Bond-Graphs + Genetic Programming: Analysis of an Automatically Synthesized Rotary Mechanical System

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ABSTRACT
Initial results of an experiment devised to combine Bond-Graph modeling and simulation with genetic programming for automated design of a simple mechatronic system are reported in [1]. Two target eigen values are specified on complex plane and a Bond-Graph model is evolved through automated design scheme outlined in [1]. As a further development this research paper presents physical design realization based on the evolved Bond-Graph model. The physical design realization yields a second order open loop system. It is analyzed from a control systems stand point to determine system's dynamic characteristics. The dynamic analysis shows that damping ratio is 0.591 so we observe underdamped transient response typical of a system with complex conjugate poles.

Categories and Subject Descriptors
I.2.2 [**Artificial Intelligence**]: Automatic Programming; J.2 [**Physical Sciences and Engineering**]: Engineering.

General Terms: Design, Experimentation, Verification.

Keywords
Bond-Graphs, Object Oriented Modeling, Genetic Programming, Unified/Automated Design, Topology Synthesis, Multi Energy Domain Dynamic or Mechatronic Systems, Rotary Mechanical Systems, Physical Design Realization, Dynamic Analysis.

1. INTRODUCTION
Mechatronic systems are multi domain dynamic systems by definition. They are mixed or hybrid systems in nature as they combine elements from different energy domains. To perform correctly mechatronic systems depend on the interaction of sensors, computers or microcontrollers and actuators. These interacting physical systems store, transport and dissipate energy among sub systems. Bond-Graphs were originated with the purpose of handling variety within multi domain dynamic systems based on energy interaction and information exchange. [2]

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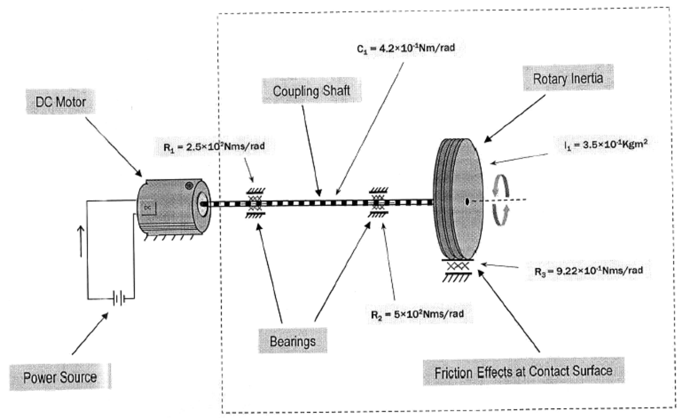

Figure 1. Rotary mechanical physical design realization of the evolved Bond-Graph model.

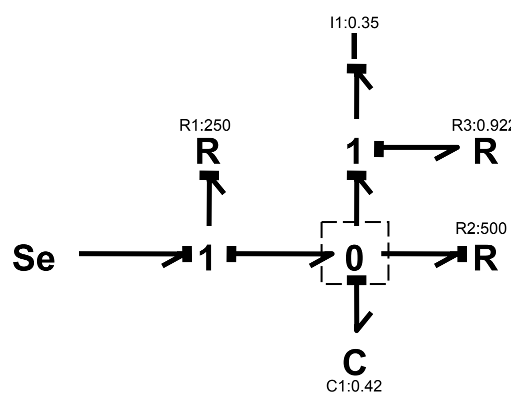

Figure 2. The evolved Bond-Graph model.

Genetic programming has emerged as one of the most promising soft computing techniques. The genetic programming paradigm is modeled on Darwinian concepts of evolution and natural selection. Genetic programming uses rooted, point labeled trees with ordered branches for representing computer programs or individuals in the process of simulated evolution. A fitness criterion is specified for evaluating performance of these computer programs in solving the problem at hand. [3][4]

Automated design/synthesis is the design or synthesis of physical systems using any of the models proposed for machine intelligence like evolutionary computation employing abundant computational resources available at present. An ideal automated design/synthesis system only receives a high level statement of the problem's requirements and attempts to create a working computer program that yields a solution for the problem. [5] In this research paper results of dynamic analysis of the physical system realized from the evolved Bond-Graph model are presented. [1] The physical system in Figure 1 is an intuitive rotary mechanical interpretation of the Bond-Graph model.

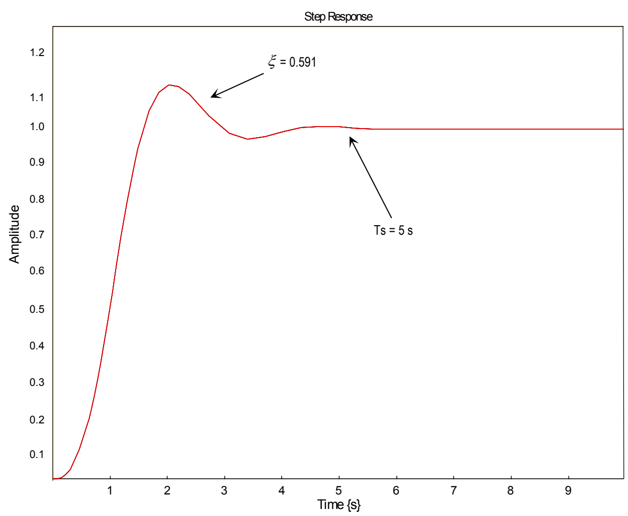

Figure 3. System response to a unit step input.

2. DYNAMIC ANALYSIS OF EVOLVED SYSTEM

The evolved Bond-Graph model of the physical system is analyzed using 20-Sim modeling and simulation software. The model contains two energy storing elements I_1 and C_1 therefore it is identified as a second order open loop system with two state variables. The evolved parameters are contained with in the periphery of the dotted square box. The general state-space representation for Bond-Graph model in Figure 2 is given as appearing in equations 1 and 2.

$$\frac{d}{dt}\{X\} = A\{X\} + B\{U\} \quad (1)$$

$$\{Y\} = C\{X\} + D\{U\} \quad (2)$$

In equations 1 and 2 $\{X\}$ is vector of states (momentum P and displacement Q), n is number of states, A is $n \times n$ square matrix, B is $n \times m$ matrix (m is the number of sources), $\{U\}$ is vector of sources (*Se* and *Sf*), $\{Y\}$ is vector of observer states (outputs), l is number of observer outputs, C is $l \times n$ matrix and D is $l \times m$ matrix. The poles of the physical system being represented by this Bond-Graph model are determined by calculating eigen values from matrix A in equation 1 using relation $| A - \lambda I | = 0$ where I is identity matrix of order $n \times n$. In due course of the procedure followed for dynamic analysis of the system source of effort *Se* is replaced with a modulated source of effort *MSe* and a motion profile tool provided by the 20-Sim modeling and simulation software is added to the workspace and connected to *MSe*. Motion profile selected is ramp with unit step as the input or excitation. Output signal is position or the observed output state is displacement *x(t)*. Values of input parameters are included in Table 1.

Table 1. Values of input parameters

Start Time	Rise Time	Stop Time	Amplitude
0 s	1 s	10 s	1

The response of the system to the unit step input is plotted in Figure 3. The values observed from the response curve appear as equations 3-6.

$$\text{Settling Time} = T_s = 5 \text{ s} \quad (3)$$

$$\text{Rise Time} = T_r = 1 \text{ s} \quad (4)$$

$$\text{Peak Time} = T_p = 2 \text{ s} \quad (5)$$

$$\text{Damping Ratio} = \xi = 0.591 \quad (6)$$

The target complex conjugate pole pair is -1 ± 2j specified in [1] where as average distance error e is calculated using equation 7.

$$e = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad (7)$$

For evolved eigen values -0.78 ± 1.063j the maximum average distance error turns out to be 0.961. The natural frequency ω_n, damped natural frequency ω_d and time period τ of the system are calculated using equations 8-11.

$$T_s = \left[\frac{1}{\xi \omega_n}\right] \ln 50 \quad (8)$$

$$\tau = \frac{2\pi}{\omega_d} \quad (9)$$

$$\omega_n = \frac{2\pi}{\tau \sqrt{1 - \xi^2}} \quad (10)$$

$$\omega_d = \omega_n \sqrt{1 - \xi^2} \quad (11)$$

Substituting values of settling time and damping ratio from equation 4 into equation 6 gives natural frequency of the system

ω_n equal to 1.319 rad/s and damped natural frequency of the system thus becomes ω_d = 1.064 rad/s. Value of time period τ is calculated as 5.904 s/rad. Value of percent overshoot $\%OS$ = 10% is determined using equation 12 same as observed from the output curve in Figure 3.

$$\%OS = e^{-\xi\pi / \sqrt{1-\xi^2}} \tag{12}$$

Relations for value of attenuation σ, rise time T_r, peak time T_p and maximum overshoot M_p appear as equations 13-17.

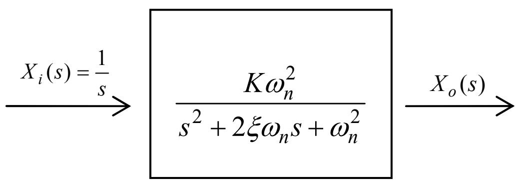

Figure 4. Step response of a second order system with value of damping ratio less than unity.

$$\sigma = \xi\omega_n \tag{13}$$

$$T_r = \frac{\pi - \beta}{\omega_d} \tag{14}$$

$$\beta = \tan^{-1}\frac{\omega_d}{\sigma} \tag{15}$$

$$T_p = \frac{\pi}{\omega_d} \tag{16}$$

$$M_p = e^{-\left(\frac{\sigma}{\omega_d}\right)\pi} \tag{17}$$

Second order system characteristic response $X_o(s)$ and second order system transfer function $G(s)$ is given by equations 18 and 19 respectively.

$$X_o(s) = \frac{K\omega_n^2}{s(s^2 + 2\xi\omega_n s + \omega_n^2)} \tag{18}$$

$$G(s) = \frac{K\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \tag{19}$$

Substituting values in equation 19 the system transfer function appears as in equation 20.

$$G(s) = \frac{(1.319)^2}{s^2 + 2(0.591)(1.319)s + (1.319)^2} \tag{20}$$

After simplification the system transfer function $G(s)$ is given by equation 21. Value of K or steady state gain is taken as 1.

$$G(s) = \frac{1.739}{s^2 + 1.56s + 1.739} \tag{21}$$

From equation 21 the characteristic equation for this particular system becomes $F(s) = s^2 + 1.56s + 1.739$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{22}$$

$$x = \frac{-1.56 \pm \sqrt{(1.56)^2 - 4(1)(1.739)}}{2(1)} \tag{23}$$

$$x_{1,2} = -0.78 \pm 1.063j \tag{24}$$

Using quadratic formula roots of the system equation or poles of the physical system $s_{1,2}$ = -0.78 ± 1.063j are determined as in equations 22 and 23 respectively. The rise time, settling time and damping ratio are typical of this type of systems. Using equations 13, 15 and 17 value of attenuation σ is determined as 0.77 rad/s, β is 54° and maximum overshoot M_p is 10.3%.

3. CONCLUSION

This dynamic analysis has been carried out to illustrate that a stable physical system can be realized from the open ended synthesis paradigm considered so inherent in all applications of automated design concept using genetic programming. For representation of the system Bond-Graphs have been used with a unified modeling approach for physical systems residing in different energy domains as is the case with mechatronic systems. Building on the experience that has been gained in physical design realization and subsequent dynamic analysis more complicated problems can be implemented by employing the same unified/automated design/synthesis methodology.

An observation pertaining to Bond-Graphs based representation is the limitation imposed due to lack of two port elements transformer *TF* and gyrator *GY* on the automated synthesis process which tends to restrict the evolutionary search and synthesis to one particular energy domain at any time without scaling. Direct transition from one energy domain to the other can be made if a genetic programming function for gyrator element is available thus extending the range of the design approach to nearly complete multi energy domain systems.

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