Topology Optimization of Compliant Mechanism using Multi-Objective Particle Swarm Optimization

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ABSTRACT

In this paper, a multi-objective particle swarm optimization approach (popularly known as MOPSO) for topology optimization of compliant mechanism is proposed. Multi-objective strategy has a great advantage, over other single objective approaches, in finding a well distributed set of non-dominated solutions in a single run which makes post-processing and decision making convenient. The stochastic multi-objective strategy also overcomes the issue of 'initialization of design space' upon which the final solutions may depend. Here, MOPSO is coupled with Material-Mask overlay strategy using honeycomb discretization to obtain optimal single-material compliant topologies that are free from the pathologies of *'checker board' and 'point flexure'*. An attempt to study the performance of proposed MOPSO is made by employing different techniques, both existing and newly proposed, of selecting the *'personal best'* and *'global best'*. In particular, a newer idea of allowing each particle to memorize all non-dominated personal best particles which it has encountered is introduced, i.e. if updated personal best position be indifferent to the old one, we keep both in the personal archive. This newly proposed strategy of particle memory seems to outperform the existing ones significantly.
Categories and Subject Descriptors: G.1.6 [Numerical Analysis]: Optimization

General Terms: Algorithms

1. INTRODUCTION

Particle Swarm Optimization *PSO* is a relatively new algorithm proposed by Kennedy and Eberhart (1995). It is a population-based stochastic optimization technique inspired by social behavior of bird flocking or fish schooling. Since PSO is originally introduced for optimization of continuous nonlinear functions, it has been successfully applied to many other problems such as discrete optimization, artificial neural network training, fuzzy system control, and other situations where the evolutionary techniques can be employed [10]. PSO similar in some respects to evolutionary algorithms (EAs), except that the potential solutions (particles) move, rather than evolve, through the search space. Each particle has a position and a velocity, and experiences linear spring-like attractions toward two attractors, namely, *personal best*- particle's best position so far, and, *global best*- best particle position in a certain neighborhood. Here best is in relation to evaluation of an objective function at that position.The *personal best* acts as an individual particle memory and the *global best* allows the particles to share information amongst them. In our study we allow particles to maintain a *personal best* archive to make a choice for *pbest*.

Most of the real-world optimization problems have more than one conflicting objectives, in recent years more and more attempts have been made to extend PSO to multi-objective problems, see e.g. [11]. These methods are called Multi-Objective Particle Swarm Optimization (MOPSO) methods, they follow the same principles as the single objective PSO, with the main difference being in the selection of the personal best (*pbest*) and the global best (*gbest*). In this attempt we study the application of MOPSO on a real world application problem and investigate the ways in which choices for *pbest* and *gbest* may effect the optimization performance. Moreover, employing PSO as an optimization procedure has certain advantages in terms of easy implementation and efficient computation [7].

GECCO'08, July 12–16, 2008, Atlanta, Georgia, USA.
ACM 978-1-60558-131-6/08/07.

2. MULTI-OBJECTIVE TOPOLOGY OPTIMIZATION AND COMPLIANT MECHANISM

Typically, a mechanism is a mechanical device used to transfer or transform motion, force, or energy [2]. Traditional rigid-body mechanisms consist of rigid links connected at movable joints. However, compliant mechanisms gain at least some of their mobility from the deflection of flexible members rather than from movable joints only. Compliant crimping mechanism [2] is a good example. Usually, minimizing mass of design structure while maintaining certain deflection and flexibility criterion are the designers goals. Thus, an overall topology optimization problem of compliant mechanism tries to derive topologies that are optimal and free from any singularities related to sub-region connectivity. An optimal topology, for example, could considered as one, which exhibits lower stiffness and requires minimum volume of the material for design. Above described problem is formulated in a multi-objective framework and then solved.

2.1 Topology Optimization and continuum Representation

Two popular models [12] for representing material in a domain that exist are *'Homogenization'* and *'SIMP'*(Solid Isotropic Material with Penalization). Both these models, are based on a rectangular shaped unit cell andsuffer from an inherit geometric pitfall of showing (i) checkerboard patterns and (ii) point flexure pathologies. These arise because two contiguous cells can be connected at a single point providing zero local bending stiffness. Checkerboard patterns are related to the over estimation of shear stress while the point flexures are associated with the strain-free rotations allowed at the locations where two solid cells are connected diagonally at a point. To over come these difficulties we employ a newly proposed [12] *honeycomb tessellation*, where each tile is a regular hexagon and all tiles are similar. Since hexagons have an edge connectivity hence such a representation provides *finite stiffness connectivity* everywhere. More over hexagonal cell can be subdivided into two four-node elements which makes the finite element analysis straight-forward.

2.2 Material Assignment

Once the whole domain is discretized with hexagonal cells (*'honeycomb tessellation'*), then using the material mask overlay strategy [12] material is allocated within the domain. For a compliant topology design problem let, $\mathbf{P}_i$, $\mathbf{P}_j$ are input loads and it may be desired to maximize deformation at point *O*, along the direction δ_{out}. Consider the masks *E1, E2, F1, F2* superimposed over the design region. Nomenclature *E* refers to empty or void while *F* refers to filled. The cells whose geometric centers are encompassed within the perimeters of *E1* and *E2* are assigned no material, while whose centers fall within *F1* and *F2* are chosen to be filled. Material assignment also depends on how the two overlaying masks interact with each another. If there is a common region between two masks, like between *E2* and *F2* in Figure 1, the topmost mask gets preference in material assignment within the intersection area. In other words, the mask that is processed later gets the preference. Since, *F2* rests over *E2* so the cells in the overlapped area are filled.

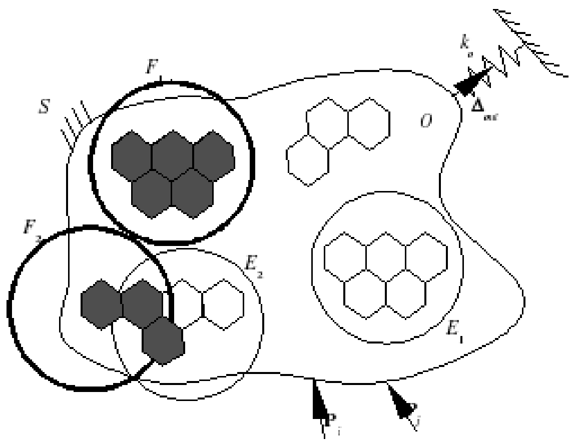

Figure 1: A generic design domain represented using a honeycomb tessellation with material masks overlaid

Given N number of masks to be overlaid in the domain, then design variables for each mask can be identified as its center coordinates (*p,q*), the radius *r* and material status *f* which can have values 0 (no material assigned) or 1 (material assigned). The material mask overlay strategy is favorable in place of controlling mass in each individual cell, when number of cells are fairly large. Thus, leading to *4N* design variables.

2.3 Design Problem and MOPSO

Here, the multi-objective problem formulation and MOPSO algorithm that is adopted as an optimization procedure are discussed. Usually, a monotonic increasing function of output deformation, sign(δ_{out}^{p-1})δ_{out}^{p}) and strain energy (e.g. SE^q) could be used as measures of flexibility and stiffness respectively [12]. Here, p and q are user specified exponents. In many cases minimizing the normalized volume is also considered. The output deformation along a prescribed direction is computed as mutual strain energy (*MSE*) using the virtual work principle by applying a unit dummy load along that direction [8] . This deformation is computed as:

$$\delta_{out} = MSE = \int \sigma_d^T \, \epsilon \, d\omega = V^T \mathbf{K}\mathbf{U} \tag{1}$$

where σ_d and $\mathbf{V}$ are the stress and displacement fields resulting from the unit dummy load, $\mathbf{K}$ is the structural stiffness matrix, and ϵ and $\mathbf{U}$ are the strain and displacement fields from the actual loads. Now, the strain energy is computed as:

$$SE = \frac{1}{2} \int \sigma^T \, \epsilon \, \omega = \frac{1}{2} U^T \mathbf{K}\mathbf{U} \tag{2}$$

where σ is the stress field resulting due to actual loads. In the finite element formulation, the equilibrium equations can be written as:

$$F_a = \mathbf{K}\mathbf{U} \tag{3}$$

$$F_d = \mathbf{K}\mathbf{V} \tag{4}$$

Where, F_a and F_d are force vectors relating to the actual and dummy loads respectively. Having known $\mathbf{U}$ and $\mathbf{V}$ from Eqs. (3) and (4), the output deformation and strain energy can be computed using Eqs. (1) and (2). The optimization problem considered in this study is stated as follows:

$\underset{\mathbf{x}}{\text{Minimize}}$: SE (*Strain Energy*)
$\underset{\mathbf{x}}{\text{Minimize}}$: V_n (*Normalized Volume*)
with: **x**= $\{\chi_i^p, \chi_i^q, \chi_i^r, \chi_i^f, i = 1, .., N\}$
such that:

1. $F_a = \mathbf{K}\mathbf{U}$
2. $\mathbf{K}$ is non-singular
3. $\mathbf{X}_L \leq \chi_i^p \leq \mathbf{X}_U$
4. $\mathbf{Y}_L \leq \chi_i^q \leq \mathbf{Y}_U$
5. $\mathbf{R}_L \leq \chi_i^r \leq \mathbf{R}_U$
6. $\chi_i^f = 0$ or 1

where χ_i^p is the *x* coordinate of the center of the *i*th mask, χ_i^q is the *y* coordinate of the same mask, χ_i^r is the radius of the *i*th mask and χ_i^f denotes the material status of the mask which can take only 0(empty) and 1(filled). $\mathbf{X}_L$ and $\mathbf{X}_U$ are the upper and the lower bounds on *x* coordinate, $\mathbf{Y}_L$ and $\mathbf{Y}_U$ are the upper and the lower bounds on *y* coordinate. The bounds are chosen such that mask center can be placed anywhere in the design region. $\mathbf{R}_L$ and $\mathbf{R}_U$ are the lower and the upper bounds on the mask radii. N is the number of masks a priory chosen by the user. To find more details about *evaluation* of objective function vector reader is referred to [12]. It is worth mentioning here that *penalty* approach [12] in cases of infeasible finite element mesh is adopted.

The proposed MOPSO algorithm in Table 1 is easy to comprehend and the reader is encouraged to refer [11]. The idea of proposing an archive based strategy for maintaining *pbest* is found to be useful. To know more about such a strategy reader is referred to [4].

Table 1: MOPSO algorithm with *pbest* archive

MOPSO Algorithm
BEGIN
Input: Optimization problem
Output: Non-dominated solutions in archive (A)

Step 1: t=0
Step 2: **Initialization:**
Initialize population P_t :
For i = 1 to N
Initialize $\bar{x}_t^i$, $\bar{v}_t^i$=$\bar{0}$ and a_t^i={$\bar{x}_t^i$ } $\bar{p}_t^i$=$\bar{x}_t^i$
End
Initialize the archive A_t :={}
Step 3: *Evaluate (P_t)*
Step 4:
(a)A_{t+1}:= *Update(P_t, A_t)*
(b) For i = 1 to N
a_{t+1}^i:= *Update($\bar{x}_t^i$, a_t^i)*
End
Step 5: P_{t+1}:=*Generate(P_t, A_t)*
For i = 1 to N
(a) $\bar{p}_t^{i,g}$ = *FindGlobalBest(A_{t+1}, $\bar{x}_t^i$)*
(b) $\bar{p}_t^i$ = *FindPersonalBest(a_{t+1}, $\bar{x}_t^i$)*
(c) For j = 1 to n
$v_{j,t+1}^i = wv_{j,t}^i + R_1(p_{j,t}^i - x_{j,t}^i) + R_2(p_{j,t}^{i,g} - x_{j,t}^i)$
$x_{j,t+1}^i = x_{j,t}^i + v_{j,t+1}^i$
End
End
Step 6 Unless a *termination criterion* is met:
t = t + 1 and *goto* Step 3
END

2.4 Selecting the Global best and Personal best

Here we briefly describe various strategies to select the global best (*gbest*) and personal best (*pbest*). The main challenge in is to pick suitable *pbest* and *gbest* to move the particles through search space. In general, a good MOPSO method must obtain solutions with (a) a good convergence, and (b) a diversity and spread along the pareto-optimal front. Typical strategies to select the *gbest* include random selection [5], selecting a particle that dominates many particles [9], or the sigma method [11]. But, the issue of selecting the personal best has not been studied thoroughly so far. In past studies a particle is allowed only to remember its latest 'best' position. The possibility of maintaining a *personal best archive* is explored for the first time here. In our study the following methods for selecting *gbest* and *pbest* can be employed.

1. **Random:** This is a simplest strategy to select randomly a non-dominated member from the global and personal archives.

2. **Wtd.:** In this approach, in order to maintain diversity, a higher weight is allotted to those criteria in which particle is already good and a weighted sum is calculated. Corresponding members in global and personal archives which have highest weighted sums are chosen [4]. This strategy helps in further improving an objective function value of a particle in which it is already good. This approach drastically favors to reach the extreme ends of a pareto front, but also has some limitations.

3. **Newest:** In this approach instead of maintaining a personal archive only one personal best is maintained. The personal best is updated as soon as new non-non-dominated position is reached. This method is only applied in selecting personal best.

4. **Indicator-based:** This is a newly proposed approach in which such a *gbest* or *pbest* is chosen which contributes most to the hyper volume with respect to a reference point. Usually, the individual itself is chosen as the reference point [1]. This strategy helps is increasing the diversity in middle parts of the pareto front, but shows a poor performance at the extreme ends.

5. **Dominance based probability:** Amongst the archive members which dominate the individual, guides are selected based on a probability distribution. Archive members which dominate greater number of individuals are assigned a higher probability of getting selected. In past, such a strategy has been successfully applied for selecting the global guides [9] and here its extension [1] for selecting *pbest* is made.

6. **Sigma-Sanaz:** Sigma method was originally proposed [11] for selecting the gbest. The idea behind this strategy it to allow the individuals to get attracted towards the non-dominated members which are closest to it. Here an extension [1] for selection of *pbest* is made.

3. RESULTS AND DISCUSSIONS

In this section first we try to analyze the solutions by using different selection schemes for lesser number of function evaluations and then try to implement the most promising strategies for larger number of function evaluations. Results obtained using the promising strategies are comparable to the ones already existing in the literature [6] and [3].

Cantiever problem [3] is one of the most standard shape design problems. As shown in Figure 2 design space is discretized using 420 hexagonal cells. The size of each cell is set to 1 unit. The left end of the plate is rigidly fixed and in the middle of right end input force of 5 N force is applied. Output is specified at the input node itself where spring constant is chosen to be 10000 N/M. Thickness of plate is selected to be 3mm. Youngs modulus of 2000 N/mm^2 and Poissons ratio of .29 are selected. The number of holes *N* is taken to be 50. Coefficients for MSE (p) and SE (q) are taken to be 1 each. MOPSO algorithm, Table 1, is then used as an optimization procedure with 30 population size, for 50 cycles with turbulence factor of 0.45. The reference point used for computation of hyper volume is [10,1000]. Two objectives considered for simultaneous minimization are a) Strain Energy(SE), flexibility criteria b) Normalized Volume, a resource criteria. Six different strategies stated in section 3.4 for selecting *pbest* and *gbest* are employed and the corresponding pareto optimal solutions obtained using these selection techniques are plotted in Figure 3. It's observed that Random-Random strategy, i.e., selecting *pbest* and *gbest* randomly, is the best performer. Dominance based probability strategy, for both *pbest* and *gbest*, seems to be next best performer. Newest-Random strategy, i.e., employing *Newest* for *pbest* and *Random* for *gbest* turns out to be third best performer. Similar conclusions can be drawn from the hyper volume curves generated for first 50 generations, as shown in Figure 4. Hyper volume curves give a better picture of performances of the selection strategies. From these curves we may conclude that *Sigma-Sigma* and Wtd.-Wtd. are the worst performers. Now, using *Random-Random* strategy, population size 30 and 600 generations, the cantilever problem is again solved to obtain a pareto optimal set. The solution corresponding to strain

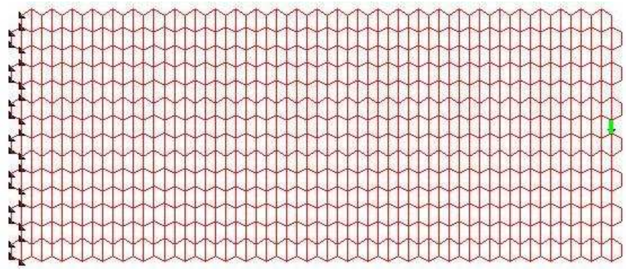

Figure 2: Discretized design space using hexagonal cells. The left end is rigidly fixed, while an input force and output displacement are specified on right end

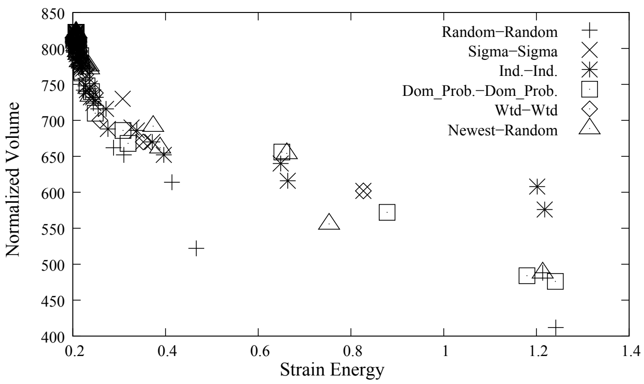

Figure 3: Pareto optimal fronts for different selection methods

energy of 5.149833 and minimum normalized volume 504.055800 is shown in Figure 5. The solution shown is having a parabolic end and is acceptable [6].

4. CONCLUSIONS

In this paper, a novel attempt has been made to apply MOPSO on a real world application problem with a high number of design variables. The proposed MOPSO is effective in finding a distributed set of pareto optimal solutions and thus goals of the study are met. Implementation of different selection strategies has brought out their relative importance, strengths and weaknesses. The study can be extended by implementation of hybrid moves, i.e., trying to find the best strategy for *pbest* and *gbest* which could give a performance boost. Other issues like handling of constraints and singularities can be dealt in a better way, thus trying to keep the solution in feasible domain and improving efficiency.

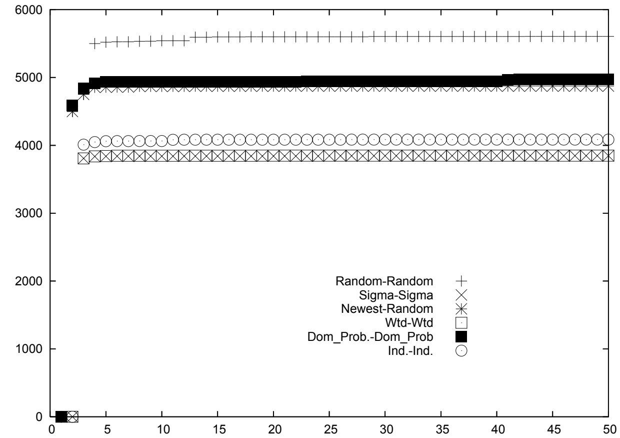

Figure 4: Hyper volume curves for different selection methods

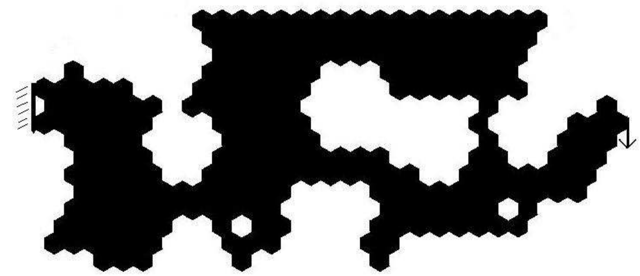

Figure 5: An optimal solution corresponding to minimum normalized volume

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