Practical and Theoretical Investigation of a Collective work

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Abstract

We study the gathering objects by agents to investigate the process of collective work.

Summary We set a simple environment of the gathering work where agents do not communicate with each other, the place to gather the objects does not exist, and a well-known simple model of gathering objects is taken. At first, we describe the basic mechanism of this model precisely. Then we simulate the model under various environments and discovered that the way of putting objects and the range of agent's operation influence to gathering work. We investigate the basic mechanism of these phenomenon and find that this model relates to an well-known problem on probability theory, the "*classical bankrupt problem[1].*" We investigate this problem and obtain an approximate solution in the general case. We treat a following well-known model of gathering work.

Simple model of gathering work (1) If an agent does not have anything and bump into an object, pick it up. (2) If an agent have an object and the agent bump into another object, put it down. We discover that the model can re-define as a *the marbles exchange game.*

Marbles exchange game Let the m players have marbles each. Each of player selects player(s) randomly and gives marbles to the player(s). If a player loses all of own marbles then the player comes out from the game.

This game is a generalization of the *bankrupt problem*[1]. It is not easy to analyze the whole process of this game theoretically. Because, in this game, if a player who lost all marbles comes out from the game then the probability of marbles exchange changes at the same time.

On the mean arrival time that players go bankrupt We assume that each player gives his coin to one of the other players randomly in each time step. Let the number of players be N. Increment of one player's coins in a single time step takes the value from -1 to N-2, because he has to give a coin away and it is possible that nobody gives him a coin or that all the others give him coins.

The increment is equal to k-1 (k=0,...,N-1) if and only if k players choose and give him coins. The probability that the increment is k-1 is easy to calculate. The answer is $(N-1;k)p^k q,^{(N-1-k),}$ where *(A;B)* is the combinatorial number to choose B among A things, and $p = 1/(N-1), q = 1-p = (N-2)/(N-1)$. It is noticed that when the value of N becomes larger, the probabilities exponential decreases, except for k=0, 1.

We obtain the mean arrival time that one player goes bankrupt as;

$$D_z = m(z,0) = A + B(\frac{q}{p})^z + \frac{z}{(q-p).} \qquad (1)$$

When the number of players becomes large, the probabilities of the incremental exponentially, therefore this is an approximate solution in general case.

Both constant A and B can be obtained through boundary condition. Because $z = 0$ denotes that a player already goes to bankrupt, we define $D_0 = 0$. And when a player gathers almost all the objects, c then the rest of players will go bankrupt, thus we define c as a cut off value, $D_c = 0$. By using these boundary conditions, we obtain,

$$D_z = \frac{z}{(q-p)} - \frac{a}{(q-p)} \cdot \frac{1-(q/p)^z}{1-(q/p)^a.} \qquad (2)$$

References

[1] W. Feller, An introduction theory and its application, vol1, Jhon Wiley & Sons, Inc, 1956.