# Evolutionary Behaviors Emerged through Strategic Interactions in the Large

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## Abstract

The purpose of this paper is to provide a new approach for investigating the competitive interactions in the large. Another purpose of this paper is to study emergent strategic behaviors and to analyze the effects of bounded rationality and the mimicry strategy in the competitive situations. We show how the society gropes for its way towards equilibrium in an imperfect world where agents are sensible but not perfectly rational. They have limited information, and there is no common knowledge among them. This paper is also about social learning and shows how the society as a whole learns even when the individuals composing it do not. Specifically, it is about the evolution of social norms. We especially examine how conventions evolve in a society that begins in an amorphous state where there is no established custom, and individuals rely on hearsay to determine what to do. With simulations, we provide specific conditions as to which conventions are most likely to emerge.

## 1 INTRODUCTION

A convention is a pattern of behavior that is customary, expected, and self-enforcing. Everyone conforms, everyone expects others to conform, and everyone wants to conform given that everyone else conforms. Familiar examples include driving on the right when others drive on the right, going to lunch at noon if others go at noon, and so forth. For each role in such asymmetric interactions there is a customary and expected behavior, and everyone prefers to follow the behavior expected of him provided that others follow the behavior expected of them. Under these circumstances we say that people follow a convention.

A convention is an equilibrium that everyone expects, but how do mutual expectations become established when there is more than one equilibrium? One explanation is that some equilibria are a priori more reasonable than others. A deductive theory of this type has been proposed by Harsanyi and Selten [3]. A second explanation, proposed by Schelling [7], is that agents focus their attention on one equilibrium because it is more prominent or conspicuous than the others. A third explanation is also possible such that, over time, expectations converge on one equilibrium through positive feedback effects. Eventually, one equilibrium becomes entrenched as the conventional one, not because it is inherently prominent or focal, but because the dynamics of the process happen to select it.

Consider, for instance, an N-person game that is played repeatedly, but by different agents. In each period, N players are drawn at random from a large finite population. Each player chooses an optimal strategy based on a sample of information about what others players have done in the past. The sampling defines a stochastic process that, for a large class of games that includes coordination games and common interest games, converges almost surely to a pure strategy Nash equilibrium. Such an equilibrium can be interpreted as the conventional way of playing the game. If, in addition, the players sometimes experiment or make mistakes, then society occasionally switches from one convention to another. As the likelihood of mistakes goes to zero, only some conventions (equilibria) have positive probability in the limit. These are known as stochastically stable equilibria. They are essentially the same as the risk dominant equilibria [3]. This concept was first defined for general evolutionary processes by Foster and Smith [9]. Subsequently it was applied to a discrete model of equilibrium selection in a pioneering paper by Kandori and his collogues [4]. They consider an evolutionary learning process defined on symmetric 2 x 2 games. In each period every player plays every other. Successful strategies are adopted with higher probability than unsuccessful ones. This evolutionary explanation for the origin of conventions has been suggested in a variety of papers [2][9][10], but the precise dynamics of the process by which expectations and behaviors evolve has not been clearly spelled out.

The purpose of this paper is to explain the problem of bounded rationality and evolution. We formalize these ideas in a model with a finite population of agents in which agents are repeatedly matched within a period to play a stage game. We only impose a weak monotonicity condition reflecting the inertia and myopia hypotheses on the dynamics, which describe the intertemporal changes in the number of agents playing each strategy. The hypotheses we employ here reflect limited ability (on the agent's part) to receive, decide, and act upon information they get in the course of interactions. Our specification of dynamics draws heavily on the biological literature. In that literature, agents are viewed as being genetically coded with a strategy and selection pressure favors animals, which are fitter (i.e., whose strategy yields a higher reproductive fitness or payoff against the population).

There are also growing literatures on the bounded rationality and the evolutional approach, the hypotheses employed in these researches reflect limited ability of each player or agent to receive, decide, and act upon information they get in the course of interactions. Specification of dynamics draws heavily on the biological literature, and agents are viewed as being genetically coded with a strategy and selection pressure favors animals, which are fitter (i.e., whose strategy yields a higher reproductive fitness or payoff against the population). Our model can be interpreted in like manner, however, we intend to combine the evolutional approach and the concept of bounded rationality. We consider the situation where a group of agents is repeatedly matched to play a game. Each agent only interacts with his neighbors, and when agents react, they react myopically (the myopia hypothesis). The following three hypotheses form the basis of our analysis [5][6].

(1) Each agent only interacts with his neighbors.

(2) When agents react, they react myopically (the myopia hypothesis).

The second hypotheses are based on the assumption that agents are completely naive and do not perform optimization calculations. Rather, agents sometimes observe the current performance of other agents, and simply mimic the most successful strategy. Note that in the first interpretation, agents are able to calculate best replies and learn the strategy distribution of play in society. In the second interpretation, players are less sophisticated in that they do not know how to calculate best replies and are using other agent's successful strategies as guides for their own choices.

## 2 STRATEGIC INTERACTION IN THE LARGE

Fig.1 shows the networks of mutual and strategic interactions in the large. In the figure, each node represents an agent, and they interact with their neighbors. These interactions can be observed everywhere in the network. We describe each mutual interaction between any two agents, which is represented by the link in the figure and it can be modeled as the strategic decision problem of each agent. That is, each agent has several strategies for the interaction. The payoffs for all the possible combinations of strategic decisions can be defined before interaction. Those strategic interactions can be repeated infinitely and none of the agents know the end of game. We classify the types of the strategic interactions into several classes based on those payoff structures, and they are given the special name in the game theory.

We observe the phenomena such as how each agent's action combines with the actions of others to produce the whole behavior and some unanticipated results. However, there are many parameters to be considered such as payoff function, noise, population structure, localization, the shadow of the future, the number of agents and so on. Among these parameters, we examine two parameters: payoff function and localization. The payoff function is generally fixed in the game but there exist many criteria of payoff function in real world, especially in economic and social systems. We especially examine the defect of payoff function and the localization. These factors are reported to affect emergence of cooperation. This paper also uses the mimicry learning strategy, which has advantage of reflection of dynamic environment.

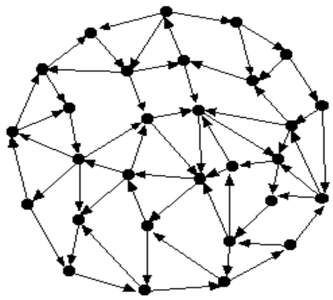

Fig 1: Competitive and local interactions in the large

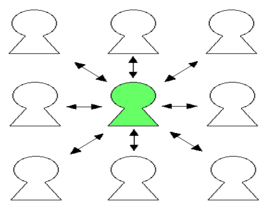

Fig 2: Localized interaction

## 3 LOCAL INTERACTION WITH MIMICRY

Each agent interacts with the agents on all eight adjacent squares and imitates the strategy of any better performing one. To make this process amenable to analysis, it must be formalized. For illustrative purposes, consider a simple structure of territories in which the entire territory is divided up so that each agent has eight neighbors as shown in Fig 2. In each generation, each agent attains a success score measured by its average performance with its eight neighbors. Then if an agent has one or more neighbors who are more successful, the agent converts to the strategy of the most successful of them. Or picks randomly among the best in case of a tie among the most successful neighbors.

Nations, businesses, tribes, and birds are examples of agents which often operate mainly within certain territories [1]. Territories can be thought of in two completely different ways. One way is in terms of geography and physical space. They interact much more with their neighbors than with those who are far away. Hence their success depends in large part on how well they do in their interactions with their neighbors. But neighbors can serve another function as well. A neighbor can provide a role model. If the neighbor is doing well, the behavior of the neighbor can be imitated. In this way successful strategies can spread throughout a population, from neighbor to neighbor. Colonization provides another mechanism in addition to imitation by which successful strategies can spread from place to place. Colonization would occur if the location of a less successful strategy were taken over by an offspring of a more successful neighbor. But whether strategies spread by imitation or colonization, the idea is the same: neighbors interact and the most successful strategy spreads to bordering locations. The individuals remain fixed in their locations, but their strategies can spread.

Territorial social structures have many interesting properties. One of them is that it is at least as easy for a strategy to protect itself from a takeover by a new strategy in a territorial structure as it is in a non-territorial structure. To see how this works, the definition of stability must be extended to include territorial systems. A strategy can invade another if it can get a higher score than the population average in that environment. In other words, a single individual using a new strategy can invade a population of natives if the newcomer does better with a native than a native does with another native. If no strategy can invade the population of natives, then the native strategy is said to be collectively stable.

To extend these concepts to territorial systems, suppose that a single individual using a defective strategy is introduced into one of the neighborhoods of a population where everyone else is using a cooperative strategy. One can say that the defective strategy territorially invades the cooperative strategy if every location in the territory will eventually convert to the new strategy. Then one can say that cooperative strategy is territorially stable if no strategy can territorially invade it. In such a case, the dynamics of the invasion process can sometimes be extremely intricate and fascinating to look at.

# 4 SIMULATION RESULTS (1): COORDINATION GAMES

In this simulation, we consider the case in which each local interaction is modeled as coordination games as shown in Table 1 and Table 2. Number of agents are 2500 (N=2500). With this game, we are especially interested in the effect of changing the parameter b, micromotives for defect for the collective behaviors in the whole society. We are mainly concerned with that how $S_1$ survive with the invasion of the $S_2$ as the changing the parameter b. Therefore, we start with only one $S_2$ in the society. We arranged agents for an area of 50*50 (N=2500 agents) with no a gap and observed how agents interact. Four corners and end of an area connect it with an opposite side. All agents interact eight agents in neighborhood.

## 4.1 COORDINATION GAMES (1)

We describe the mutual interaction by coordination game as shown in Table 1.

| | $S_1$ | $S_2$ |
|---|---|---|
| $S_1$ | b, b | 0, 0 |
| $S_2$ | 0, 0 | 1, 1 |

Table 1: Payoff matrix of the coordination game (1)

**CASE 1: (b=0.624) 75% of randomly chosen agents adopt $S_1$, and the rest (25%) adopt $S_2$ at the beginning.**

At the beginning, the rates of agents who adopt $S_1$ are 75%, and that of agents who adopt $S_2$ are 25%. Fig 3(a) shows the rates of both strategies over generation. This figure shows that the rates of $S_1$ increases at the beginning and then it gradually decreases and reaches to 0 finally. This implies the whole society is occupied by the superior strategy $S_2$. At the 15th generation, all agent in the society adopt $S_2$ wholly. Fig 3(b) represents the average payoff of each agent at every generation. As the number of agent who adopt $S_2$ increases, the payoff of each agent increases and finally the payoff becomes to be 1 at the 15th generation.

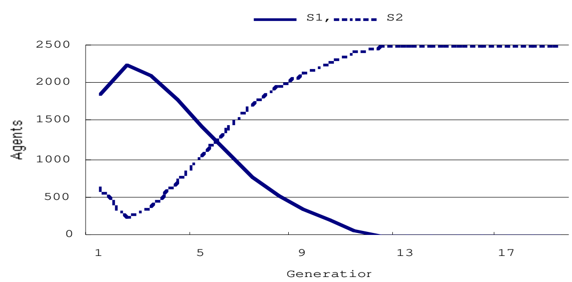


Fig 3(a): The proportion of $S_1$ and $S_2$ over generation

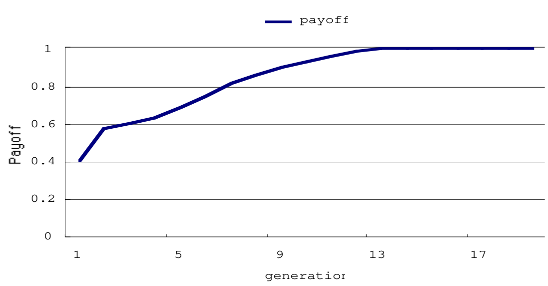


Fig 3(b): The average payoff of each agents

**CASE 2: (b=0.626) 75% of randomly chosen agents adopt $S_1$, and the rest (25%) adopt $S_2$ at the beginning.**

Fig 4(a) shows the rates of both strategies over generation and the majority of the society adopt $S_1$. As shown in Fig 4(b), represents the average payoff, Fig 4(c) represents distribution map of the strategies $S_1$ and $S_2$ in the society. A white square represents the strategy $S_1$ and black square represents the strategy $S_2$. This implies, if the parameter b increases slightly from b=0.624 to 0.626, then the society is almost occupied by the inferior strategy $S_1$.

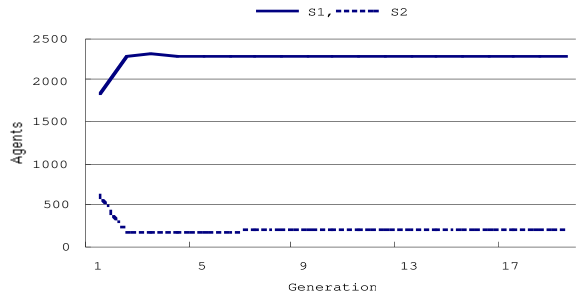


Fig 4(a): The proportion of $S_1$ and $S_2$ over generation

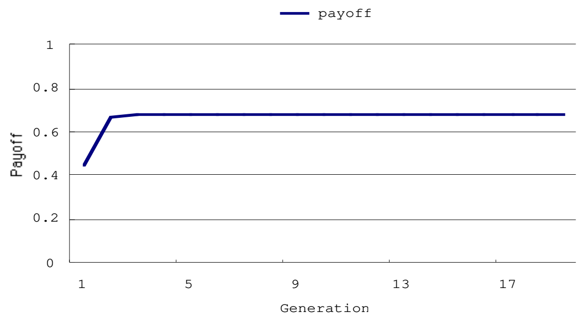


Fig 4(b): The average payoff of each agents

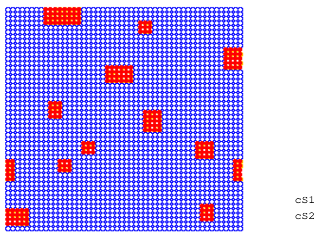


Fig 4(c): The distribution of $S_1$ and $S_2$

In the coordination game in Table 1, if b<1, $S_2$ is superior to $S_1$. To make the superior strategy becomes $S_2$ invade the society, neighboring agents should also adopts $S_2$. When a small group composed of six who adopt $S_2$ and if b<0.625, this group can survive and also invades the whole society. However, if b<0.625, this group can survive, but can't invade.

## 4.2 COORDINATION GAMES (2)

We describe the mutual interaction by another coordination game as shown in Table 2.

| | $S_1$ | $S_2$ |
|---|---|---|
| $S_1$ | 1 \ 1 | 0 \ b |
| $S_2$ | b \ 0 | 1+b \ 1+b |

Table 2: Payoff matrix of the coordination game (2)

(1) b > 1

In this the strategy, $S_2$ is always more superior to $S_1$. We also consider the case such that a only one agent adopts $S_2$ at the beginning. One agent who adopts $S_2$ invades neighbors who adopt $S_1$, and then whole society becomes to be occupied by $S_2$. As agent who adopt $S_2$ exist more and more, invasion speed becomes fast. Even though neighbors adopt $S_1$ or $S_2$, agent who adopt $S_2$ get more payoff than that of adopting $S_1$.

(2) b = 1

When only one agent who adopts $S_2$ exist, he does only survive, can't invade the society. But if an agent who adopts $S_2$ exists even one in the neighborhood, the invasion of one agent who adopts $S_2$ is possible. As a result, whole society becomes to be occupied by $S_2$. We represents this mechanism in shown Fig 5. Two agents who adopt $S_2$ in center getting payoff 8b+1, these payoff is higher than neighboring payoff 6 or 7. After all, when only one agent who adopts $S_2$ can't invade a society, but if two agents who adopt $S_2$ cooperate, they can invade the whole society.

| | | | | | |
|---|---|---|---|---|---|
| 8 | 8 | 8 | 8 | 8 | 8 |
| 8 | 7 | 6 | 6 | 7 | 8 |
| 8 | 7 | 8b+1 | 8b+1 | 7 | 8 |
| 8 | 7 | 6 | 6 | 7 | 8 |
| 8 | 8 | 8 | 8 | 8 | 8 |

Fig 5: The mechanism of invation in the coordination games (2)

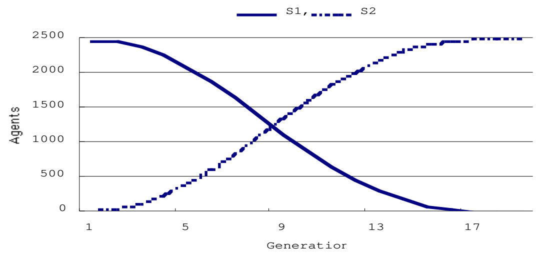


Fig 6(a): The proportion of $S_1$ and $S_2$ over generation

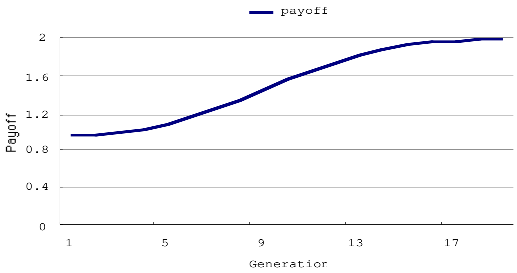


Fig 6(b): The average payoff of each agents

Fig 6(a) shows that only one small group composed of two agents who adopt $S_2$ invades the whole society very fast. At 18th generation, the society becomes to be occupied by $S_2$ wholly. At first, almost agents adopt $S_1$, so average payoff is about 1. But as generation proceeds, agent who adopts $S_2$ increase gradually, finally, average payoff changes to be 2 at 18th generation.

(3) b < 1

If an agent who adopt $S_2$ exist only one and there isn't only one agent who adopt $S_2$ in his neighborhood in the society, this agent change to $S_1$ rapidly, and the society becomes to be occupied by $S_1$ rapidly. But if agents who adopt $S_2$ make a small group composed of some, they can invade a whole society.

Concretely, if b>0.375 and agent who adopt $S_2$ make a group composed of six, this group invade the society wholly. As a result, the society becomes to be occupied by $S_2$.

**CASE 3: (b=0.376) 80% of randomly chosen agents adopt $S_1$, and the rest (20%) adopt $S_2$ at the beginning.**

At the beginning, the rates of agents who adopt $S_2$ are 20%. As shown in $S_2$-curve at Fig 7(a), the rates of $S_2$ are 20% at 1st generation, and decreasing suddenly at 3th generation and then increasing after next all generation. This phenomenon represents follows: in this society, because of existing one at least that agent who adopt $S_2$ makes a group composed of six, this group can survive and invade this society. On the other hand, agent who adopt $S_2$ makes a group composed of less than six can't survive and they changes to $S_1$ rapidly.

As shown in Fig 7(b), average payoff is about 0.75 at 1st generation. But as generation proceeds, agent who adopt $S_2$ increasing gradually, finally average payoff becomes to be 1.4 at 12th generation.

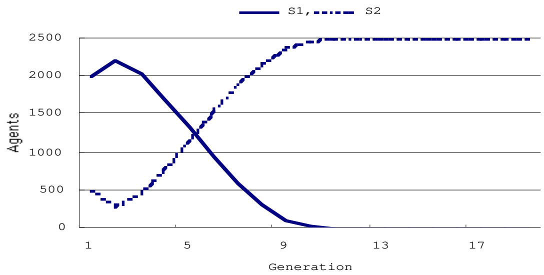


Fig 7(a): The proportion of $S_1$ and $S_2$ over generation

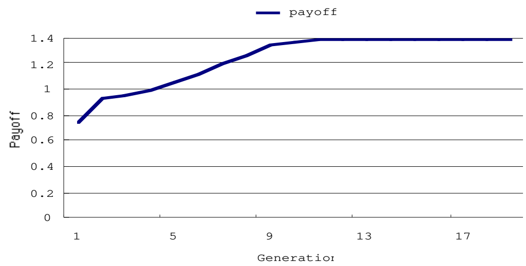


Fig 7(b): The average payoff of each agents

**CASE 4: (b=0.374) 80% of randomly chosen agents adopt $S_1$, and the rest (20%) adopt $S_2$ at the beginning.**

At the beginning, there are seven groups exist composed of more than six that agent who adopt $S_2$ in this society. However, because of b<0.375, these group make colonies and only can survive (as shown in Fig 8(c)). Fig 8(b) represents average payoff. If this society becomes to be $S_2$ wholly, average payoff change to value of 1.4, however these groups who adopt $S_2$ could only survive. Therefore, average payoff change to value of 1.

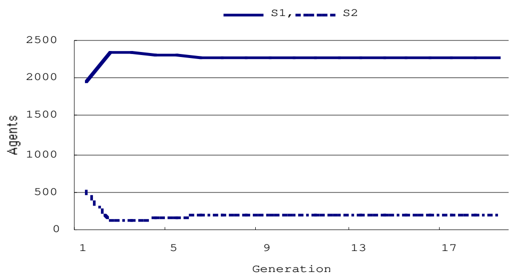


Fig 8(a): The proportion of $S_1$ and $S_2$ over generation

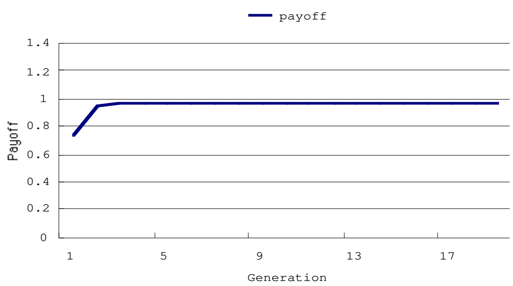


Fig 8(b): The average payoff of each agents

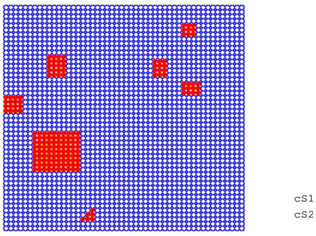


Fig 8(c): The distribution of $S_1$ and $S_2$

## 5 SIMULATION RESULTS (2): CHICKEN GAMES

In this simulation, we consider the case that each mutual interaction is modeled as so called chicken game as shown in Table 3. In chicken game, the most suitable strategy that agents adopt is alternately adopt $S_1$ and $S_2$ strategies. For instance, when two drivers meet at an intersection, there is no rational basis for deciding who should yield unless there is an established custom (or law) for dealing with the matter. Moreover, there is not too much room for learning here since one mistake can cut learning short. Even in less precarious circumstances, people do not tend to solve such problems by relying on their own limited experience. They learn through others' mistakes, and come to know from vicarious experience what the accepted pattern of behavior is. After, we call this strategy a shift strategy. Does a society change to adopt a shift strategy or not? We think this situation that only one agent in the center $S_2$, and remaining agents $S_1$ at the beginning.

| | $S_1$ | $S_2$ |
|---|---|---|
| $S_1$ | 0 \ 0 | 1 \ b |
| $S_2$ | b \ 1 | 0 \ 0 |

Table 3: Payoff matrix of the chicken game

**CASE 5: (b=0.7) Only one agent in the center $S_2$, and remaining agents $S_1$ at the beginning.**

As shown in Fig 9(a), the society accomplishes a dynamic inflection even 30 generation. A point that should be emphasized is that the majority of agents who adopt $S_1$ in the society changes to shift strategy. The society reached an equilibrium state at 60th generation, then at this time, the rates of agents who adopt shift strategy are 60%, the rates of agents who persists $S_1$ are 25%, and the rates of agents who persists $S_2$ are 15% in the society. Fig 9(b) represents average payoff. At an equilibrium state, average payoff vibrate around the value of 0.25. If all agent adopt shift strategy, agents can get more payoff. So payoff average should be 0.85(=(1+b)/2), but agents gets small payoff the value of about 0.25. This is the reason why agents who persists $S_1$ or $S_2$ exist in the society and these interactions of same strategies (($S_1$, $S_1$) or ($S_2$, $S_2$)) gets payoff the value of 0, so agents who adopt shift strategy only gets payoffs in the society.

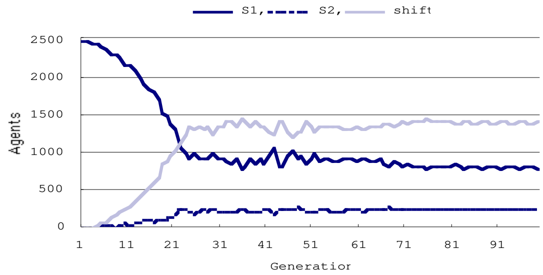


Fig 9(a): The proportion of $S_1$ and $S_2$ over generation

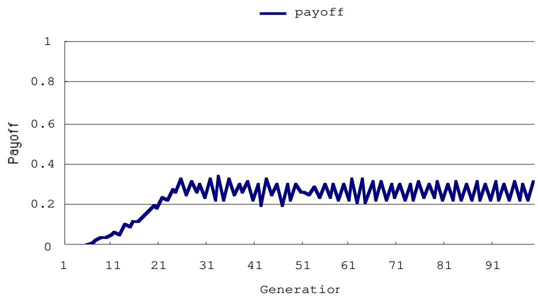


Fig 9(b): The average payoff of each agents

**CASE 6: (b=1.0) Only one agent in the center $S_2$, and remaining agents $S_1$ at the beginning.**

At this case, even though, agent who adopts $S_1$ or $S_2$, a payoff that agent get is equal. As a result of this simulation, the rates of agents who persist $S_1$ or $S_2$ are 40% in the society (the rates of agents who persisted $S_1$ are 20% and the rates of agents who persisted for $S_2$ are 20%). And the rates of agents who adopt shift strategy are 60% in the society. This implies that differences of a beginning state of the rates of agents who adopt $S_1$ or $S_2$ don't give an influence to equilibrium state, it is value of b to have an influence on rather.

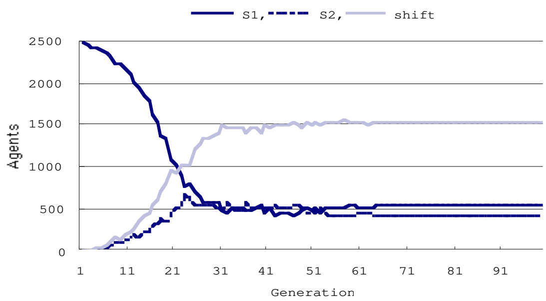


Fig 10(a): The proportion of $S_1$ and $S_2$ over generation

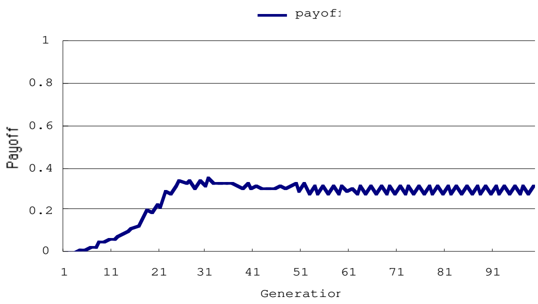


Fig 10(b): The average payoff of each agents

**CASE 7: (b=1.2) Only one agent in the center $S_2$, and remaining agents $S_1$ at the beginning.**

As a generation proceeds, agents who adopt $S_2$ increases and agents who adopt $S_1$ decreases more and more in the society. The society reached an equilibrium state at 24th generation, then at this time, the rates of agents who adopt $S_1$ are 5%, the rates of agents who adopt $S_2$ are 85% and the rates of agents who adopt shift strategy are only 10%. In this society, we can say that a social dilemma occurred.

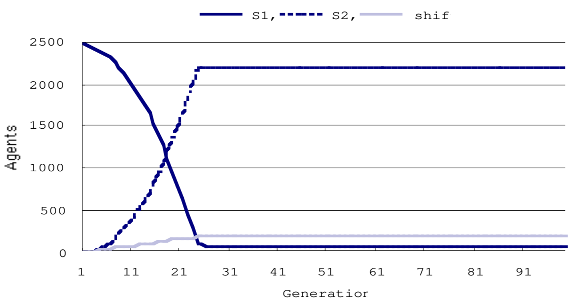


Fig 11(a): The proportion of $S_1$ and $S_2$ over generation

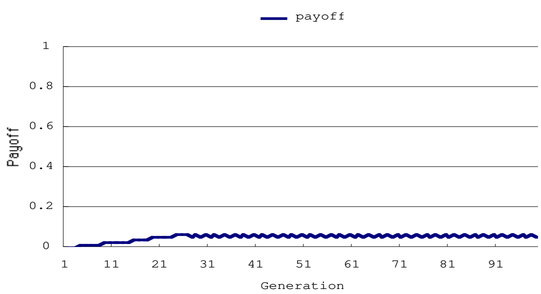


Fig 11(b): The average payoff of each agents

## 6 THE MECHANISM OF EVOLUTION OF SOCIAL NORMS

By changing the value of b in the chicken game in Table 3, we investigate how a shift strategy emergent in the whole society. Fig 12, Fig 13 and Fig14 represent how a society evolve as a generation proceeds. Each square represents the payoffs of each agent. With the color of square white, it also represents the agent who adopt $S_1$ and that of gray represents agent who adopt $S_2$.

(1) $0 < b < 0.125$

One agent who adopt $S_2$ in center and the another agents who adopt $S_1$. Fig 12 shows the condition of an agent who adopt $S_2$ exist at t-th generation. In this case, an agent who adopt $S_2$ in center changes to $S_1$ at next t+1-th generation. That is, when a value of b is very small, even though there are some agents who adopt $S_2$ exist, they change to $S_1$ and then all agents change to $S_1$ in the society. This means that when the value of b is very small, the society can't emerge a shift strategy.

| | | | | |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 0 |
| 0 | 1 | 8b | 1 | 0 |
| 0 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 |

Fig 12: The payoff of each agent at the t-th generation

(2) $0.125 < b < 0.4$

In this case, center agent who adopt $S_2$ can survive, and neighbors changes to $S_2$. As a result, state of the society at t-th generation shift to t+1-th generation as shown in Fig 13. Then state of the society at t+1-th generation shift to t+2-th generation as shown in Fig 13. This state at t+2-th generation is same as that of t-th generation. The society will repeat this phenomenon. As a result, neighboring agents of a central agent adopt $S_1$ and $S_2$ alternately as a generation proceeds. So this society emerged a shift strategy, if it says. This means that when $0.125<b<0.4$, if there are some agents who adopt $S_2$ exist, neighboring agents of them change to shift strategy.

(3) $0.4 < b < 0.9$

An inflection to the t+1-th generation in the same as (2). Aspect of an inflection of the society as shown in Fig 14. In this case, center agent who adopt $S_2$ gives an influence neighbor agent, and center agent makes neighboring agents change $S_2$. And then, at t+2-th generation, agents who adopt $S_2$ increases for five. These five of agents who adopt $S_2$ act in the same way of center agent who adopt $S_2$ at t-th generation. Thus, agents who adopt $S_2$ spreads in the society wholly. Further more, neighboring agents in the place of agent who adopt $S_2$ takes a shift strategy. As a result, almost agents will adopt shift strategy in this society. This means that when $0.4<b<0.9$, agents don't persist $S_1$ or $S_2$ strategy and many agent will adopt shift strategy.

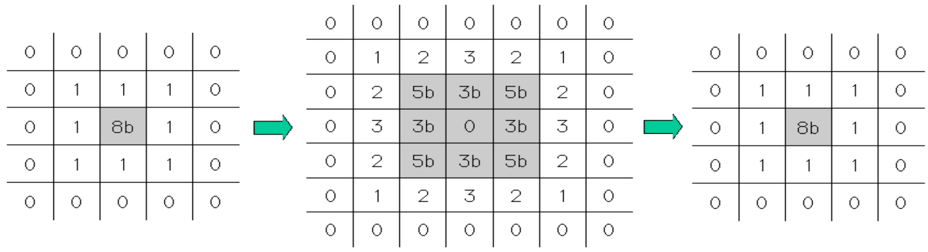


Fig 13: The t-th generation The t+1-th generation The t+2-th Generation

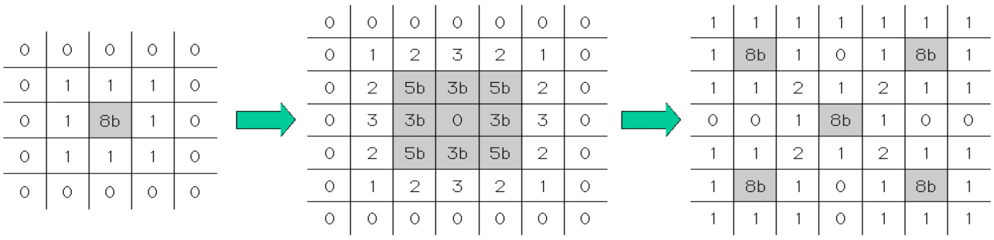


Fig 14: The t-th generation The t+1-th generation The t+2-th Generation

## 7 CONCLUSION

We analyzed the competitive interactions in a finite population of agents in which agents are repeatedly matched within a period to play a stage game. We only imposed a weak monotonicity condition reflecting the inertia and myopia hypotheses on the dynamics, which describe the intertemporal changes in the number of agents playing each strategy. The hypotheses we employed here reflect limited ability (on the agent's part) to receive, decide, and act upon information they get in the course of interactions. Our specification of dynamics draws heavily on the biological literature. And we analyzed also about social learning and shown how the society as a whole learns even when the individuals composing it do not. Specifically, it is about the evolution of social norms. We especially examined how conventions evolve in a society that begins in an amorphous state where there is no established custom, and individuals rely on hearsay to determine what to do. With simulations, we provided specific conditions as to which conventions are most likely to emerge.

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