Effectiveness of Scalability Improvement Attempts on the Performance of NSGA-II for Many-Objective Problems

Hisao Ishibuchi, Noritaka Tsukamoto, Yasuhiro Hitotsuyanagi, and Yusuke Nojima
Graduate School of Engineering, Osaka Prefecture University
1-1 Gakuen-cho, Naka-ku, Sakai, Osaka 599-8531, Japan
Phone: +81-72-254-9350
{hisaoi@, nori@ci., hitotsu@ci., nojima@}cs.osakafu-u.ac.jp

ABSTRACT
Recently a number of approaches have been proposed to improve the scalability of evolutionary multiobjective optimization (EMO) algorithms to many-objective problems. In this paper, we examine the effectiveness of those approaches through computational experiments on multiobjective knapsack problems with two, four, six, and eight objectives. First we briefly review related studies on evolutionary many-objective optimization. Next we explain why Pareto dominance-based EMO algorithms do not work well on many-objective optimization problems. Then we explain various scalability improvement approaches. We examine their effects on the performance of NSGA-II through computational experiments. Experimental results clearly show that the diversity of solutions is decreased by most scalability improvement approaches while the convergence of solutions to the Pareto front is improved. Finally we conclude this paper by pointing out future research directions.

Categories and Subject Descriptors
I.2.8 [**Artificial Intelligence**]: Problem Solving, Control Methods, and Search – *Heuristic Methods.*

General Terms
Algorithms.

Keywords
Evolutionary multiobjective optimization (EMO), many-objective optimization, Pareto dominance, crowding distance, knapsack problems, balance between diversity and convergence.

1. INTRODUCTION
Evolutionary multiobjective optimization (EMO) algorithms have been successfully used in a wide range of real-world application tasks [1], [5], [6], [8]. Whereas EMO algorithms usually work very well on two-objective problems, their search ability is severely deteriorated by the increase in the number of objectives. Well-known Pareto dominance-based EMO algorithms such as SPEA [33] and NSGA-II [9] do not work well on many-objective problems with four or more objectives. This is because almost all individuals in each population become non-dominated with each other when they are compared using many objectives. That is, almost all individuals have the same fitness with respect to Pareto dominance-based criteria. As a result, Pareto dominance-based EMO algorithms can not have a strong selection pressure toward the Pareto front of a many-objective optimization problem.

A simple approach for increasing the selection pressure toward the Pareto front is to introduce different ranks to non-dominated solutions in each population. Another simple approach is to modify dominance relation in order to decrease the number of non-dominated solutions in each population. Almost the same effect can be obtained by the modification of objective functions to increase the correlation among them. In this paper, we examine the effectiveness of these approaches by incorporating them into NSGA-II. We also examine two simple tricks for decreasing the diversity maintenance effect in NSGA-II. One is to assign a zero distance instead of an infinity distance to extreme solutions as their crowding distance. The crowding distance of the other solutions is calculated in the same manner as NSGA-II. Another trick is to assign a random value to each solution as their crowding distance.

Experimental results on many-objective knapsack problems show that all of these approaches significantly improve the performance of NSGA-II in terms of the convergence of solutions toward the Pareto front. These approaches, however, severely decrease the diversity of solutions. We also demonstrate that the hybridization of NSGA-II with local search clearly improves the diversity of solutions along the Pareto front while it slightly improves the convergence property of NSGA-II.

In this paper, we briefly review related studies on evolutionary many-objective optimization in Section 2 (see [21] for a review on this area). Next we explain why EMO algorithms do not work well on many-objective problems in Section 3. Then we examine the effectiveness of scalability improvement approaches by combining them into NSGA-II in Section 4. Finally we conclude this paper by pointing out some future research directions in the field of evolutionary many-objective optimization in Section 5.

2. RELATED STUDIES
The deterioration of the search ability of EMO algorithms by the increase in the number of objectives has already been pointed out

GECCO'08, July 12-16, 2008, Atlanta, Georgia, USA

in a number of studies. Examples of early studies are [24], [27]. It was clearly shown by [15], [23] that multiple runs of single-objective evolutionary algorithms outperformed EMO algorithms when they were applied to many-objective problems.

In Pareto dominance-based EMO algorithms such as SPEA [33] and NSGA-II [9], the fitness of each solution is usually evaluated by two criteria: a primary criterion based on the Pareto dominance relation and a secondary criterion based on the concept of crowding. A Pareto dominance-based primary criterion is used to generate a selection pressure toward the Pareto front while a crowding-based secondary criterion is used to increase the diversity of solutions along the Pareto front.

As we have already explained, the increase in the number of objectives weakens the selection pressure toward the Pareto front in EMO algorithms through the increase in the number of non-dominated solutions in each population. Thus the scalability improvement of EMO algorithms to many-objective problems can be realized by strengthening the selection pressure toward the Pareto front. One approach in this direction is to assign different ranks to non-dominated solutions [7], [13], [25], [26], [29]. Another approach in the same direction is to modify dominance relation to decrease the number of non-dominated solutions in each population [28]. Almost the same effect as the modification of dominance relation is obtained by the modification of objective functions to increase the correlation among them [2], [19].

Another direction for the scalability improvement is the use of different fitness evaluation mechanisms (instead of Pareto dominance). One approach in this direction is the use of an indicator function such as hypervolume to evaluate the quality of solution sets [20], [31]. This class of EMO algorithms is often called IBEAs (indicator-based evolutionary algorithms). Another approach is to use a number of different scalarizing functions for fitness evaluation of each solution [15], [16], [18], [19], [23], [32].

In the above-mentioned approaches, EMO algorithms are applied to many-objective problems. On the other hand, the number of objectives is decreased in dimensionality reduction [3], [4], [10], [11]. It is much easier for EMO algorithms to search for Pareto-optimal solutions with respect to a small number of selected objectives after dimensionality reduction than the search with respect to a large number of original objectives.

The incorporation of preference of the decision maker into EMO algorithms has also been proposed to handle many-objective problems [12], [14], [30]. Preference information is used to concentrate the search by EMO algorithms on a small region of the Pareto front.

3. MANY-OBJECTIVE OPTIMIZATION

In this section, we explain why many-objective optimization is difficult for EMO algorithms through computational experiments on multiobjective knapsack problems. Throughout this paper, we use NSGA-II as a representative algorithm of Pareto dominance-based EMO algorithms.

3.1 Test Problems

As test problems, we used 500-item knapsack problems with two, four, six and eight objectives. We denote each problem as a *k-n* test problem where *k* is the number of objectives and *n* is the number of items (i.e., 2-500, 4-500, 6-500, 8-500 test problems). The 2-500 and 4-500 test problems are exactly the same as those in Zitzler and Thiele [33]. On the other hand, we generated our 6-500 and 8-500 test problems in the same manner as in [33].

Our *k-n* test problem is written in a general form as follows:

$$\text{Maximize } \mathbf{f}(\mathbf{x}) = (f_1(\mathbf{x}),\ f_2(\mathbf{x}),\ ...,\ f_k(\mathbf{x}))\ , \tag{1}$$

$$\text{subject to } \sum_{j=1}^{n} w_{ij} x_j \le c_i\ ,\quad i = 1, 2, ..., k\ , \tag{2}$$

$$x_j = 0 \text{ or } 1,\quad j = 1, 2, ..., n\ , \tag{3}$$

$$\text{where}\quad f_i(\mathbf{x}) = \sum_{j=1}^{n} p_{ij} x_j\ ,\quad i = 1, 2, ..., k\ . \tag{4}$$

In this formulation, **x** is an *n*-dimensional binary vector, p_{ij} is the profit of item *j* according to knapsack *i*, w_{ij} is the weight of item *j* according to knapsack *i*, and c_i is the capacity of knapsack *i*. Each solution **x** is handled as a binary string of length *n* (i.e., 500).

3.2 Performance Measures

We used the following three performance measures to monitor the convergence of solutions toward the Pareto front and their diversity along the Pareto front during the execution of NSGA-II:

Maximum sum of the objective values: MaxSum

In each generation, we calculated the maximum value of the sum of the objective functions as follows:

$$\text{MaxSum}(\Psi) = \max_{\mathbf{x}\in\Psi} \sum_{i=1}^{k} f_i(\mathbf{x})\ , \tag{5}$$

where Ψ denotes the current population in each generation.

This measure was used to evaluate the convergence of solutions toward the center region of the Pareto front in the objective space.

Sum of the maximum objective values: SumMax

The sum of the maximum value of each objective function was calculated in each generation as follows:

$$\text{SumMax}(\Psi) = \sum_{i=1}^{k} \max_{\mathbf{x}\in\Psi} f_i(\mathbf{x})\ . \tag{6}$$

This measure was used to evaluate the convergence of solutions toward the *k* edges of the Pareto front in the objective space.

Sum of the ranges of the objective values: Range

The sum of the range of the objective values of each objective was calculated in each generation as follows:

$$\text{Range}(\Psi) = \sum_{i=1}^{k} [\max_{\mathbf{x}\in\Psi}\{f_i(\mathbf{x})\} - \min_{\mathbf{x}\in\Psi}\{f_i(\mathbf{x})\}]\ . \tag{7}$$

This measure was used to evaluate the diversity of solutions in the objective space.

We used these performance measures to examine the effect of scalability improvement approaches on the convergence property and the diversity maintenance ability of NSGA-II. The simplicity of the calculation is a large advantage of these performance measures over more sophisticated measures such as hypervolume especially when they are used for many-objective problems.

3.3 Conditions of Computational Experiments

We applied NSGA-II [9] to each of our test problems using the following parameter specifications:

Population size: 100,
Crossover probability: 0.8 (Uniform crossover),
Mutation probability: 1/500 (Bit-flip mutation),
Stopping condition: 100,000 generations,
Number of runs for each test problem: 10 runs.

We performed computational experiments for an unusually large number of generations (i.e., 100,000 generations) to examine a long-term behavior of NSGA-II. Due to such a heavy computation load for each run, we applied NSGA-II to each test problem only ten times. Reported results in this paper are average results over those ten runs.

In the execution of NSGA-II, infeasible solutions were often generated. In order to transform an infeasible solution into a feasible one, we used a repair procedure based on a maximum profit/weight ratio as in Zitzler and Thiele [33]. More specifically, we removed items from an infeasible solution in ascending order of the following maximum profit/weight ratio of each item until all the constraint conditions of each test problem were satisfied:

$$q_j = \max\{p_{ij}/w_{ij} \mid i = 1, 2, ..., k\}, \ \ j = 1, 2, ..., n. \quad (8)$$

3.4 Experimental Results

In Figs. 1-3, we show average results over ten runs of NSGA-II. Experimental results were normalized so that the average result of each measure at the initial generation becomes 100 for each test problem in Figs. 1-3. We always used this normalization procedure for the three performance measures throughout this paper (i.e., we always used the average result at the initial generation as the baseline value 100).

The MaxSum measure in Fig. 1 shows the convergence property of NSGA-II toward the center region of the Pareto front. From Fig. 1, we can see that the convergence of solutions to the Pareto front was slowed down by the increase in the number of objectives. One interesting observation is that the MaxSum measure first increased then decreased during the execution of NSGA-II for the 4-500 and 6-500 test problems in Fig. 1.

The SumMax measure in Fig. 2 shows the convergence property of NSGA-II toward the edges of the Pareto front. Of course, this measure implicitly shows the diversity of solutions. In Fig. 2, the average values of the SumMax measure were gradually improved during the execution of NSGA-II over a large number of generations. This observation suggests the difficulty in finding non-dominated solutions that cover the entire Pareto front within a small number of generations of NSGA-II.

The Range measure in Fig. 3 shows the diversity of solutions. We can observe in Fig. 3 that the increase in the number of objectives led to the increase in the diversity of solutions. This is because the crowding distance-based secondary criterion instead of the Pareto sorting-based primary one had a dominant effect on the fitness evaluation of each solution when the number of objectives was large (i.e., when almost all solutions in each population were non-dominated). The Pareto sorting-based primary criterion had a dominant effect on the fitness evaluation only in early generations where the diversity of solution was decreased.

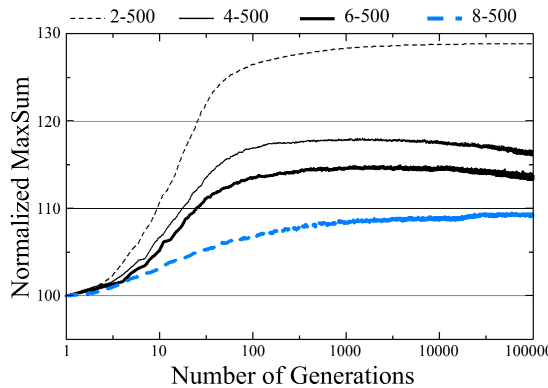


Figure 1. Convergence toward the center region of the Pareto front (NSGA-II).

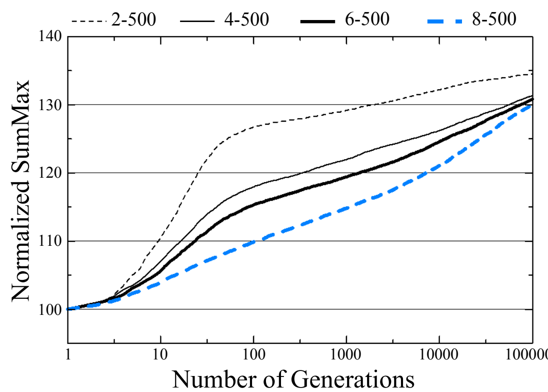


Figure 2. Convergence toward the edges of the Pareto front (NSGA-II).

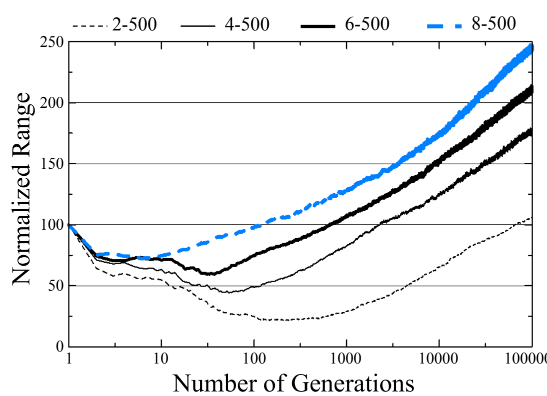


Figure 3. Diversity of solutions (NSGA-II).

To further examine the search behavior of NSGA-II in Figs. 1-3, we counted the number of non-dominated solutions in the current and offspring populations just before the generation update in each generation. Since we specified the population size as 100, 100 offspring were generated from the current population of 100 solutions in each generation. We counted the number of non-dominated solutions among those 200 solutions in the current and offspring populations. Average results are shown in Fig. 4 in the same manner as in Sato et al. [28]. In the generation update phase of NSGA-II, the best 100 solutions were chosen as the next population from 200 solutions in the current and offspring populations in each generation.

In Fig. 4, we can see that almost all solutions after the generation update were non-dominated with each other except for very early generations. When the number of non-dominated solutions was smaller than 100 (i.e., smaller than the population size) in Fig. 4, the Pareto sorting-based primary criterion had a large effect on the fitness evaluation in both the parent selection phase and the generation update phase. As a result, the convergence was improved in Fig. 1 and the diversity was decreased in Fig. 3 in early generations. On the other hand, the parent selection phase was governed by only the crowding distance-based secondary criterion when the number of non-dominated solutions was larger than 100 in Fig. 4. As a result, the convergence improvement was slowed down in Fig. 1 and the diversity was increased in Fig. 3 in later generations.

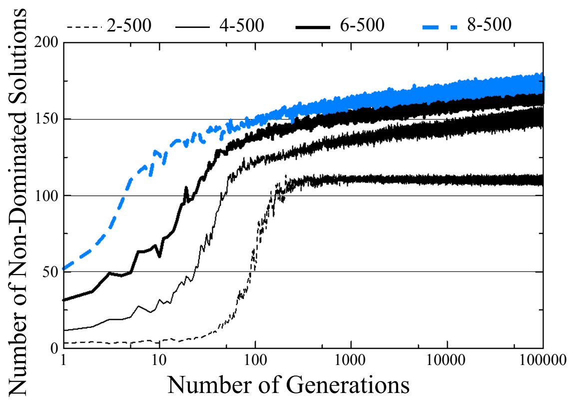


Figure 4. Average number of non-dominated solutions in the current and offspring populations.

4. SCALABILITY IMPROVEMENT

In this section, we examine the effect of scalability improvement approaches on the performance of NSGA-II.

4.1 Modification of Crowding Distance

As we have already explained, the increase in the number of objectives biases the convergence-diversity balance toward the increase in the diversity of solutions. Thus the decrease in the diversity maintenance effect may have a positive effect on the performance of NSGA-II on many-objective problems. A simple way is to assign a zero distance (instead of an infinity distance) to extreme solutions with maximum or minimum objective values as the crowding distance, which was suggested in [31]. We also examined the random assignment of the crowding distance to each solution. This has the same effect on the performance of NSGA-II as the assignment of the same crowding distance to all solutions.

In Figs. 5-7, we show experimental results with the assignment of a zero distance as the crowding distance to extreme solutions. For the sake of comparison, we show the corresponding results at the 100,000th generation of NSGA-II on the right side of each figure by short horizontal lines. We can see from Fig. 5 that the convergence of solutions toward the Pareto front was improved by the assignment of a zero distance to extreme solutions. On the other hand, the diversity of solutions was severely decreased in Fig. 7. As a result, the convergence toward the edge of the Pareto front in Fig. 6 was also severely degraded. This means that good solutions were not obtained along a wide range of the Pareto front (i.e., good solutions were obtained only around its center region).

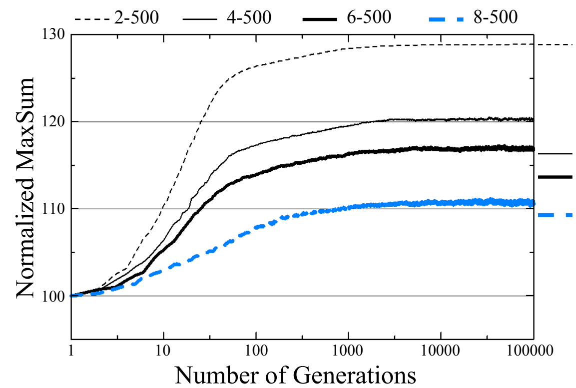


Figure 5. Convergence toward the center (Zero distance).

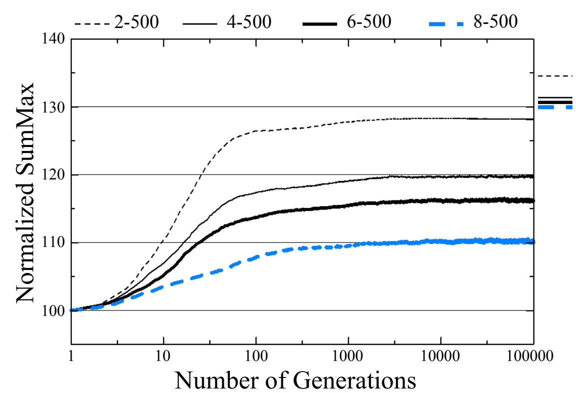


Figure 6. Convergence toward the edges (Zero distance).

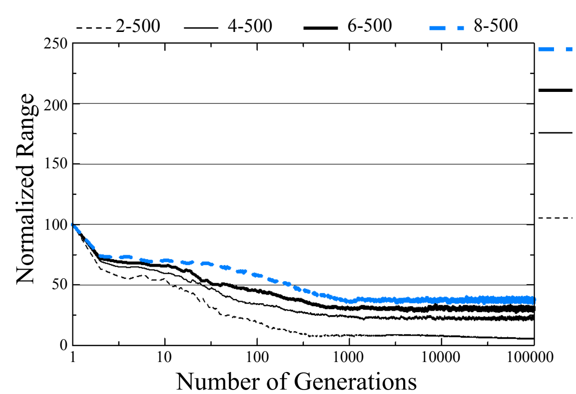


Figure 7. Diversity of solutions (Zero distance).

In Fig. 8 and Fig. 9, we show experimental results with the random assignment of the crowding distance to all solutions. In this case, solutions with the same rank (with respect to Pareto sorting) were randomly ordered in their fitness evaluation for parent selection and generation update. Since we totally removed the crowding distance calculation (i.e., we removed the crowding distance-based secondary criterion) from NSGA-II, the diversity of solutions could not be maintained in Fig. 9 while the convergence toward the Pareto front was improved in Fig. 8 for many-objective problems.

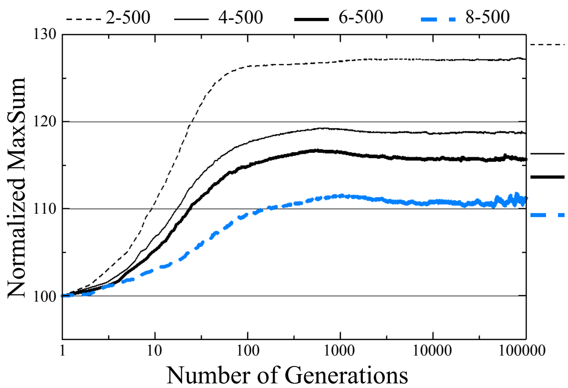


Figure 8. Convergence toward the center (Random distance).

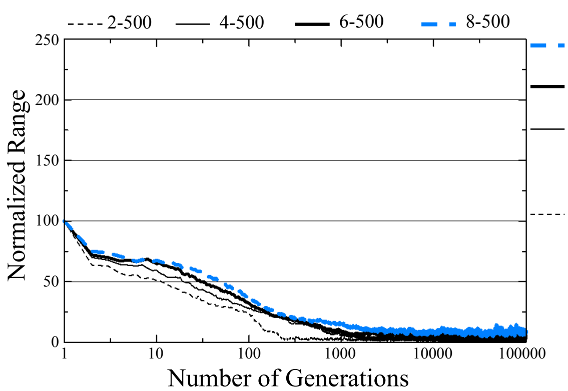


Figure 9. Diversity of solutions (Random distance).

4.2 Introduction of Different Ranks

Let us assume that the shaded region of each pentagon in Fig. 10 is an objective vector of a five-objective maximization problem. These three objective vectors are non-dominated with each other. Whereas the Pareto sorting-based primary criterion assigns the same rank to these three objective vectors, one may think that Solution A (and Solution B) seems to be better than Solution C. A number of ranking methods have been proposed to assign different ranks to non-dominated solutions in the literature.

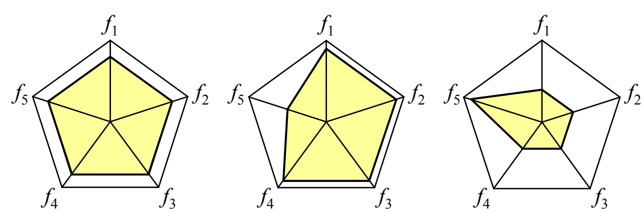
Drechsler et al. [13] proposed the use of a relation called *favour* to differentiate between non-dominated solutions for the handling of many-objective problems. They defined the relation *favour* based on the number of objectives for which one solution is better than the other. More specifically, a solution **z** is viewed as being better than another solution **y** under the relation *favour* when the following relation holds:

$$\left|\{j : f_j(\mathbf{z}) < f_j(\mathbf{y}), 1 \le j \le k\}\right| < \left|\{i : f_i(\mathbf{y}) < f_i(\mathbf{z}), 1 \le i \le k\}\right|. \quad (9)$$

The relation *favour* was modified in Sülflow et al. [29] by taking into account not only the number of objectives for which one solution is better than the other but also the difference in objective values between the two solutions.

Various ranking methods were compared with each other in [7], [25], [26]. For example, Corne and Knowles [7] reported that the best results were obtained from a simple average ranking method than more complicated ranking schemes. In the average ranking method, first a rank for each objective is assigned to each solution based on the ranking of its objective value among non-dominated solutions in the current population. Thus each solution has k ranks, each of which is based on one of the k objectives. Then the average rank is calculated for each solution as its overall rank. In Kukkonen and Lampinen [26], the average and minimum ranking methods were examined. Köppen and Yoshida [25] examined more complicated ranking methods based on ε-dominance and fuzzy Pareto dominance.

Experimental results with the average ranking method in [7] are shown in Fig. 11 and Fig. 12. The convergence of solutions toward the Pareto front was improved by the use of the average ranking method in Fig. 11. The diversity of solutions, however, was severely decreased in Fig. 12. As a result, the SumMax measure was also severely deteriorated. Similar results were reported for the minimum ranking method in Kukkonen and Lampinen [26]. They suggested the use of an additional scheme for diversity improvement together with a ranking method.



(a) Solution A. **(b) Solution B.** **(c) Solution C.**

Figure 10. Three non-dominated objective vectors.

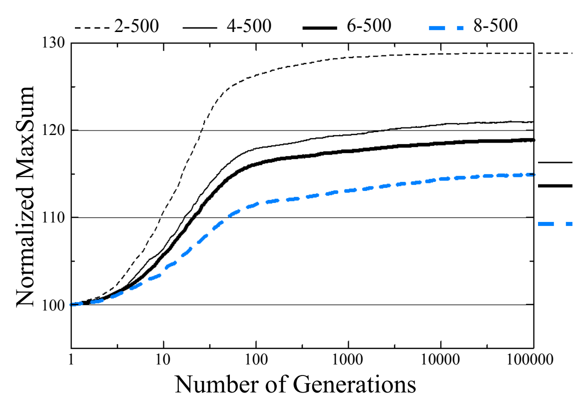


Figure 11. Convergence toward the center (Average rank).

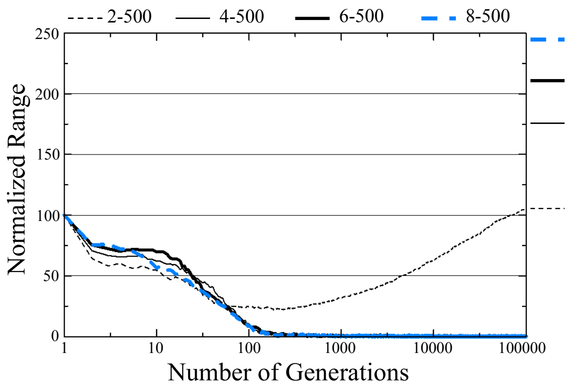
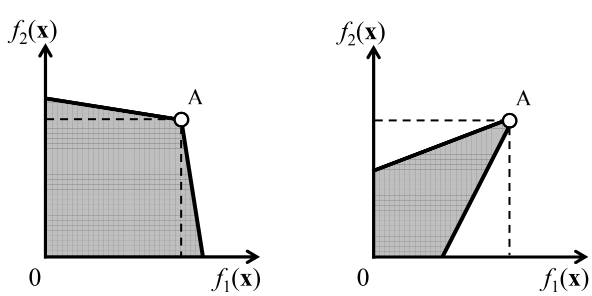


Figure 12. Diversity of solutions (Average rank).

4.3 Modification of Pareto Dominance

Sato et al. [28] proposed the modification of Pareto dominance relation to adjust the diversity-convergence balance in EMO algorithms. We illustrate their idea in Fig. 13 where the shaded region in each plot shows the dominated region by Solution A. The dominated region is widened in Fig. 13 (a). In this case, more solutions are dominated by other solutions. Thus the number of non-dominated solutions is decreased. On the other hand, the dominated region is narrowed in Fig. 13 (b). In this case, fewer solutions are dominated by other solutions. As a result, the number of non-dominated solutions is increased. Whereas the modification of Pareto dominance relation had already been proposed in the literature (e.g., [2], [17]), Sato et al. [28] is one of the first studies that clearly demonstrated the effectiveness of this idea on many-objective problems. They also proposed that the dominated region should be narrowed for two-objective problems as in Fig. 13 (b) while it should be widened for many-objective problems as in Fig. 13 (a) using a parameter S. As shown in Fig. 13, $S>0.5$ means narrowed dominated regions while $S<0.5$ means widened ones (for details, see Sato et al. [28]).



(a) Widened region (S=0.45). **(b) Narrowed region (S=0.65).**

Figure 13. Illustration of modified dominated regions.

In Fig. 14 and Fig. 15, we show experimental results with a widened dominated region in Fig. 13 (a). The value of the parameter S in Sato et al. [28] was specified as $S=0.45$ for all the four test problems for the sake of convenience in this paper. As we have already explained, the use of the widened dominated region decreases the number of non-dominated solutions in each generation. This leads to the increase in the selection pressure toward the Pareto front. As a result, the convergence property of NSGA-II is improved. Actually we obtained improved results in Fig. 14 with respect to the convergence of solutions toward the center region of the Pareto front. At the same time, the diversity of solutions was decreased in Fig. 15.

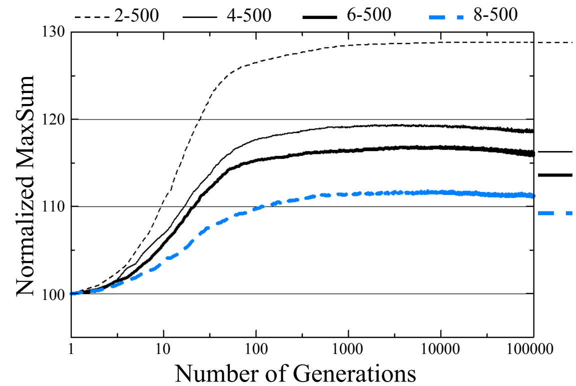


Figure 14. Convergence toward the center (Modified dominance).

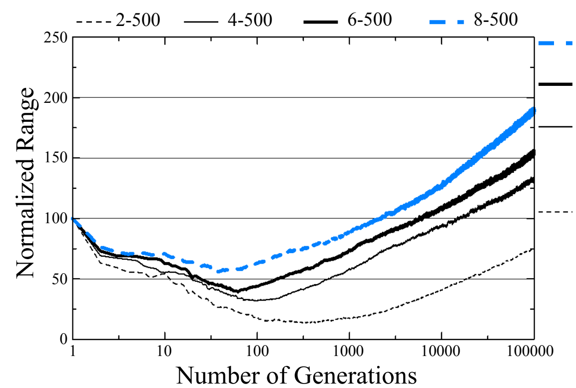


Figure 15. Diversity of solutions (Modified dominance).

4.4 Modification of Objective Functions

Almost the same effect as the modification of Pareto dominance in the previous subsection can be realized by linear transformation of objective functions [2], [19]. Let us consider the following simple linear transformation:

$$g_i(x) = f_i(x) + \beta \times \sum_{j=1}^{k} f_j(x)\,,\ i = 1, 2, ..., k, \tag{10}$$

where β is a prespecified constant ($\beta = 1$ in this paper).

Experimental results are shown in Fig. 16 where the convergence of solutions toward the center region of the Pareto front was clearly improved by the linear transformation for many-objective problems. The diversity of solutions, however, was decreased by the linear transformation as in the case of the widened dominated region in Fig. 15. (Experimental results are not shown due to the page limitation for the case of the linear transformation.) It should be noted that the diversity-convergence balance can be adjusted by the value of β in the case of the linear transformation (as S in the dominance modification in [28]).

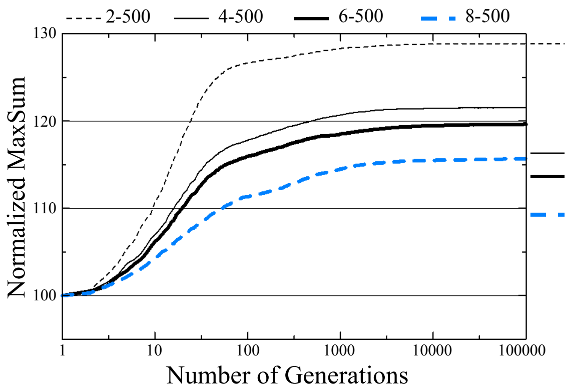


Figure 16. Convergence toward the center (Linear transformation of objective functions).

4.5 Hybridization with Local Search

We also examined a hybrid algorithm of NSGA-II with a weighted sum-based local search scheme of Jaszkiewicz [22]. Experimental results are shown in Figs. 17-19. Contrary to the above-mentioned results, local search significantly improved the diversity (Fig. 19) and slightly improved the convergence (Fig. 17) for many-objective problems. Better solutions around the edges of the Pareto fronts were also obtained by the hybrid algorithm (Fig. 18).

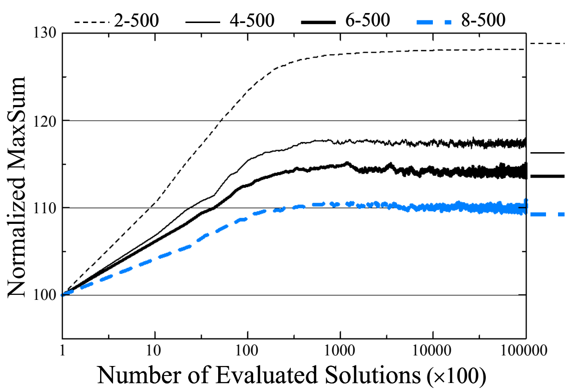


Figure 17. Convergence toward the center (Local search).

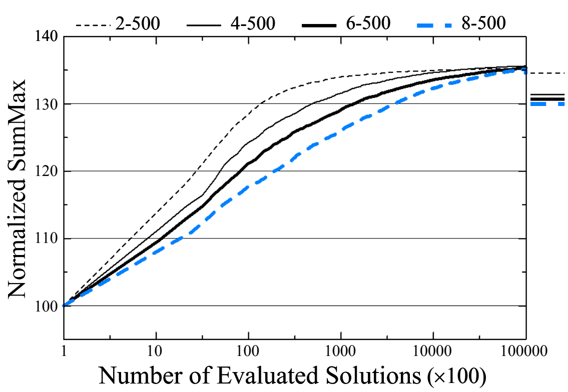


Figure 18. Convergence toward the edges (Local search).

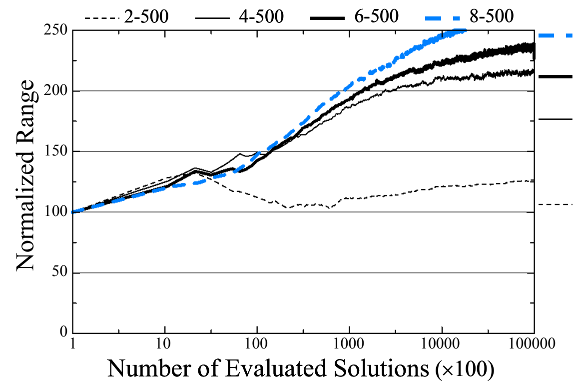


Figure 19. Diversity of solutions (Local search).

5. CONCLUDING REMARKS

We showed that scalability improvement approaches improved the convergence property of NSGA-II for knapsack problems with many objectives. Most of them, however, had a severe side-effect on the diversity of solutions. IBEAs (indicator-based evolutionary algorithms) have a potential ability to find good solution sets for many-objective problems with respect to both the convergence and the diversity of solutions [31]. The main difficulty in the application of IBEAs to many-objective problems is their heavy computation load. Another promising approach is the utilization of scalarizing functions [16], [18], [19], [32] including the hybridization with local search. As we have explained, local search has a potential ability to improve the performance of EMO algorithms for many-objective problems. One clear advantage of the use of scalarizing functions is their computational efficiency. Comparison among Pareto dominance-based EMO algorithms, IBEAs and scalarizing function-based algorithms for many-objective problems is left for future research.

6. REFERENCES

[1] Abraham, A., Jain, L. C., and Goldberg, R. (eds.) Evolutionary Multiobjective Optimization: Theoretical Advances and Applications. Springer, Berlin (2005).

[2] Branke, J., Kaußler, T., and Schmeck, H. Guidance in evolutionary multi-objective optimization. *Advances in Engineering Software* 32, 6 (2001) 499-507.

[3] Brockhoff, D., and Zitzler, E. Are all objectives necessary? On dimensionality reduction in evolutionary multiobjective optimization. *Lecture Notes in Computer Science 4193: Parallel Problem Solving from Nature - PPSN IX*, Springer, Berlin (2006) 533-542.

[4] Brockhoff, D., and Zitzler, E. Improving hypervolume-based multiobjective evolutionary algorithms by using objective reduction methods. *Proc. of 2007 IEEE Congress on Evolutionary Computation* (2007) 2086-2093.

[5] Coello, C. A. C., and Lamont, G. B. *Applications of Multi-Objective Evolutionary Algorithms*. World Scientific, Singapore (2004).

[6] Coello, C. A. C., van Veldhuizen, D. A., and Lamont, G. B. *Evolutionary Algorithms for Solving Multi-Objective Problems*. Kluwer, Boston (2002).

[7] Corne, D., and Knowles, J. Techniques for highly multiobjective optimization: Some non-dominated points are better than others. *Proc. of 2007 Genetic and Evolutionary Computation Conference* (2007) 773-780.

[8] Deb, K. *Multi-Objective Optimization Using Evolutionary Algorithms*. John Wiley & Sons, Chichester (2001).

[9] Deb, K., Pratap, A., Agarwal, S., and Meyarivan, T. A fast and elitist multiobjective genetic algorithm: NSGA-II. *IEEE Trans. on Evolutionary Computation* 6, 2 (2002) 182-197.

[10] Deb, K., and Saxena, D. K. On finding Pareto-optimal solutions through dimensionality reduction for certain large-dimensional multi-objective optimization problems. *KanGAL Report*, No. 2005011, Kanpur Genetic Algorithms Laboratory (KanGAL), Indian Institute of Technology Kanpur (2005).

[11] Deb, K., and Saxena, K. Searching for Pareto-optimal solutions through dimensionality reduction for certain large-dimensional multi-objective optimization problems. *Proc. of 2006 IEEE Congress on Evolutionary Computation* (2006) 3353-3360.

[12] Deb, K., and Sundar, J. Preference point based multi-objective optimization using evolutionary algorithms. *Proc. of 2006 Genetic and Evolutionary Computation Conference* (2006) 635-642.

[13] Drechsler, N., Drechsler, R., and Becker, B. Multi-objective optimization based on relation favour. *Lecture Notes in Computer Science 1993: Evolutionary Multi-Criterion Optimization - EMO 2001*, Springer, Berlin (2001) 154-166.

[14] Fleming, P. J., Purshouse, R. C., and Lygoe, R. J. Many-objective optimization: An engineering design perspective. *Lecture Notes in Computer Science 3410: Evolutionary Multi-Criterion Optimization - EMO 2005*, Springer, Berlin (2005) 14-32.

[15] Hughes, E. J. Evolutionary many-objective optimization: Many once or one many?. *Proc. of 2005 IEEE Congress on Evolutionary Computation* (2005) 222-227.

[16] Hughes, E. J. MSOPS-II: A general-purpose many-objective optimizer. *Proc. of 2007 IEEE Congress on Evolutionary Computation* (2007) 3944-3951.

[17] Ikeda, K., Kita, H., and Kobayashi, S. Failure of Pareto-based MOEAs: Does non-dominated really mean near to optimal?. *Proc. of 2001 IEEE Congress on Evolutionary Computation* (2001) 957-962.

[18] Ishibuchi, H., Doi, T., and Nojima, Y. Incorporation of scalarizing fitness functions into evolutionary multiobjective optimization algorithms. *Lecture Notes in Computer Science 4193: Parallel Problem Solving from Nature - PPSN IX*, Springer, Berlin (2006) 493-502.

[19] Ishibuchi, H., and Nojima, Y. Optimization of scalarizing functions through evolutionary multiobjective optimization. *Lecture Notes in Computer Science 4403: Evolutionary Multi-Criterion Optimization - EMO 2007*, Springer, Berlin (2007) 51-65.

[20] Ishibuchi, H., Tsukamoto, N., and Nojima, Y. Iterative approach to indicator-based multiobjective optimization. *Proc. of 2007 IEEE Congress on Evolutionary Computation* (2007) 3697-3704.

[21] Ishibuchi, H., Tsukamoto, N., and Nojima, Y. Evolutionary many-objective optimization: A short review. *Proc. of 2008 IEEE Congress on Evolutionary Computation* (in press).

[22] Jaszkiewicz, A. On the performance of multiple-objective genetic local search on the 0/1 knapsack problem - A comparative experiment. *IEEE Trans. on Evolutionary Computation* 6, 4 (2002) 402-412.

[23] Jaszkiewicz, A. On the computational efficiency of multiple objective metaheuristics: The knapsack problem case study. *European Journal of Operational Research* 158, 2 (2004) 418-433.

[24] Khara, V., Yao, X., and Deb, K. Performance scaling of multi-objective evolutionary algorithms. *Lecture Notes in Computer Science 2632: Evolutionary Multi-Criterion Optimization - EMO 2003*, Springer, Berlin (2004) 367-390.

[25] Köppen, M. and Yoshida, K. Substitute distance assignments in NSGA-II for handling many-objective optimization problems. *Lecture Notes in Computer Science 4403: Evolutionary Multi-Criterion Optimization - EMO 2007*, Springer, Berlin (2007) 727-741.

[26] Kukkonen, S., and Lampinen, J. Ranking-dominance and many-objective optimization. *Proc. of 2007 IEEE Congress on Evolutionary Computation* (2007) 3983-3990.

[27] Purshouse, R. C., and Fleming, P. J. Evolutionary many-objective optimization: An exploratory analysis. *Proc. of 2003 IEEE Congress on Evolutionary Computation* (2003) 2066-2073.

[28] Sato, H., Aguirre, H. E., and Tanaka, K. Controlling dominance area of solutions and its impact on the performance of MOEAs. *Lecture Notes in Computer Science 4403: Evolutionary Multi-Criterion Optimization - EMO 2007*, Springer, Berlin (2007) 5-20.

[29] Sülflow, A., Drechsler, N., and Drechsler, R. Robust multi-objective optimization in high dimensional spaces. *Lecture Notes in Computer Science 4403: Evolutionary Multi-Criterion Optimization - EMO 2007*, Springer, Berlin (2007) 715-726.

[30] Thiele, L., Miettinen, K., Korhonen, P. J., and Molina, J. A preference-based interactive evolutionary algorithm for multiobjective optimization. *Helsinki School of Economics*, *Working Paper* (2007) W-412.

[31] Wagner, T., Beume, N., and Naujoks, B. Pareto-, aggregation-, and indicator-based methods in many-objective optimization. *Lecture Notes in Computer Science 4403: Evolutionary Multi-Criterion Optimization - EMO 2007*, Springer, Berlin (2007) 742-756.

[32] Zhang, Q., and Li, H. MOEA/D: A multiobjective evolutionary algorithm based on decomposition. *IEEE Trans. on Evolutionary Computation* 11, 6 (2007) 712-731.

[33] Zitzler, E., and Thiele, L. Multiobjective evolutionary algorithms: A comparative case study and the strength Pareto approach. *IEEE Trans. on Evolutionary Computation* 3 , 4 (1999) 257-271.