A Domain-Specific Crossover and a Helper Objective for Generating Minimum Weight Compliant Mechanisms

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ABSTRACT
While designing the Compliant Mechanisms (CM), an equal attention is required on both the problem formulation and the optimization algorithm used. Authors of this paper have successfully proposed the formulation of CM tracing user-defined paths based on the precision points [3, 5]. In this paper, authors modify the NSGA-II algorithm by incorporating (i) a helper objective and (ii) a domain specific crossover which assist in generating a diverse set of non-dominated solutions. First, the single-objective optimization problem of minimizing the weight of structure is solved and named the reference design. Thereafter, a bi-objective optimization problem is dealt to evolve ‘trade-off’ solutions based on precision points and stress. A bi-objective formulation is given below:

Minimize: Weight of structure,  
Maximize: Diversity of structure,  
such that:

\[ 1 - \eta \frac{\sqrt{(x_{i2} - x_{i1})^2 + (y_{i2} - y_{i1})^2}}{\sqrt{((x_{i1} - x_{i2})^2 + (y_{i1} - y_{i2})^2)}} \geq 0, \quad i = 1, 2, ..., N \]

where \( x_{i1} \) and \( x_{i2} \) are the \( i \)th precision point and the corresponding point on actual path after FE analysis of structure, \( \eta = 15\% \) is allowed deviation (kept fixed in this paper), and \( \sigma_{\text{flexural}} \) and \( \sigma \) are flexural yield strength of material and maximum stress developed in the structure, respectively.

The design domain (DD) of CM is shown in Figure 1. It is discretized by 4 node rectangular elements and each element is represented either by 1 (filled with material) or 0 (void). Before the FE analysis, the DD is categorized into the three regions of interest. The I\( ^{st} \) region is called support region where the nodes of structure are restrained with zero displacement whereas, in the II\( ^{nd} \) region (loading region) some input displacement is applied. The output region is the III\( ^{rd} \) region, that is, the fixed point on the structure which traces out the desired path defined by user. Here, a binary string is made of two sets. The I\( ^{st} \) set represents the shape of structure whereas, the decoded value of the II\( ^{nd} \) set defines the required boundary conditions, for example, support and loading locations within their respective regions, and the input displacement magnitude ranging from 1 mm to 16 mm at a step of 1 mm [4].

We modify the NSGA-II algorithm by incorporating a domain specific crossover which assists in generating a diverse set of non-dominated solutions. This crossover works by exchanging the sub-domains between the two selected parents. As it is shown in Figure 2, points P1, P2 and P3 are randomly generated from three regions on precision points and stress. A bi-objective formulation is given below:

\[ \text{Minimize: Weight of structure, } \quad \text{Maximize: Diversity of structure, } \]

\[ \text{subject to: } \quad 1 - \eta \frac{\sqrt{(x_{i2} - x_{i1})^2 + (y_{i2} - y_{i1})^2}}{\sqrt{((x_{i1} - x_{i2})^2 + (y_{i1} - y_{i2})^2)}} \geq 0, \quad i = 1, 2, ..., N \]

\[ \sigma_{\text{flexural}} - \sigma \geq 0, \]

\[ 3 \times \text{weight of reference design} - \text{weight of structure} \geq 0, \]

where \( x_{i1} \) and \( x_{i2} \) are the \( i \)th precision point and the corresponding point on actual path after FE analysis of structure, \( \eta = 15\% \) is allowed deviation (kept fixed in this paper), and \( \sigma_{\text{flexural}} \) and \( \sigma \) are flexural yield strength of material and maximum stress developed in the structure, respectively.

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of interest. The domain is divided into the sub-domains by joining these three points by the straight lines. Thereafter, on an average two of the four sub-domains are swapped between the parents. Mutation is a standard bit-wise operator.

2. RESULTS AND COMPARISON

The design domain of CM (50 mm by 50 mm) is discretized with 50 by 50 rectangular elements in x and y directions respectively. A material with Young’s modulus of 3.3 GPa, flexural yield stress of 6.9 MPa, density of 1.114 gm/cm$^3$ and Poisson ratio of 0.40, is assumed for the synthesis of CM. A population of 240, crossover probability of 0.95 and mutation probability of (1/string length) are assigned and NSGA-II is run for a maximum of 100 generations.

2.1 Reference Design

By solving a minimum-weight single-objective problem with a GA having binary tournament selection and above crossover & mutation operators, a reference design (Figure 3) of 0.6985 gms is evolved. It is supported and loaded at their respective region’s elements at 4 mm and 44 mm from the origin, respectively. It requires 6 mm of input displacement for tracing a user-defined curvilinear path.

2.2 Bi-Objective Optimization Analysis

The solutions of single and bi-objective analyses are shown in Figure 5. A significant improvement of the representative NSGA-II solutions can be seen after the local search. But after the local search, the solutions of bi-objective study are grouped into two regions. One region signifies minimum weight solutions whereas, other indicates maximum diverse structures with respect to the reference design. When the solutions of both studies are compared, solutions 1 and 2 come out to be lighter in weight with respect to the reference design. It reveals a fact that the helper objective not only generates the ‘trade-off’ solutions but the added diversity in population helps in evolving even a smaller weight solutions than that by a single-objective study. The evolved designs of solutions 1 and 2 as shown in Figure 4 are supported and loaded at their respective side’s elements which are 10 mm and 32 mm away from the origin, respectively and require 7 mm of input displacement.

2.3 Comparison With Another Study

A comparative study of NSGA-II solutions is done using the proposed crossover and another crossover operator which works by exchanging the rows/columns [2] as oppose to ‘functional’ exchange proposed in this paper. As Figure 6 shows that the proposed crossover based study generates a better distributed set of NSGA-II solutions. This distributed set of NSGA-II solutions provides a good plat-

3. CONCLUSIONS

This study successfully implemented the domain specific crossover with NSGA-II and a helper objective to evolve ‘trade-off’ solutions based CM designs which were better than the solution of the single-objective study (called as reference design) in terms of weight. A detailed study of this paper can be found in [4].

4. ACKNOWLEDGMENTS

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5. REFERENCES


