Genetic Algorithms as Random Sytems with Complete Connections

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Abstract

There is a long history for the Markov chain analysis of *canonical* GAs and a lot of empirical work has been done on their *adaptive* variants; although, 'stochastic modeling' and 'adaptive GAs' seemed to be two non-overlapping fields of research. This is due to the natural limits of the Markovian dependence; making a step ahead, we introduces *random systems with complete connections* – another class of stochastic processes, powerful enough for providing a 'classification of states' and a globally convergence condition for a binary p_m*-adaptive* GA, namely: ***the algorithm is convergent iff each non-optimal state is innesential***.

1 RANDOM SYSTEMS WITH COMPLETE CONNECTIONS

A *random system with complete connections* - RSCC, (Iosifescu, 1990) is a quadruple $\{(W, \mathbf{W}), (X, \mathbf{X}), u, P\}$, where
- $(W, \mathbf{W})$ and $(X, \mathbf{X})$ are arbitrary measurable spaces;
- $u : W \times X \rightarrow W$ is a $(\mathbf{W} \otimes \mathbf{X}, \mathbf{W})$ measurable map;
- P is a *transition probability function* from $(W, \mathbf{W})$ to $(X, \mathbf{X})$ – analogous to the transition probability matrix in finite Markov chains.

The RSCC can be used in modeling a p_m*-adaptive* GA if considering the following interpretation:

- X - the set of all GA populations;
- W – the set of mutation (rate) vectors of length ℓ - the chromosome length;
- u – a function determining the changes in the mutation vector, during the evolution;
- P – a *t.p.f.* modeling transition between populations.

2 GA MODELING

The central element of our model is u, as it involves the mutation's adaptation rule for the GA to follow in case of stagnation. Actually, u provides a passing rule from the current to the next mutation vector, involving also the current population: the mutation rate on position k will decrease if the diversity (measured by the average number of 'ones' on that position in the current population) is 'satisfactory', and will increase otherwise.

Following (Davis & Principe, 1993), we successively define the probabilistic model for the *one/two/three-operators* GA, respectively. Following the Markovian models already introduced for canonical GA - (Rudolph, 1997) and (Agapie, 1998) - we make the connection between the algorithm's convergence and the classification of states (elements of X, in this case).

As long as the t.p.f. P depends strongly on the selection operator (which depends respectively on the objective function), the only way for a theoretical (but non-statistical) analysis of GA convergence on *black box* problems would be a state identification of the following type:

Optimal populations ~ states of type A

Non-optimal populations ~ states of type B

with A and B defining a partition of the total set X.

In Markov modeling, A was identified to the class of *essential (recurrent)* states, while B corresponds to the *inessential (transient)* ones.

As the RSCC theory allows (under some specific conditions) an essential-inessential classification of states we can prove the following:

<u>Theorem.</u> The p_m-adaptive GA modeled by an RSCC is globally convergent *iff* each non-optimal population is inessential.

References

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