A Robust Evolutionary Framework for Multi-Objective Optimization

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ABSTRACT

Evolutionary multi-objective optimization (EMO) methodologies, suggested in the beginning of Nineties, focussed on the task of finding a set of well-converged and well-distributed set of solutions using evolutionary optimization principles. Of the EMO methodologies, the elitist non-dominated sorting genetic algorithm or NSGA-II, suggested in 2000, is now probably the most popularly used EMO procedure. NSGA-II follows three independent principles – domination principle, diversity preservation principle and elite preserving principle – which make NSGA-II a flexible and robust EMO procedure in the sense of solving various multi-objective optimization problems using a common framework. In this paper, we describe NSGA-II through a functional decomposition following the implementation of these three principles and demonstrate how various multi-objective optimization tasks can be achieved by simply modifying one of the three principles. We argue that such a functionally decomposed and modular implementation of NSGA-II is probably the reason for it's popularity and *robustness* in solving various types of multi-objective optimization problems.

Categories and Subject Descriptors

I.2.8 [**Computing Methodologies**]: Problem Solving, Control Methods, and Search

General Terms

Algorithms

Keywords

Multi-objective optimization, Functional decomposition, Decision-making, Evolutionary optimization.

1. INTRODUCTION

Evolutionary multi-objective optimization (EMO) methodologies are now being developed and applied for the past 15 years [4, 5]. Besides finding a set of trade-off near-optimal solutions, these methodologies are also applied to solve other optimization problems (such as single-objective optimization, goal programming problems) which are traditionally not solved using a multi-objective optimization algorithm [18]. The presence of multiple conflicting objectives and the need of using decision-making principles cause a number of different problem scenarios to emerge in practice. For example, if the user is interested in finding a particular preferred region on the Pareto-optimal front, instead of the entire frontier, or if the user is interested in finding a set of Pareto-optimal solutions with certain pre-defined properties in them, or if the user is interested in finding only 'knee'-like points on the Pareto-optimal front, or if the user is interested in finding multi-modal Pareto-optimal solutions, what changes the user must make to an existing EMO methodology? Could an existing methodology be used with some simple change in its search, or an entirely new methodology is called for? If one particular methodology can be modified slightly to achieve many different problem-solving abilities, it is probably the best and most desired for a user, as (s)he is then required to know only one methodology for achieving different tasks needed in multi-objective optimization.

In this paper, we analyze a popularly-used EMO methodology – the elitist non-dominated sorting genetic algorithm or NSGA-II [6] – and discuss that the NSGA-II procedure can be functionally decomposed into three main operations. They are (i) elite preservation to achieve faster and reliable convergence towards better solutions, (ii) emphasis to non-dominated solutions for achieving a progress towards the entire Pareto-optimal front, and (iii) emphasis of less-crowded solutions for maintaining a diversity in solutions. These three operations are implemented in a *modular* manner, so that each can be modified independently to allow the NSGA-II to solve different types of multi-objective optimization problems. In this paper, for the first time, we discuss this functional decomposition aspect of NSGA-II and show how, over the years, various extensions of NSGA-II through a modification of each of these three aspects were able to find a better distribution of solutions, a partial frontier, knee points, global Pareto-optimal front, robust and reliable frontiers, estimate the nadir point, and help choose a single solution. This modular aspect makes NSGA-II framework a useful tool for research in multi-objective optimization. This aspect of modularity, along with NSGA-II's need for no additional parameter, is probably the reason for NSGA-II's popularity among EMO researchers and applicationists.

GECCO'08, July 12–16, 2008, Atlanta, Georgia, USA.

2. EVOLUTIONARY MULTI-OBJECTIVE OPTIMIZATION (EMO)

EMO methodologies work with two main goals:

1. Find a set of Pareto-optimal solutions, and
2. find a set of diverse solutions to make a better representation of the Pareto-optimal front.

In this respect, EMO methodologies belong to the class of a posteriori multiple criterion decision making (MCDM) methods [21].

2.1 Elitist Non-Dominated Sorting GA (NSGA-II)

The NSGA-II procedure [6] is one of the popularly used EMO procedures which attempt to find multiple Pareto-optimal solutions in a multi-objective optimization problem and has the following three features:

1. It uses an elitist principle,
2. it uses an explicit diversity preserving mechanism, and
3. it emphasizes non-dominated solutions.

At any generation t, the offspring population (say, Q_t) is first created by using the parent population (say, P_t) and the usual genetic operators. Thereafter, the two populations are combined together to form a new population (say, R_t) of size $2N$. Then, the population R_t classified into different non-domination classes, leading to a process called *non-dominated sorting*. It begins by identifying solutions which are not dominated by any other member of the population. The domination between two solutions is defined as follows [5, 21]:

Definition 1. *A solution* $\mathbf{x}^{(1)}$ *is said to dominate solution* $\mathbf{x}^{(2)}$*, if both are true:*

1. $\mathbf{x}^{(1)}$ *is no worse than* $\mathbf{x}^{(2)}$ *in all objectives.*
2. $\mathbf{x}^{(1)}$ *is strictly better than* $\mathbf{x}^{(2)}$ *in at least one objective.*

After solutions of the first non-dominated class are identified, they are discounted and another round of identification of non-dominated solutions is made. The corresponding solutions belong to second class of non-domination. This process is continued till all solutions are classified into a non-dominated class.

Once the non-domination sorting is over, the new population is filled by points of different non-domination fronts, one at a time. The filling starts with the first non-domination front (of class one) and continues with points of the second non-domination front, and so on. Since the overall population size of R_t is $2N$, not all fronts can be accommodated in N slots available for the new population. All fronts which could not be accommodated are deleted. When the last allowed front is being considered, there may exist more points in the front than the remaining slots in the new population. This scenario is illustrated in Figure 1. Instead of arbitrarily discarding some members from the last front, the points which will make the diversity of the selected points the highest are chosen. The crowded-sorting of the points of the last front which could not be accommodated fully is achieved in the descending order of their *crowding distance values*

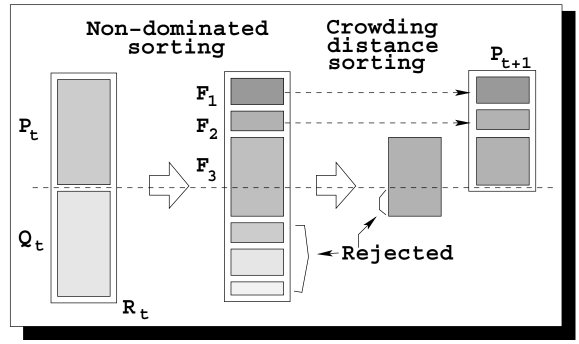

Figure 1: Schematic of the NSGA-II procedure.

and points from the top of the ordered list are chosen. The crowding distance d_i of point i is a measure of the objective space around i which is not occupied by any other solution in the population. Here, we simply calculate this quantity d_i by estimating the perimeter of the cuboid (Figure 2) formed by using the nearest neighbors in the objective space as the vertices (we call this the *crowding distance*).

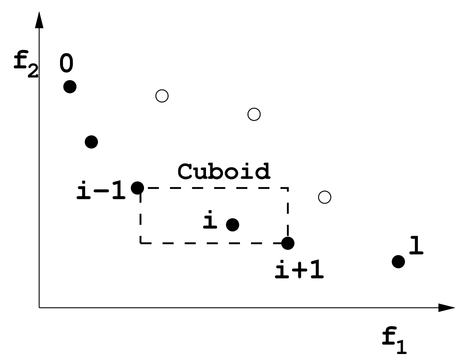

Figure 2: The crowding distance calculation.

NSGA-II provides a flexible and modular framework which is capable of solving various kinds of multi-objective optimization problems – (i) the emphasis of non-dominated solutions allows convergence towards Pareto-optimal front, (ii) the emphasis of least-crowded solutions allows NSGA-II to find a diverse set of solutions, and (iii) the emphasis of elite preservation allows a reliable and monotonically non-decreasing performance of NSGA-II. It is worth mentioning here that despite the popularity of NSGA-II in scientific and application-oriented applications, all EMO algorithms including NSGA-II are not free from their weakness in dealing with more than four objective optimization problems [11]. With an increase in number of objectives, the number of points needed to represent a higher-dimensional Pareto-optimal front must be increased exponentially. Moreover, the usual domination principle demands an exponentially large population size to be effective [5]. However, EMO methodologies can be effective if a preferred region on the Pareto-optimal front is the target, rather than the entire front, even in the case of 10 or 20 objectives [12].

In the following sections, we discuss how these three principles of NSGA-II can be modified independently one at a time to achieve different efficient algorithms for solving different types of multi-objective optimization tasks.

3. MODIFYING DOMINATION PRINCIPLE

First, we describe modified NSGA-II procedures which simply change the domination principle and keep the diversity and elite preserving principles the same as before.

3.1 Guided Domination

In this approach [3], a weighted function of the objectives is defined as follows:

$$\Omega_i(\mathbf{f}(\mathbf{x})) = f_i(\mathbf{x}) + \sum_{j=1, j\neq i}^{M} a_{ij} f_j(\mathbf{x}), \quad i = 1, 2, \ldots, M. \quad (1)$$

where a_{ij} is the amount of gain in the j-th objective function for a loss of one unit in the i-th objective function. Now, we define a different domination concept for minimization problems as follows.

Definition 2. *A solution* $\mathbf{x}^{(1)}$ *dominates another solution* $\mathbf{x}^{(2)}$, *if* $\Omega_i(\mathbf{f}(\mathbf{x}^{(1)})) \leq \Omega_i(\mathbf{f}(\mathbf{x}^{(2)}))$ *for all* $i = 1, 2, \ldots, M$ *and the strict inequality is satisfied at least for one objective.*

Figure 3(b) shows the contour lines corresponding to the above two linear functions passing through a solution A in the objective space. All solutions in the hatched region are

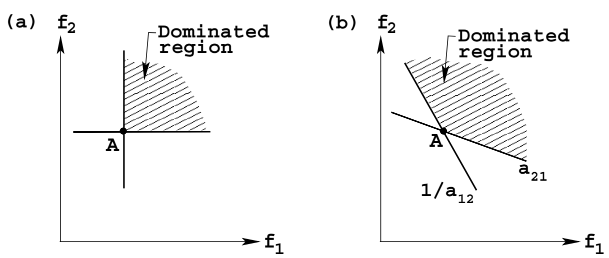

Figure 3: (a) the dominated region using the usual definition (b) the dominated region using guided domination.

dominated by A according to the above definition of domination. It is interesting to note that when using the usual definition of domination (Figure 3(a)), the region marked by a horizontal and a vertical line will be dominated by A. Thus, it is clear from these figures that the modified definition of domination allows a larger region to become dominated by any solution than the usual definition. Since a larger region is now dominated, the complete Pareto-optimal front (as per the original domination definition) may not be non-dominated according to this new definition of domination. In order to demonstrate the working of the above procedure, we apply an NSGA with the modified domination principle to the SCH1 problem [5]. With $a_{12} = a_{21} = 0.75$, the obtained partial front is shown in Figure 4. The complete frontier is much wider, as shown with a dashed line.

3.2 Epsilon Domination

In the ϵ-MOEA proposed elsewhere [10], dominance definition is changed to make sure a solution dominates another solution with at least ϵ_i difference in i-th objective [20]. A solution $\mathbf{x}^{(1)}$ ϵ-dominates another solution $\mathbf{x}^{(2)}$, if $f_i(\mathbf{x}^{(1)}) \leq f_i(\mathbf{x}^{(2)}) + \epsilon_i$ for all objectives and if the strict inequality is true for at least one objective. Although such a consideration requires users to set ϵ_i parameters, this provide a flexibility on the part of the user to find a well-distributed set of Pareto-optimal solutions. This concept

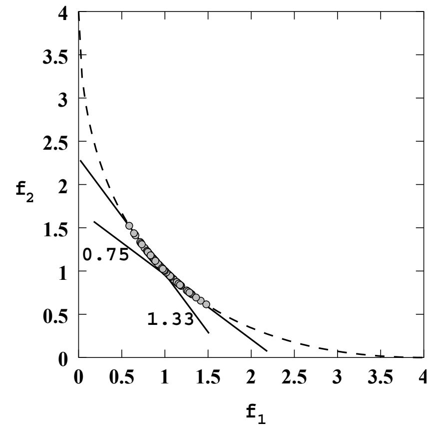

Figure 4: An intermediate portion of the Pareto-optimal region for the problem SCH1.

also has a practical aspect in that the decision-maker can now specify a minimum difference in objective values before he or she is interested in evaluating two trade-off solutions. To illustrate the effect of the ϵ-dominance, we show simulation results on the three-objective DTLZ1 problem which has a linear Pareto-optimal front. In comparison to a standard NSGA-II simulation run (Figure 5), the ϵ-MOEA with $\epsilon_i = 0.2$ (Figure 6) seems to find a better distribution of points on the Pareto-optimal front.

3.3 Other Domination and Fuzzy Dominance

Other principles based on proper Pareto-optimality conditions [15], fuzzy dominance [14] are certainly possible to be implemented by simply replacing the usual domination principle coded in NSGA-II.

For handling a large number of objectives, a study used a simpler domination strategy in which a solution having a larger number of better objective values is declared to dominate the other solution. Thus, between two solutions $\mathbf{x}^{(1)}$ and $\mathbf{x}^{(2)}$, if $f_i(\mathbf{x}^{(1)}) < f_i(\mathbf{x}^{(2)})$ for more objectives than $f_i(\mathbf{x}^{(2)}) < f_i(\mathbf{x}^{(1)})$, then solution $\mathbf{x}^{(1)}$ dominates solution $\mathbf{x}^{(2)}$. It is likely that this new definition will only identify intermediate solutions in a higher-dimensional Pareto-optimal front. It is interesting to extrapolate the idea and include some preference information in the comparison. Based on a priori preference information about important objectives, a hierarchy or more weightage can be allocated for preferred objectives. In this aspect, the weighted domination approach [22] is interesting and can be tried to implement with the NSGA-II procedure.

The fuzzy-dominance concept determines dominance of a solution based on a fuzzy-logic based comparison scheme. Thus, a solution winning with a larger margin can be sure of being a non-dominated solution. A solution with marginally better function values may dominate a solution or may get dominated by another solution with a probability less than one. The effect of such domination schemes is that the final trade-off region is not crisp, but defined with a fuzzy boundary [14].

3.4 Constrained Domination

The constraint handling method modifies the domination definition to include feasibility of two comparing solutions.

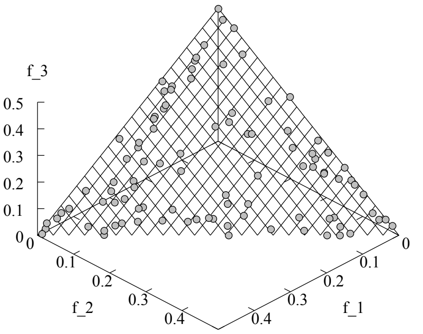

Figure 5: NSGA-II distribution on DTLZ1.

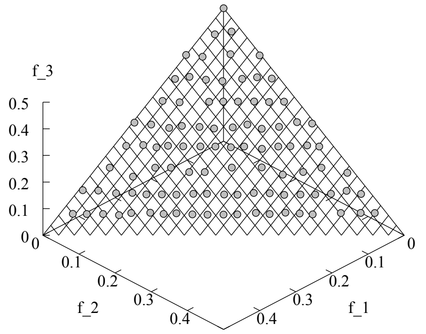

Figure 6: ϵ-MOEA distribution on DTLZ1.

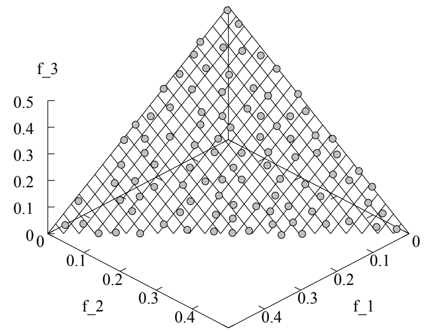

Figure 7: C-NSGA-II distribution on DTLZ1.

We simply redefine the domination principle as follows:

Definition 3. *A solution $\mathbf{x}^{(i)}$ is said to 'constrained-dominate' a solution $\mathbf{x}^{(j)}$ (or $\mathbf{x}^{(i)} \preceq_c \mathbf{x}^{(j)}$), if any of the following conditions are true:*

1. *Solution $\mathbf{x}^{(i)}$ is feasible and solution $\mathbf{x}^{(j)}$ is not.*
2. *Solutions $\mathbf{x}^{(i)}$ and $\mathbf{x}^{(j)}$ are both infeasible, but solution $\mathbf{x}^{(i)}$ has a smaller constraint violation, which can be computed by adding the normalized violation of all constraints:*

$$\mathrm{CV}(\mathbf{x}) = \sum_{j=1}^{J} \langle \bar{g}_j(\mathbf{x}) \rangle + \sum_{k=1}^{K} \mathrm{abs}(\bar{h}_k(\mathbf{x})),$$

where $\langle \alpha \rangle$ is $-\alpha$, if $\alpha < 0$ and is zero, otherwise. The normalization is achieved with the population minimum ($\langle g_j \rangle_{\min}$) and maximum ($\langle g_j \rangle_{\max}$) constraint violations: $\bar{g}_j(\mathbf{x}) = (\langle g_j(\mathbf{x}) \rangle - \langle g_j \rangle_{\min})/(\langle g_j \rangle_{\max} - \langle g_j \rangle_{\min})$.

3. *Solutions $\mathbf{x}^{(i)}$ and $\mathbf{x}^{(j)}$ are feasible and solution $\mathbf{x}^{(i)}$ dominates solution $\mathbf{x}^{(j)}$ in the usual sense (Definition 1).*

The above change in the definition requires a minimal change in the NSGA-II procedure described earlier. Figure 8 shows the non-domination fronts on a six-membered population due to the introduction of two constraints (the minimization problem is described as CONSTR elsewhere [5]). In the absence of the constraints, the non-domination fronts (shown by dashed lines) would have been `((1,3,5), (2,6), (4))`, but in their presence, the new fronts are `((4,5), (6), (2), (1), (3))`. Such a simple change in definition of domination allows NSGA-II to make a proper emphasis among feasible and infeasible solutions, so that feasible and optimal solutions can be found using NSGA-II.

3.5 Robust Domination Handling Uncertainties

In many applied multi-objective optimization problems, problem parameters and decision variables can be uncertain. In such scenarios, instead of finding the deterministic Pareto-optimal front, a *robust frontier*, on which every solution is relatively insensitive to the uncertainties, is usually the target. In such cases, instead of performing domination on the objective function values, it can be performed on the neighborhood-average function values. Since average values are minimized, the obtained frontier is likely to be a robust

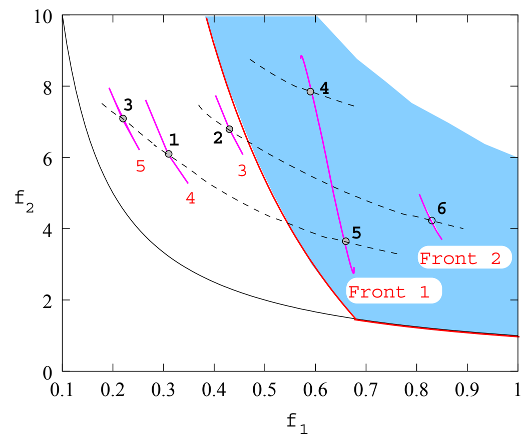

Figure 8: Non-constrained-domination fronts.

frontier. Figure 9 shows the simulation result of NSGA-II on a test problem. The modification has enabled NSGA-II to find a different trade-off frontier (robust frontier) which depends on the chosen neighborhood size (δ) and is different from the original Pareto-optimal frontier.

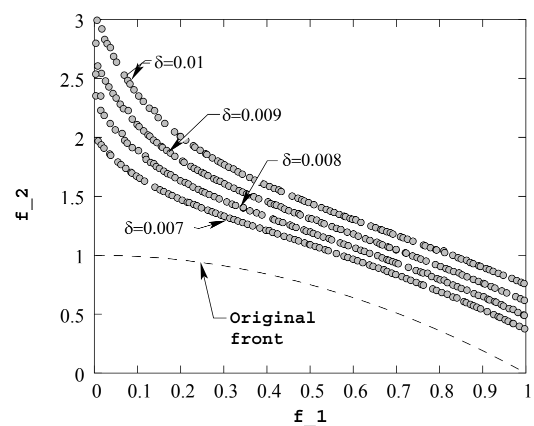

Figure 9: Robust solutions are found using NSGA-II with modified dominance.

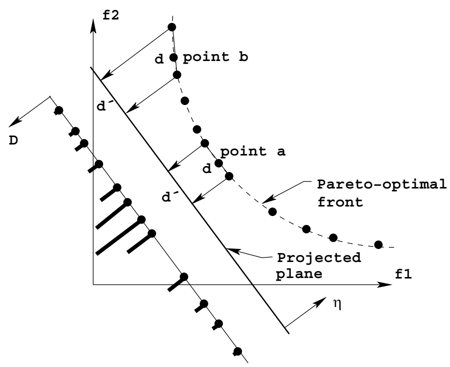

Figure 10: The biased crowding approach in NSGA-II.

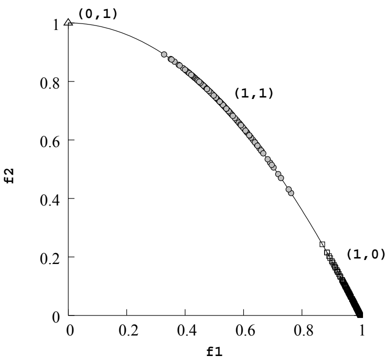

Figure 11: Biased NSGA-II with three different planes on ZDT2.

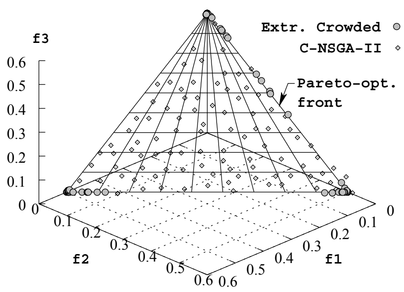

Figure 12: NSGA-II population using extremized crowded and original NSGA-II for DTLZ1.

4. MODIFYING DIVERSITY PRESERVING PRINCIPLE

Next, we describe modified NSGA-II procedures which changed the diversity preserving principle and kept domination and elite preserving principles the same. There exist a number of modifications under this category, as by modifying the diversity preserving principle, focus can be placed in finding specific Pareto-optimal solutions, instead of the complete Pareto-optimal frontier.

4.1 Clustered NSGA-II

In this approach, the crowding distance operator of NSGA-II is replaced with a K-mean clustering approach [10]. Each solution of the last front is first considered to lie on a separate cluster. Thereafter, the inter-cluster Euclidean distance is computed and the two clusters with the minimum distance are merged together. This process is repeated till the required number of clusters remain. For a cluster having multiple solutions, an average distance to another cluster is computed. Finally, from each cluster a solution close to the centroid of the cluster is chosen and others are deleted. Although this requires a larger computational time, the clustered NSGA-II is expected to find a better distributed set of Pareto-optimal solutions than the original NSGA-II. Figure 7 shows a better distribution with this clustering approach than that obtained in Figure 5 with the original NSGA-II.

Recently, pruning strategies are suggested in which instead of selecting a block of solutions simultaneously by the crowding distance operator, solutions are chosen one by one and crowding distance values are re-computed after each update [19]. This certainly resulted in finding a better distribution of solutions.

4.2 Projection Based Diversity Preservation

In the projection-based diversity preservation method, a biased crowding distance measure is used [1]:

$$D_i = d_i \left(\frac{d_i'}{d_i}\right)^{\alpha}, \qquad (2)$$

where d_i and d_i' are the original crowding distance and the crowding distance calculated based on the locations of the individuals projected onto the (hyper-)plane with direction vector η. Figure 10 illustrates the concept. As a result, for a solution in a region of the Pareto-optimal front more or less parallel to the projected plane (such as solution 'a'), the original crowded distance d_a and projected crowding distance d_a' are more or less the same, thereby making the ratio d_a'/d_a close to one. Solutions in such regions will be preferred by this biased NSGA-II. Figure 11 shows three different runs, each simulated with a different plane indicated on the plot with $\alpha = 100$. Every time, the biased NSGA-II converges to a different part of the non-convex Pareto-optimal front.

4.3 Distributed Domination

The cone dominance concept facilitates NSGA-II to be used on a distributed computing platform to find the complete frontier by distributing the task among multiple processors. Each processor uses a different but non-overlapping cone, so that the Pareto-optimality of each processor is different from others. By using occasional migration of individuals from one processor to the other, an overall faster optimization task is achieved in finding the complete frontier in an adaptive manner. Figure 13 shows the results obtained by three processors to find a three-dimensional Pareto-optimal front.

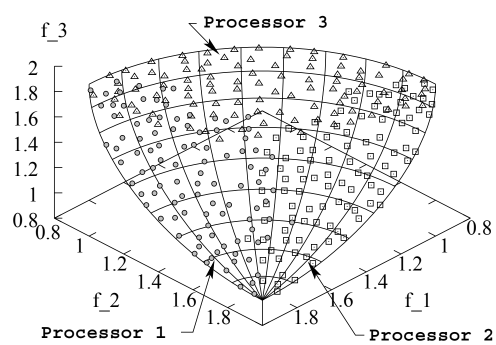

Figure 13: Parallel NSGA-II solutions with three processors in solving DTLZ2.

4.4 Multi-modal Diversity Preservation

In some problems, one Pareto-optimal point in the objective space ($\mathbf{f} \in R^M$) may correspond to a number of solutions in the decision space ($\mathbf{x} \in R^n$). In such problems, a goal may be to find multiple solutions corresponding to each Pareto-optimal point. This task is similar to finding multiple optimal solutions in a multi-modal single objective optimization problem [16]. We extend the *niching* concept for handling multi-modal problems for the multi-objective optimization case here.

First, we delete the duplicate solutions from each non-domination set in R_t. Thereafter, each set is accepted as usual till the last front F_l which can be accommodated. Let us say that solutions remaining to be filled before this last front is considered is N' and the number of non-duplicate solutions in the last front is N_l ($> N'$). We also compute the number of distinct objective solutions in the set F_l and let us say it is n_l (obviously, $n_l \leq N_l$). This procedure is illustrated in Figure 14. If $n_l \geq N'$ (the top case shown in

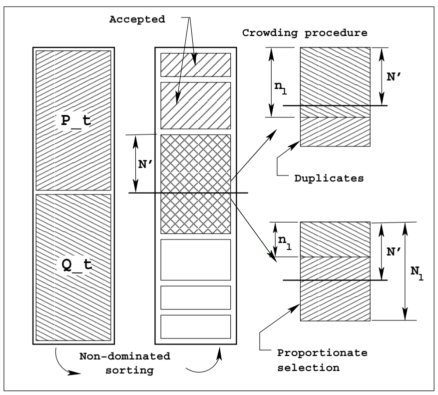

Figure 14: Schematic of the multi-modal NSGA-II procedure.

the figure), we use the usual crowding distance procedure to choose N' most dispersed and distinct solutions from n_l solutions. Otherwise, we change the procedure as follows. We choose a strategy in which every distinct objective solution is allowed to have a proportionate number of multi-modal solutions as they appear in F_l. To avoid loosing any distinct objective solutions, we first allocate one copy of each distinct objective solution, thereby allocating n_l copies. Thereafter, the proportionate rule is applied to the remaining solutions ($N_l - n_l$) to find the accepted number of solutions for the i-th distinct objective solution as follows:

$$\alpha_i = \frac{N' - n_l}{N_l - n_l}(m_i - 1), \tag{3}$$

where m_i is the number of multi-modal solutions of the i-th distinct objective solution in F_l, such that $\sum_{i=1}^{n_l} m_i = N_l$. The final task is to choose $(\alpha_i + 1)$ multi-modal solutions from m_i copies for the i-th distinct objective solution. In the rare occasions of having less than N non-duplicate solutions in R_t, new random solutions are used to fill up the population.

The multi-modal NSGA-II is applied to a bioinformatics problem of identifying gene classifiers for achieving minimum number of mismatches in classification of training samples of microarray data on two types of leukemia samples and simultaneously achieving the task with the smallest classifier size. In this task, 50 genes are considered and the objective space is discrete. Weak Pareto-optimal solutions are attempted to find for getting a comprehensive idea of optimal classifiers. Figure 15 shows a part of the feasible objective space and the obtained Pareto-optimal front. Multiplicities of the solutions with no mismatches in classification are also shown. Such a consideration brings out important insights about the problem: (i) There are eight different five-gene classifiers which cause 100% correct classification, (ii) only five to 10-gene classifiers are capable of making 100% correct classifications.

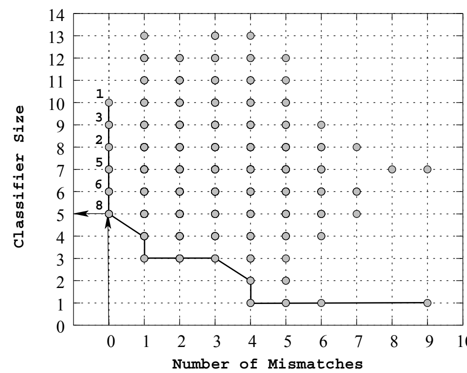

Figure 15: Multiple weak Pareto-optimal solutions found for the 50-gene leukemia problem.

4.5 Omni-optimizer

In this extension of NSGA-II, the crowding distance measure is computed in both objective and decision variable spaces and a combined method is suggested [13]. As expected, such an implementation was able to find a well-distributed set of solutions in both spaces thereby solving multi-modal problems in single and multi-objective optimization problems alike. In single objective problems, this means finding multiple global optimal solutions simultaneously and in multi-objective optimization problems this means simultaneously finding multiple solutions corresponding a single Pareto-optimal point. Due to space restrictions, we do not show the results here.

4.6 Extreme Point Preference for Nadir Point Estimation

Nadir point corresponds to the worst objective values of the Pareto-optimal front. Nadir point is important to know in a multi-objective optimization problem solving task because of a number of reasons. Together with the ideal point, it provides a way to normalize the objectives so certain multi-objective optimization algorithms can be used. In an earlier study [7], NSGA-II's diversity preservation operator is modified to find only extreme points in a Pareto-optimal front so that the nadir point can be estimated. The crowding distance measure is replaced with a ranking scheme in which solutions having the best and worst individual objective values are given the highest rank and the importance gets less for intermediate solutions. With such a strategy applied to NSGA-II resulted in a population shown in Figure 12 with circles, whereas the original NSGA-II resulted in the population marked with diamonds. Since the modified NSGA-II can find only extreme solutions, the estimation of the nadir point becomes an easier and quicker task using this approach.

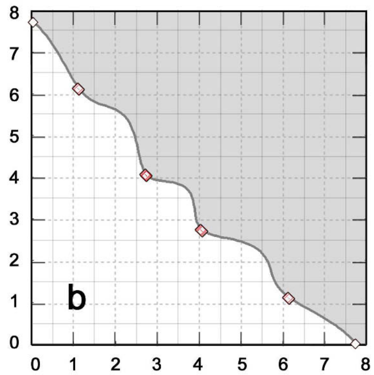

Figure 16: Knee based NSGA-II finds solutions near knee points.

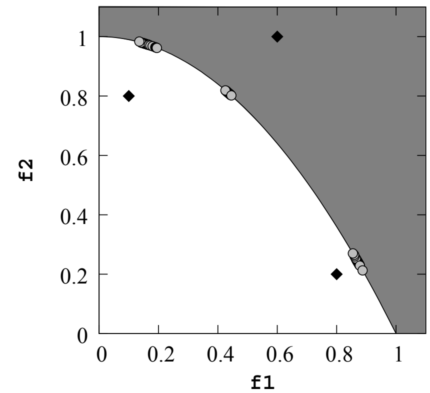

Figure 17: Preferred NSGA-II solutions with three reference points on ZDT2.

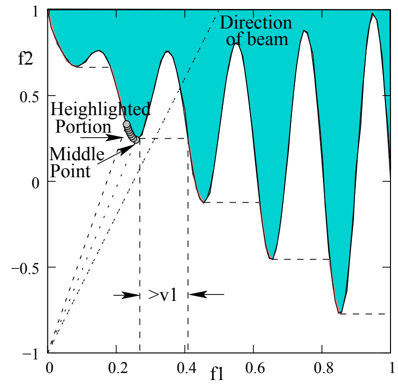

Figure 18: LBS based NSGA-II on ZDT3.

4.7 Knee Point Preference

Knee points refer to points from which a small gain in one objective requires a large sacrifice in at least one other objective. Knee points are important in multi-objective optimization, as once found there is not much motivation for a decision-maker to move out of these points. If knee points exist in a problem, it may be desirable to find only these points. For this task, we can again modify NSGA-II's crowding distance operator and aim at emphasizing solutions portraying knee-like properties [2]. A previous study considered two different implementations for this purpose. Simply stated, in the utility function based approach, several linear utility functions are chosen and solutions corresponding to the maximum values of more utility functions are emphasized in the population. Since knee points will correspond to optimum of many linear utility functions, such a strategy ends up finding only the knee points, as shown in Figure 16.

4.8 Reference Point Based NSGA-II

In an effort to combine EMO procedures with a multi-criterion decision-making aid for choosing a single preferred Pareto-optimal solution, an earlier study proposed a hybrid reference point based NSGA-II [12]. Instead of finding the entire Pareto-optimal front, the focus is to find the region (through a set of points) which corresponds to the optimum of the achievement scalarizing function formed with one or more reference points. Again, NSGA-II's crowding distance operator is changed to emphasize solutions with larger values of the achievement scalarizing function. Thereafter, to maintain a range of solutions, an ϵ-dominance principle is incorporated to further emphasize which are at least ϵ distance away from each other. The rest of NSGA-II implementation remains the same. Figure 17 shows that this reference point based NSGA-II is able to find three different regions on the Pareto-optimal front corresponding to the three different reference points supplied by the user. More results can be found in the original study [12].

4.9 Light Beam Search Based NSGA-II

In another study [9], the classical light beam search approach [17] is combined with NSGA-II so as to find only a part of the Pareto-optimal front lighted by a beam. Usually, the beam is started from the ideal point or a desired reference point and aimed towards the nadir point. The diverging angle of the beam gets determined by the shape of the Pareto-optimal frontier and a number of user-defined trade-off parameter, called veto thresholds. By modifying the crowding distance operator to emphasize solutions near the augmented achievement scalarizing function solution, this task can be achieved easily, even for multiple light beams.

Figure 18 shows the result on ZDT3 having a disconnected Pareto-optimal frontier. One side of the beam does not light up any part of the Pareto-optimal front in this problem. The modified NSGA-II is able to display this fact by finding only the lighted side of the Pareto-optimal frontier. More results can be found in the original study [9].

5. MODIFYING ELITE PRESERVING PRINCIPLE

The elite preserving principle can be modified to emphasize or de-emphasize solutions on the best non-dominated frontier. There is at least one such implementation which we describe in the next subsection.

5.1 Controlled NSGA-II

Most studies in EMO concentrated on emphasizing diversity preservation along the current non-dominated frontier. However, the lateral diversity is also an important matter which may enhance the performance of an EMO particularly in difficult optimization problems. In an earlier study [8], non-dominated fronts of second, third and higher levels (which are worse than the first-level non-dominated solutions) are deliberately kept in the population in a geometrically reduced manner. In ZDT4 problem which has many local Pareto-optimal frontiers, such a reduced and controlled elite preservation helped find better non-dominated frontiers, as shown in Figure 19. In this problem, the smaller the value of $g()$, the closer is the frontier to the true Pareto-optimal frontier. It is clear that the original NSGA-II was equipped with a too strong an elite preservation and as the emphasis on higher-level frontiers are made by increasing the geometric progression parameter r, the performance gets better. At around $r = 0.65$, the performance of controlled NSGA-II is the best.

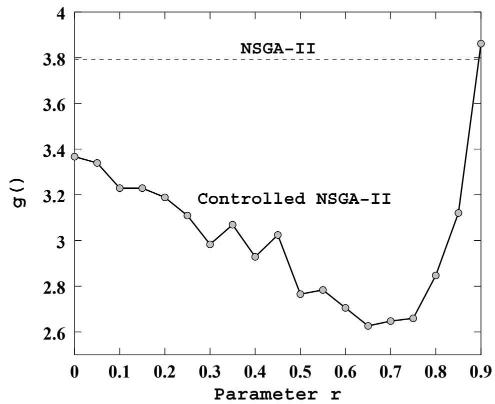

Figure 19: The controlled elite-preserving procedure in NSGA-II.

6. CONCLUSIONS

In this paper, we have functionally decomposed a well-known evolutionary multi-objective optimization algorithm (NSGA-II) for its three main aspects: elite preservation, dominance consideration and diversity preservation. NSGA-II algorithm was suggested in 2000 and since then the procedure has been modified to solve various multi-objective problem solving tasks by simply modifying the one or more of these three aspects. In this paper, for the first time, we have demonstrated how each of these extensions was made possible. This suggests that NSGA-II framework is modular and is an ideal platform to extend to achieve different goals by understanding each of the three functionalities. Such a flexible framework makes NSGA-II an ideal candidate to be coded in a software which can turn itself into several multi-objective optimizers by a simple modification to one or more of its operations. It will also be interesting to investigate other popular EMO algorithms (such as SPEA2, PESA and others) for their modularity and ability to provide a similar flexibility as demonstrated in this paper with NSGA-II in solving different types of multi-objective optimization problems. Other extensions than what has been discussed here are certainly possible and this paper should provide a motivation for readers to try other ideas, may be by changing more than one aspects at a time, with NSGA-II and other EMO methodologies.

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