

Gecco 2008 Grammatical Evolution Tutorial

R. Muhammad Atif Azad and Conor Ryan

Biocomputing and Developmental Systems Group
Department of Computer Science and Information Systems
University of Limerick

July 12, 2008

Outline

- 1 Introduction
- 2 Grammatical Evolution
- 3 Genetic Operators
- 4 GAuGE
- 5 Chorus
- 6 Degeneracy
- 7 Wrapping
- 8 Search Techniques

Copyright is held by the author/owner(s).
GECCO'08, July 12, 2008, Atlanta, Georgia, USA.
ACM 978-1-60558-131-6/08/07.

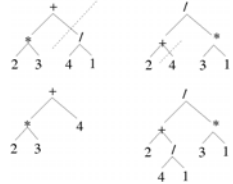
Issues with GP

- Function/terminal set must have "closure"

- Single types only
- Trees grow, or "bloat"

Issues with GP

- Function/terminal set must have "closure"



- Single types only
- Trees grow, or "bloat"

Introduction

Biological Phenomena

- No simple one to one mapping
 - Genes produce proteins
 - Proteins combine to create phenotype
- Linear strings
 - Genomes are always held on strings
- Unconstrained search
 - Repair not performed

Grammatical Evolution

Grammatical Evolution

- Grammatical Evolution (GE)
 - GA to evolve programs
 - Morphogenetic Effect:
 - Genotype mapped to phenotype
 - Phenotype is a compilable program
- Genome governs mapping of a BNF/attribute grammar definition to the program

Grammatical Evolution

Grammatical Evolution

- Here genome (a binary string) is mapped to compilable C code
- Can potentially evolve programs in any language, with arbitrary complexity
- Any structure than be specified with a grammar, e.g. graphs, neural networks, etc.

Grammatical Evolution Grammars

Language Definition

- Backus Naur Form (BNF)
 - Notation for expressing a languages grammar as Production Rules
- BNF Grammar consists of the tuple $\langle T, N, P, S \rangle$ where
 - T is Terminals set
 - N is Non-Terminals set
 - P is Production Rules set
 - S is Start Symbol (a member of N)
- BNF Example

$$T = \{Sin, Cos, Tan, Log, +, -, /, *, X, (,)\}$$
$$S = \langle expr \rangle$$

Grammatical Evolution Grammars

BNF Definition

- $N = \{expr, op, pre_op\}$
- And P can be represented as:
 - $\langle expr \rangle ::= \langle expr \rangle \langle op \rangle \langle expr \rangle$ (A)
 $| (\langle expr \rangle \langle op \rangle \langle expr \rangle)$ (B)
 $| \langle pre_op \rangle (\langle expr \rangle)$ (C)
 $| \langle var \rangle$ (D)
 - $\langle op \rangle ::= +$ (A)
 $| -$ (B)
 $| /$ (C)
 $| *$ (D)

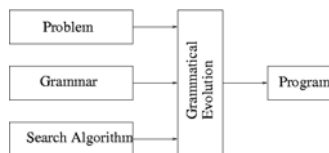
Grammatical Evolution Grammars

BNF Definition

- $\langle pre_op \rangle ::= \text{Sin}$ (A)
 $| \text{Cos}$ (B)
 $| \text{Tan}$ (C)
 - $\langle var \rangle ::= X$ (A)
- A Genetic Algorithm is used to control choice of production rule

Grammatical Evolution Architecture

Architecture



Grammatical Evolution Comparison

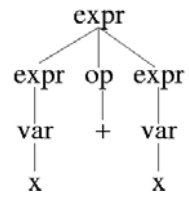
Related GP Systems

Name	Genome	Representation
Koza	Tree	Direct
Banzhaf et al	Linear	Direct
Gruau	Tree	Graph Grammar
Whigham	Tree	Derivation Tree
Wong & Leung	Tree	Logic Grammars
Paterson	Linear	Grammar

- Repair mechanisms..
- Koza - none needed
- Banzhaf - required for syntactically legal individuals
- Gruau - none needed
- Whigham - all crossovers subject to repair
- Wong & Leung - all crossovers subject to repair
- Paterson - under/overspecification.

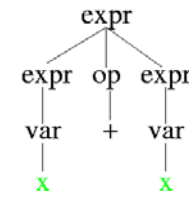
Grammatical Evolution Comparison

Repair



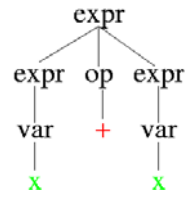
Grammatical Evolution Comparison

Repair



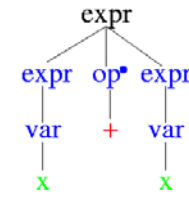
Grammatical Evolution Comparison

Repair



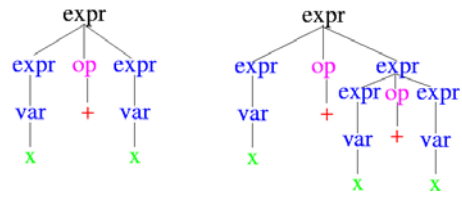
Grammatical Evolution Comparison

Repair



Grammatical Evolution Comparison

Repair



Grammatical Evolution Comparison

Grammatical Evolution

- In contrast GE uses
 - BNF - Paterson/Whigham/Wong etc.
 - Variable Length Linear Chromosomes - Koza/Gruau/Banzhaf
 - Genome encodes pseudo-random numbers
 - Degenerate Genetic Code
 - Several genes map to same phenotype
 - Wrap individuals
- Use 8 bit codons
 - Each codon represents at least one Production Rule
 - Gene contains many codons
- Pseudo-random numbers determine what production rule will be used

Grammatical Evolution Comparison

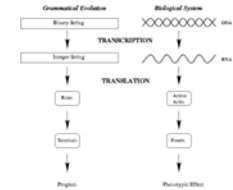
Grammatical Evolution

- Expression of a *Codon* results in an *Amino Acid* (choice in the derivation sequence)
 - *Amino acids* can combine to form a functional protein (i.e. Terminals such as +, X or *Sin*, can combine)

Grammatical Evolution Comparison

Grammatical Evolution

- Expression of a *Codon* results in an *Amino Acid* (choice in the derivation sequence)
 - *Amino acids* can combine to form a functional protein (i.e. Terminals such as +, X or *Sin*, can combine)



Grammatical Evolution Example

Example Individual

- To complete BNF definition for a function written in a subset of C we include.....


```

<func> ::= <header>
<header> ::= float symb(float X) <body>
<body> ::= <declarations><code><return>
<declarations> ::= float a;
<code> ::= a = <expr>;
<return> ::= return (a);
      
```
- Note implementation details.....
 - Function is limited to a single line of code
 - If required can get GE to generate multi-line functions.....modify


```

<code> ::= <line>;
          | <line>; <code>
              
```

University of Limerick Grammatical Evolution July 12, 2008 16 / 82

Grammatical Evolution Example

Example Individual

- In this subset of C all individuals of the form


```

float symb(float x)
{
    float a;
    a = <expr>;
    return(a);
}
      
```
- Only `<expr>` will be evolved
- Each non-terminal is mapped to a terminal before any others undergo a mapping process

University of Limerick Grammatical Evolution July 12, 2008 17 / 82

Grammatical Evolution Example

Example Individual

- Given the individual

220	203	51	123	2	45
-----	-----	----	-----	---	----

what will happen?
- `<expr>` has 4 production rules to choose from


```

(1) <expr> ::= <expr> <op> <expr> (A)
          | ( <expr> <op> <expr> ) (B)
          | <pre-op> ( <expr> ) (C)
          | <var> (D)
      
```

 - Taking first codon 220 we get $220 \text{ MOD } 4 = 0$
 - Gives `<expr><op><expr>`
- Next choice for the first `<expr>`
 - Taking next codon 203 we get $203 \text{ MOD } 4 = 3$
 - Gives `<var><op><expr>`

University of Limerick Grammatical Evolution July 12, 2008 18 / 82

Grammatical Evolution Example

Example Individual

- `<var>` involves no choice
 - Mapped to X...only one production
 - Now have `X <op><expr>`

220	203	51	123	2	45
-----	-----	----	-----	---	----
- Read next codon to choose `<op>`
 - Next is third codon, value 51, so get $51 \text{ MOD } 4 = 3$
 - Now have `X* <expr>`
- Next choice for `<expr>`
 - Next codon is 123 so get $123 \text{ MOD } 4 = 3$
 - Now have `X* <var>`
- Again `<var>` involves no choice
 - Finally we get `X * X`
- The extra codons at end of genome are simply ignored in mapping the genotype to phenotype

University of Limerick Grammatical Evolution July 12, 2008 19 / 82

Grammatical Evolution Mapping

Example Mapping Overview

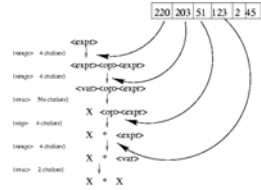


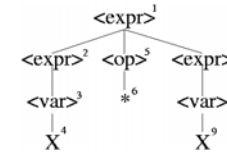
Figure: Example Mapping Outline

$\langle \text{expr} \rangle ::= \langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle \mid (\langle \text{expr} \rangle \langle \text{op} \rangle \langle \text{expr} \rangle) \mid \langle \text{pre} \rangle \mid \langle \text{var} \rangle$

Grammatical Evolution Mapping

Derivation Tree Structure

1	2	5	7		
220	203	51	123	2	45



- Not all nodes require a choice!

Grammatical Evolution Mapping

Codons are polymorphic

- When mapping $\langle \text{expr} \rangle$, we calculate $220 \bmod 4$
- However, if we were mapping $\langle \text{pre} - \text{op} \rangle$ with 220, we would calculate $220 \bmod 3$ because there are just three choices
- Meaning of a codon depends on its *context*

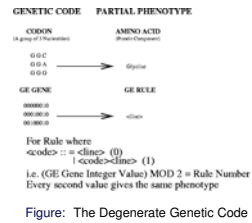
Grammatical Evolution Mapping

Mapping Process

- No simple one to one mapping in GE
- Mapping Process to generate programs
 - Separate Search and Solution Spaces
 - Ensure validity of individuals
 - Remove language dependency
 - Maintain diversity

Grammatical Evolution Mapping

Genetic Code Degeneracy



Grammatical Evolution Mapping

Genetic Code Degeneracy

- Neutral Mutations
 - Mutations having no effect on Phenotype Fitness
- Help preserve individual validity
- Gradual accumulation of mutations without harming functionality
 - Revisit later

Grammatical Evolution Initialisation

Initialisation

- Individuals are strings of random numbers
 - No guarantee that they will terminate
 - Individuals can be very short.
- ```

<expr> ::= <expr> <op> <expr>
 | (<expr> <op> <expr>)
 | <pre-op> (<expr>)
 | <var>

```
- Production  
 $\langle \text{expr} \rangle \rightarrow \langle \text{var} \rangle$   
 always leads to termination
  - $\langle \text{expr} \rangle$   
 is the start symbol
    - On average, a quarter of all individuals are just one point

Grammatical Evolution Initialisation

## Sensible Initialisation

- Generate a spread of individual sizes.
  - Based on *Ramped Half and Half* initialisation in GP
    - For all tree depths from 2 to maximum size
    - Generate an equal number of trees of that size
    - Use *full* for 50%
    - Use *grow* for 50%
- Similar in GE, but generate *derivation trees* of equivalent size



- Record which number choice was made for each step
- Perform an "unmod" on list of choices
  - Produce a number between 0 and 255 that produces the original number when moded by the number of choices for that productionrule
- Ensures that *all* individuals are valid
- Reduces the number of clones (easier to detect)
- Eliminates single point individuals (if desired)

- Perform unconstrained Evolutionary Search
- GE employs standard operators of Genetic Algorithms
  - Point mutation, one-point crossover etc.
- Sometimes modified version of one-point crossover, Sensible Crossover, is used:
  - Effective length
  - Actual length

- Perform unconstrained Evolutionary Search
- GE employs standard operators of Genetic Algorithms
  - Point mutation, one-point crossover etc.
- Sometimes modified version of one-point crossover, Sensible Crossover, is used:
  - Effective length
  - Actual length

a b c d e f g h i j

A B C D E F G

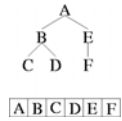
- What actually happens in crossover?
- Preliminary : Visualisation.

• Crossover is performed at *genotypic* level

Genetic Operators Crossover

## Crossover

- What actually happens in crossover?
- Preliminary : Visualisation.

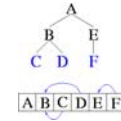


- Crossover is performed at *genotypic* level

Genetic Operators Crossover

## Crossover

- What actually happens in crossover?
- Preliminary : Visualisation.

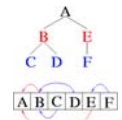


- Crossover is performed at *genotypic* level

Genetic Operators Crossover

## Crossover

- What actually happens in crossover?
- Preliminary : Visualisation.

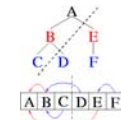


- Crossover is performed at *genotypic* level

Genetic Operators Crossover

## Crossover

- What actually happens in crossover?
- Preliminary : Visualisation.



- Crossover is performed at *genotypic* level

Genetic Operators | Ripple Crossover

## Ripple Crossover

- Analyse 1-point crossover in terms of derivation & syntax trees
- Use a *closed* grammar

```

E ::= (+ E E) {0}
 | (- E E) {1}
 | (* E E) {2}
 | (% E E) {3}
 | X {4}
 | Y {5}

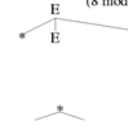
```

- No polymorphism, because there is only one non-terminal, i.e. one *context*

Genetic Operators | Ripple Crossover

## Different Views of Crossover

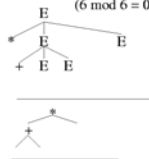
8 | 6 | 4 | 5 | 9 | 4 | 5 | 2 | 0  
 E (8 mod 6 = 2)



Genetic Operators | Ripple Crossover

## Different Views of Crossover

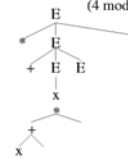
8 | 6 | 4 | 5 | 9 | 4 | 5 | 2 | 0  
 E (6 mod 6 = 0)

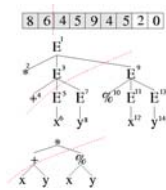
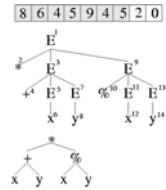


Genetic Operators | Ripple Crossover

## Different Views of Crossover

8 | 6 | 4 | 5 | 9 | 4 | 5 | 2 | 0  
 E (4 mod 6 = 4)





- Parent left with "spine"



- Tail swapped with other parent  
4 5 9 4 5 2 0 5 2 2
- Unmapped E terms must be mapped
- Use tail from other parent

- With more than one non-terminal, a codon could be used differently in the offspring

1 0 0 2 0 1 1 0 0 2 0 1 1 0 0 2 0 1

```

expr ::= var | expr op expr
opr ::= + | * | - | %
var ::= x | y

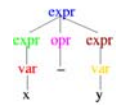
```

Genetic Operators | Ripple Crossover

### Intrinsic Polymorphism

- With more than one non-terminal, a codon could be used differently in the offspring

1 0 0 2 0 1 | 1 0 0 2 0 1 | 1 0 0 2 0 1



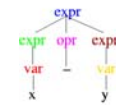
expr ::= var | expr op expr  
opr ::= + | \* | - | %  
var ::= x | y

Genetic Operators | Ripple Crossover

### Intrinsic Polymorphism

- With more than one non-terminal, a codon could be used differently in the offspring

1 0 0 2 0 1 | 1 0 0 2 0 1 | 1 0 0 2 0 1



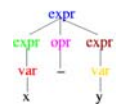
expr ::= var | expr op expr  
opr ::= + | \* | - | %  
var ::= x | y

Genetic Operators | Ripple Crossover

### Intrinsic Polymorphism

- With more than one non-terminal, a codon could be used differently in the offspring

1 0 0 2 0 1 | 1 0 0 2 0 1 | 1 0 0 2 0 1



expr ::= var | expr op expr  
opr ::= + | \* | - | %  
var ::= x | y

Genetic Operators | Ripple Crossover

### Effects of Ripple Crossover

- Symbolic Regression Grammars

Closed Grammar

E ::= x  
| (+ E E) | (\* E E)  
| (- E E) | (/ E E)

And the context free grammar:

Exp ::= Var | Exp Op Exp  
Var ::= x  
Op ::= + | \* | - | /

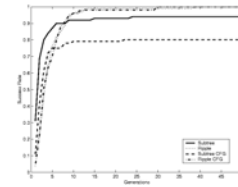
- Santa Fe ant trail grammars

**Closed grammar**

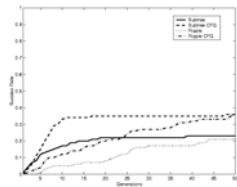
```
E ::= move() | left() | right()
 | iffloodahead(E E) | prog2(E, E)
```

**Context free grammar:**

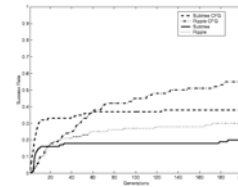
```
Code ::= Line | prog2(Line, Code)
Line ::= Condition | Action
Action ::= move() | right() | left()
Condition ::= iffloodahead(Code, Code)
```



Both ripple crossovers start more slowly, but reach higher fitness.



Both ripple crossovers again start more slowly, but reach similar fitness.



Success rates on the Santa Fe ant trail problem, averaged over 100 runs, for 250 generations. Ripple crossovers start slowly, but reach higher fitness.

Genetic Operators Alternative Crossovers

### Other types of Crossover?

- Homologous Crossover
  - Try not to cross in identical areas
- Uniform
- Same size homologous
- Same size two point

Genetic Operators Alternative Crossovers

### Homologous Crossover - First point

- Record rule histories for each individual

```

Child Integer 2 13 48 1 3 248 100 23
Rule 0 1 0 1 1 3 0 3 PARENT 1

Child Integer 2 13 48 7 4 3 1 100
Rule 0 1 0 4 0 2 1 0 PARENT 2

```

- Align rule histories of parents

Genetic Operators Alternative Crossovers

### Homologous Crossover - First point

- Record rule histories for each individual

```

Child Integer 2 13 48 1 3 248 100 23
Rule 0 1 0 1 1 3 0 3 PARENT 1

Child Integer 2 13 48 7 4 3 1 100
Rule 0 1 0 4 0 2 1 0 PARENT 2

```

- Align rule histories of parents

```

Rule 0 1 0 1 1 3 0 3 PARENT 1
Rule 0 1 0 4 0 2 1 0 PARENT 2

```

Genetic Operators Alternative Crossovers

### Homologous Crossover - Second Point

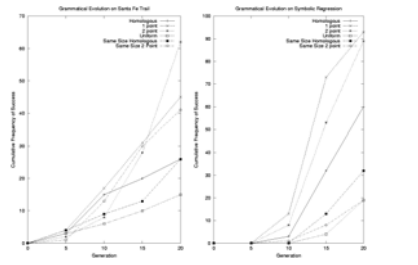
- Choose second point outside of area of similarity

```

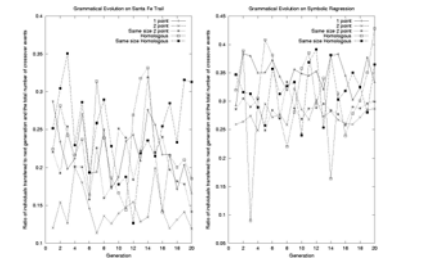
Rule 0 1 0 1 1 3 0 3 PARENT 1
Rule 0 1 0 4 0 2 1 0 PARENT 2

```

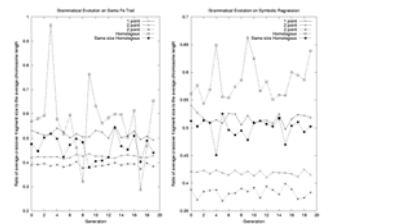
Genetic Operators Alternative Crossovers  
**Crossover comparisons (Cumulative Freq. Success)**



Genetic Operators Alternative Crossovers  
**Productivity of Operators (Ratio of successes)**



Genetic Operators Alternative Crossovers  
**Relative size of crossover fragments**



Ratio of the average fragment size being swapped and the average chromosome length at each generation averaged over 20 runs.

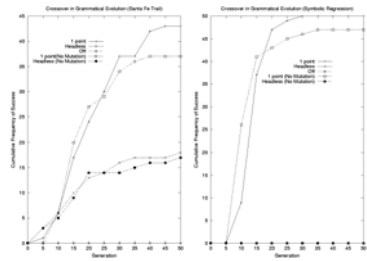
Genetic Operators Alternative Crossovers  
**Headless Chicken - Crossover or Macromutation**

- Appears Crossover works
- 50% material exchange with 1-point over entire runs
- If useful material exchanged then swapping random fragments should degrade performance?



Genetic Operators | Alternative Crossovers

### Headless Chicken Comparison



Genetic Operators | Explanation

### Why does crossover work?

- Take a cue from GP crossover - The "Eve" Effect :
  - All individuals in the final generation tend to evolve from the same ancestor

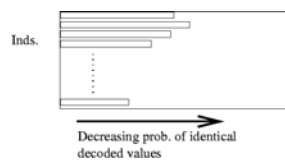


- The upper parts of individuals tend to come from the same individual

Genetic Operators | Explanation

### GE View of Eve Effect?

- Individuals grow from left to right



Genetic Operators | Explanation

### Size of region of similarity increases over time

- Area immediately beyond region of similarity is "region of discovery" :



Genetic Operators Explanation

### Size of region of similarity increases over time

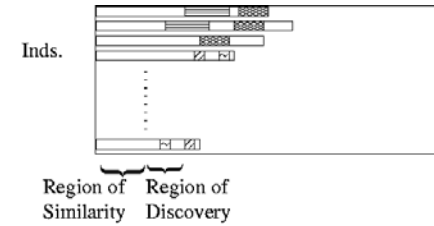
- Area immediately beyond region of similarity is "region of discovery" :



Genetic Operators Explanation

### Size of region of similarity increases over time

- Area immediately beyond region of similarity is "region of discovery" :



GAuGE Introduction

### The GAuGE System

Genetic Algorithms using Grammatical Evolution

Purpose:

- Position independent genetic algorithm;
- No under- or over-specification;
- Independent of search engine.

Based on mapping process (similar to GE):

- Specify position and value of each variable at genotypic level;
- Map genotype strings into functional phenotype strings.

GAuGE Mapping

### Mapping in the GAuGE System

Transform binary string into integer string:

- Problem has 4 variables ( $\ell = 4$ ), with range  $0 \dots 7$ ;
- Choose *position field size* ( $pfs = 2$ );
- Choose *value field size* ( $vfs = 4$ );
- Calculate binary string length:

$$L = (pfs + vfs) \times \ell = (2 + 4) \times 4 = 24 \text{ bits}$$

GAuGE Mapping

### Mapping in the GAuGE System

Transform binary string into integer string:

- Problem has 4 variables ( $\ell = 4$ ), with range  $0 \dots 7$ ;
- Choose *position field size* ( $pfs = 2$ );
- Choose *value field size* ( $vfs = 4$ );
- Calculate binary string length:

$$L = (pfs + vfs) \times \ell = (2 + 4) \times 4 = 24 \text{ bits}$$

GAuGE Mapping

### Mapping in the GAuGE System

Transform binary string into integer string:

- Problem has 4 variables ( $\ell = 4$ ), with range  $0 \dots 7$ ;
- Choose *position field size* ( $pfs = 2$ );
- Choose *value field size* ( $vfs = 4$ );
- Calculate binary string length:

$$L = (pfs + vfs) \times \ell = (2 + 4) \times 4 = 24 \text{ bits}$$

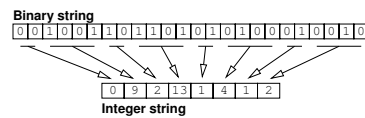
GAuGE Mapping

### Mapping in the GAuGE System

Transform binary string into integer string:

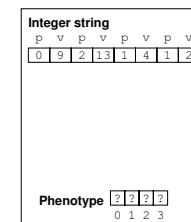
- Problem has 4 variables ( $\ell = 4$ ), with range  $0 \dots 7$ ;
- Choose *position field size* ( $pfs = 2$ );
- Choose *value field size* ( $vfs = 4$ );
- Calculate binary string length:

$$L = (pfs + vfs) \times \ell = (2 + 4) \times 4 = 24 \text{ bits}$$

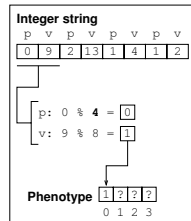


GAuGE Mapping

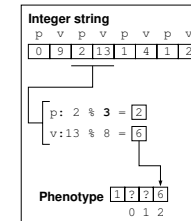
### Calculating Phenotype



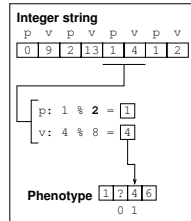
GAuGE Mapping  
Calculating Phenotype



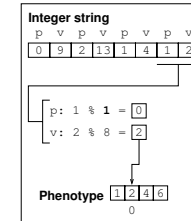
GAuGE Mapping  
Calculating Phenotype



GAuGE Mapping  
Calculating Phenotype



GAuGE Mapping  
The GAuGE System



- GAuGE adapts the representation to the problem
  - Useful where interactions between genes not known
- GAuGE is cheap
  - Far less complicated than algorithms that try to model gene interactions/relationships
- GAuGE discovers saliency
  - Most important genes end up on left side of strings

- Mapping Independent Codons - no ripple effect
- Codon % *Total* number of rules in the grammar
- Competition between the Genes
- Concentration Table
- Variable length binary strings
- 8 bit codons

```

S= <expr>
(0) <expr> ::= <expr> <op> <expr>
(1) | (<expr> <op> <expr>)
(2) | <pre-op> (<expr>)
(3) | <var>
(4) <op> ::= +
(5) | -
(6) | *
(7) | /
(8) <pre-op> ::= Sin
(9) | Cos
(A) | Exp
(B) | Log
(C) <var> ::= 1.0
(D) | X

```

Four non-terminals:

- <expr> 0..3, <op> 4..7, <pre-op> 8..B, <var> C..D

209 102 190 55 65 15 255 87  
 D 4 8 D 9 1 3 3

|     |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
|-----|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
|     | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | A | B | C | D |
| <e> | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Chorus Introduction  
**Mapping - 2**

Four non-terminals:

- $\langle \text{expr} \rangle 0..3, \langle \text{op} \rangle 4..7, \langle \text{pre-op} \rangle 8..B, \langle \text{var} \rangle C..D$

209 102 190 55 65 15 255 87  
 D 4 8 D 9 1 3 3

|                                                         | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | A | B | C | D |
|---------------------------------------------------------|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\langle e \rangle$                                     | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\langle e \rangle \langle o \rangle \langle e \rangle$ | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 2 |

Chorus Introduction  
**Mapping - 3**

Four non-terminals:

- $\langle \text{expr} \rangle 0..3, \langle \text{op} \rangle 4..7, \langle \text{pre-op} \rangle 8..B, \langle \text{var} \rangle C..D$

209 102 190 55 65 15 255 87  
 D 4 8 D 9 1 3 3

|                                                         | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | A | B | C | D |
|---------------------------------------------------------|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\langle e \rangle$                                     | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\langle e \rangle \langle o \rangle \langle e \rangle$ | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 2 |
| $\langle v \rangle \langle o \rangle \langle e \rangle$ | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 2 |

Chorus Introduction  
**Mapping - 4**

Four non-terminals:

- $\langle \text{expr} \rangle 0..3, \langle \text{op} \rangle 4..7, \langle \text{pre-op} \rangle 8..B, \langle \text{var} \rangle C..D$

209 102 190 55 65 15 255 87  
 D 4 8 D 9 1 3 3

|                                                         | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | A | B | C | D |
|---------------------------------------------------------|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\langle e \rangle$                                     | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\langle e \rangle \langle o \rangle \langle e \rangle$ | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 2 |
| $\langle v \rangle \langle o \rangle \langle e \rangle$ | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 2 |
| $X \langle o \rangle \langle e \rangle$                 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 |

Chorus Introduction  
**Mapping - 5**

Four non-terminals:

- $\langle \text{expr} \rangle 0..3, \langle \text{op} \rangle 4..7, \langle \text{pre-op} \rangle 8..B, \langle \text{var} \rangle C..D$

209 102 190 55 65 15 255 87  
 D 4 8 D 9 1 3 3

|                                                         | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | A | B | C | D |
|---------------------------------------------------------|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\langle e \rangle$                                     | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\langle e \rangle \langle o \rangle \langle e \rangle$ | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 2 |
| $\langle v \rangle \langle o \rangle \langle e \rangle$ | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 2 |
| $X \langle o \rangle \langle e \rangle$                 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 |
| $X \langle + \rangle \langle e \rangle$                 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 |

Chorus Introduction  
**Mapping - 6**

Four non-terminals:

- <expr> 0..3, <op> 4..7, <pre-op> 8..B, <var> C..D

209 102 190 55 65 15 255 87  
 D 4 8 D 9 1 3 3

|           | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | A | B | C | D |   |
|-----------|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| <e>       | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| <e><o><e> | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 2 | 0 |
| <v><o><e> | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 2 | 0 |
| X<o><e>   | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 |
| X+<e>     | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 |
| X+<v>     | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 |

Chorus Introduction  
**Mapping - 7**

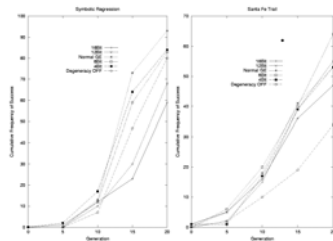
Four non-terminals:

- <expr> 0..3, <op> 4..7, <pre-op> 8..B, <var> C..D

209 102 190 55 65 15 255 87  
 D 4 8 D 9 1 3 3

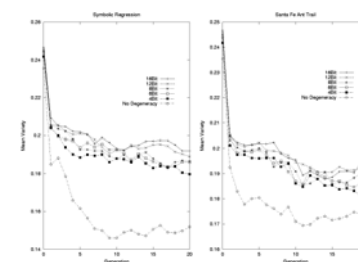
|           | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | A | B | C | D |   |
|-----------|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| <e>       | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| <e><o><e> | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 2 | 0 |
| <v><o><e> | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 2 | 0 |
| X<o><e>   | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 |
| X+<e>     | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 |
| X+<v>     | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 |
| X+X       | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |

Degeneracy Performance  
**Cumulative Freq. with and without degeneracy**



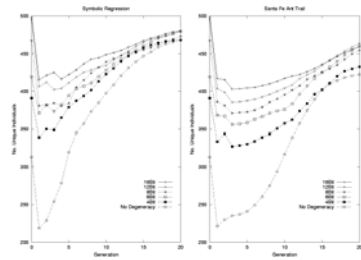
- No huge difference : Normal, 4- and 6-bit top three in both

Degeneracy Variety  
**Mean Variety - Any degeneracy helps!**



Degeneracy Variety

## Unique Individuals



Degeneracy Variety

## Conclusions

- Conclusions:
  - Improves genetic diversity
  - Improves frequency of success on Santa Fe ant trail
  - Tuneable/Evolvable Degeneracy a good idea?

Wrapping Number of individuals wrapped

## Number of individuals wrapped

- Wrap Count & Invalid Individuals

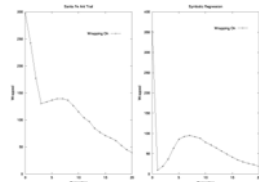


Figure: Number of individuals wrapped on the symbolic regression and Santa Fe trail problems.

Wrapping Number of individuals wrapped

## Wrapping and Invalid Individuals

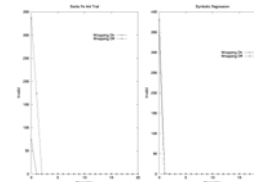


Figure: The number of invalid individuals for each generation in the presence and absence of wrapping.



Wrapping Performance

## Performance

- Freq. of Success

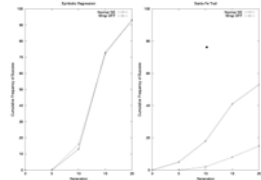


Figure: Figure shows the cumulative frequency of success measures on both problems with and without the presence of wrapping.

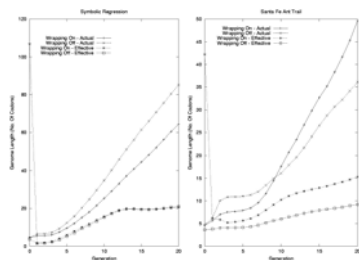
Wrapping Genome Lengths

## Lengths (Some Definitions)

- Actual length
  - Entire length of individual
- Effective length
  - Number of codons used
  - (Note! Can be less than or greater than actual length)

Wrapping Genome Lengths

## Genome Lengths



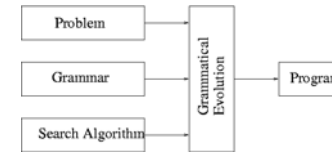
Wrapping Summary

## Summary

- For SR (left) wrapping off has the longest actual length
- Effective length virtually the same
- For SF (right) wrapping on longer in both cases.
- Conclusions:
  - Wrapping improves frequency of success on Santa Fe ant trail
  - No effect on Symbolic Regression cumulative frequency
  - Provides some constraint on genome lengths

- Removing both....
  - Cumulative frequency of success degrades
  - Genome lengths increase over 60% on Symbolic Regression
  - Genetic diversity no worse than without degeneracy alone

Search Techniques



- Other techniques
  - Simulated Annealing
  - Hill Climbing
  - Random Search

- Three standard GP problems
  - Santa Fe trail
  - Symbolic Integration (integrate  $\cos(x) + 2x + 1$ )
  - Symbolic regression  $x^4 + x^2 + x^2 + x$

| Problem              | Metaheuristic |    |     |      |
|----------------------|---------------|----|-----|------|
|                      | RS            | HC | SA  | GA   |
| Santa Fe             | 54%           | 7% | 14% | 81%  |
| Symbolic Integration | 66%           | 4% | 3%  | 100% |
| Symbolic Regression  | 0%            | 0% | 0%  | 59%  |

- Evolving machine code (Machine Code Grammatical Evolution - MCGE)
- The Grammar (Attribute Grammars)
- Search & Evolutionary Dynamics
- Applications
- Newest Code Release
  - <http://www.grammaticalevolution.org/libGE>

## Opportunities

- Programmer
  - EUR 31,000 - EUR 33,000
  - Possible to register for part time PhD
- PhD Students
  - EUR 24,000