

# Stress-based Crossover Operator for Structure Topology Optimization using Small Population Size and Variable Length Chromosome

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## ABSTRACT

This paper talks about genetic algorithm (GA) with a stress-based crossover operator (SX) to multi-constrained structural topology optimization. By bitstring chromosome representation, the design variables are very large that result in long computation time. Population size, as the key to cut computation time, is discussed through a number of small population sized experiments. Moreover, we introduce a variable length chromosome to further optimize the topology hierarchically.

**Categories and Subject Descriptors:** G.2.3 Applications: Genetic Algorithms

**General Terms:** Algorithms.

**Keywords:** genetic algorithm, stress-based crossover, variable length chromosome, structure topology optimization.

## 1. INTRODUCTION

Main approaches to structural topology optimization (STO) include homogenization[1], solid isotropic microstructure with penalization (SIMP)[2], evolutionary structural optimization (ESO)[3], bi-directional evolutionary structural optimization (BESO)[4] and evolution computation methods, such as, genetic algorithm (GA), multi-objective GA, etc.

This paper presents a stress-based crossover operator (SX)[5], by which neighboring elements connectivity are considered during the procedure. Population size, as the key factor to this problem, is discussed in this paper. Moreover, researches demonstrate SX can find out the general shape even using coarse meshes. Therefore, a variable length chromosome is introduced to optimize the structure hierarchically.

## 2. STRESS-BASED CROSSOVER OPERATOR

In this section, the procedures of stress-based crossover operator are introduced in detail. Firstly, the nomenclatures used in this operator are explained.

- $P(t) = \{p_i(t) | i \in \{1 \dots n\}\}$  is population of generation  $t$ ,  $n$  is the population size.
- $p_i(t)$  is one individual.
- $p_i(t).weight$  is number of "1" in chromosome.
- $p_i(t).code[k] \in \{0, 1\}$  is one gene, where  $k \in \{1 \dots N\}$ ,  $N$  is chromosome length.

- $p_i(t).stress[k]$  is stress of element  $k$ .
- $p'_i(t).ability[k]$  is ability of gene  $k$  of child individual  $p'_i(t)$ .

### Procedures of SX:

- (1) Randomly select two individuals,  $p_i, p_j$  from  $P(t)$ .
- (2) Add up the element stress on each gene of  $p_i$  and  $p_j$  by formula (1). Naming this value as the ability of each gene of child individual  $p'_i(t)$ .

$$p'_i(t).ability[k] = p_i(t).stress[k] + p_j(t).stress[k], k = 1 \dots N \quad (1)$$

- (3) Sort the  $p'_i(t).ability[k], k = 1 \dots N$  from big to small.
- (4) According to the *ability* value of each gene, the bigger ability valued genes will be set "1". Namingly, divide the genes into two groups,  $U1$  and  $U0$ .  $U1$  is group of the front  $m$  genes.  $U0$  is group of the last  $N - m$  genes. In this study,  $m$  is defined by formula (2). Generate a child individual by formula (3).

$$p'_i(t).weight = \frac{p_i(t).weight + p_j(t).weight}{2} \quad (2)$$

$$p'_i(t).code[k] = \begin{cases} 1, & \text{if } p'_i(t).ability[k] \in U1 \\ 0, & \text{if } p'_i(t).ability[k] \in U0 \end{cases} \quad (3)$$

Applying these four steps on population  $P(t)$  to generate new individuals. After SX operation, uniform mutation operator is applied to each gene of each individual on a small rate.

## 3. OBJECTIVE FUNCTION

The numerical example is the mbb-beam problem as shown in Figure 1. The objective function is defined to minimize the weight as maximal stress  $Stress_{max} < Stress_{lim} = 3.3 \times 10^9 (N/mm^2)$  and maximal displacement  $Disp_{max} < Disp_{lim} = 0.33 (mm)$ .

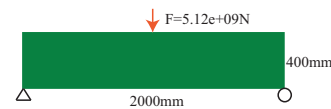


Figure 1: MBB beam problem

The following material properties are assumed: Young's modulus  $E = 206 GPa$  for "1" and  $E = 103 MPa$  for "0", Poisson's ratio  $\nu = 0.3$ .

## 4. POPULATION SIZE DISCUSSION

In this section we will discuss these questions. Which is the best population size for SX to structure topology optimization? How does the population size affect to the solution for SX to structure topology optimization problem? Does the population size relate to mesh size (in other word, design variable number)?

Population sizes of 2, 4, 6 and 8 are experimented, respectively. Two mesh sizes, (100, 100) mm and (50, 50) mm, are adopted.

### 4.1 Experiments Results and Discussion

For each experiment we run four trials on same parameters. The best solutions for each experiment are showed in Figure 2 and Figure 3. Where,  $Psize$  represents population size. The fitness evolution histories are compared in Figure 4 and Figure 5.

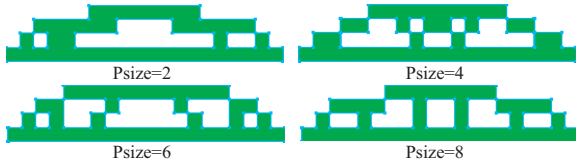


Figure 2: Solutions by Mesh size (100, 100) mm

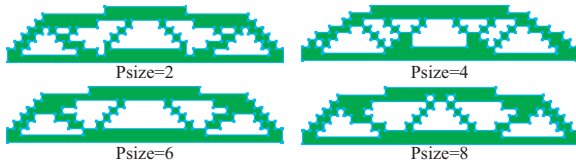


Figure 3: Solutions by Mesh size (50, 50) mm

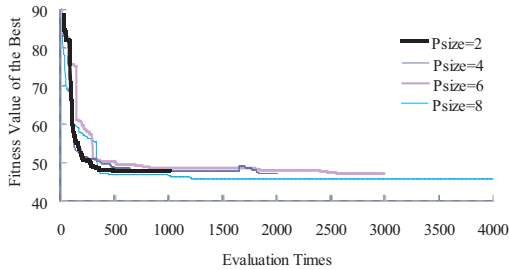


Figure 4: Fitness Evolution Histories Comparison on Mesh size (100, 100) mm

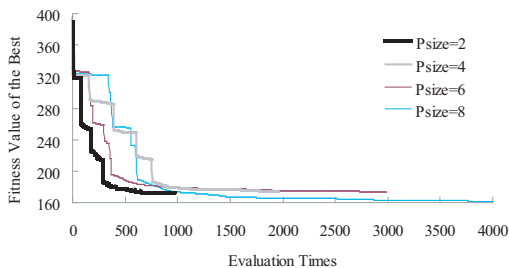


Figure 5: Fitness Evolution Histories Comparison on Mesh Size (50, 50) mm

The geometric results demonstrate SX can search out a solution on population size 2. Fitness evolution histories comparisons in Figure 4 and Figure 5 show final solution's weight on population size 8 is smaller, which indicates big population drives to more optimal solution.

## 5. VARIABLE LENGTH CHROMOSOME

To further smooth the solution obtained on coarse mesh, each mesh is quadrupled by subdividing each element into four smaller elements like Figure 6. After that, the outside boundary elements are appended to solid element as shown in Figure 6 (b). This structure is named mask structure -  $p_{mask}$ . The new initial population is generated randomly. After that, a logic "and" operator is applied on each individual and the mask individual -  $p_{mask}$  that aims to eliminate the elements outside the mask structure.

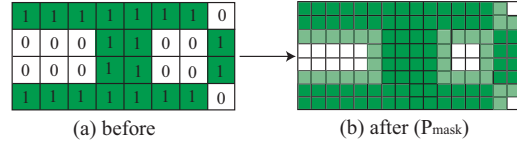


Figure 6: Variable Length Chromosome (a) before and (b) after subdivision

### 5.1 Experiment Results and Discussion

At this section we experiment on the same constraints defined in section 3. Mesh size (100, 100) mm is used in the internal genetic procedures. Mesh size (50, 50) mm is used in the external procedures. The elite solution of internal procedures is shown in Figure 7. The elite solution of external procedures is shown in Figure 8. The numerical properties of both solutions are listed in Table 1.



Figure 7: Elite Solution of Internal GA



Figure 8: Elite Solution of External GA

Table 1: Numerical Results of Figure 7, 8

Index	Weight(%)	$Stress_{max}$	$Disp_{max}$
Figure 7	55.0%	1.644e+07	0.322
Figure 8	52.2%	2.861e+07	0.319

Experiment presents the boundary becomes smooth and the numerical results are also more optimal.

## 6. CONCLUSION

Small population is effective for SX to STO. However, big design domain problem needs big population for a good solution. Experiment of hierarchical coded GA shows the solution, which is obtained by a coarse mesh, can be further optimized. Moreover, the computation time can also be reduced.

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