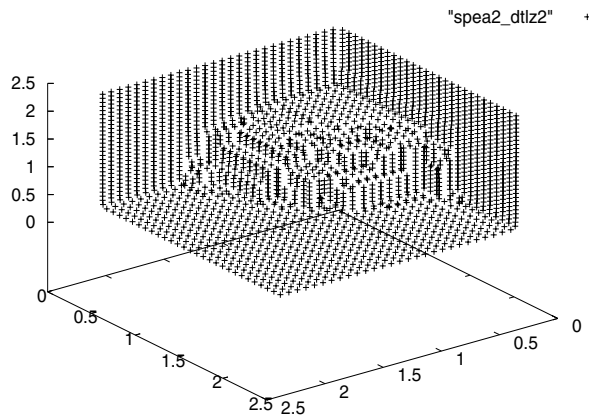

"spea2_dtlz2"   +

Figure 15: Problem DTLZ2: Best attainment surface for SPEA2 after 100 generations

A "predomination" relation is proposed in order to be used during the selection for survival process.

Experimental results indicate that GHEA performs well for various static optimization problems.

The target hyperplane guides the search gradually towards the promising regions of the search space leading to good approximations of the Pareto frontier.

Statistical tests made on the selected performance metrics indicate that GHEA outperforms SPEA2 and NSGA-II for the considered experimental set-ups.

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