Improving the Scalability of Dynastically Optimal Forma Recombination by Tuning the Granularity of the Representation

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1 DYNASTICALLY OPTIMAL RECOMBINATION (DOR)

Let x and y be two individuals from a solution space $\mathcal{S}$. A recombination operator X can be defined as a function $X : \mathcal{S} \times \mathcal{S} \times \mathcal{S} \to [0, 1]$, where $X(x, y, z)$ is the probability of generating z when recombining x and y using X. Clearly,

$$\forall x \in \mathcal{S}, \forall y \in \mathcal{S} \ : \ \sum_{z \in \mathcal{S}} X(x, y, z) = 1 \tag{1}$$

The *Dynastic Potential* of x and y is defined as

$$\Gamma_{\{x,y\}} = \{z \mid \forall \xi \in \Xi : z \in \xi \Rightarrow (x \in \xi) \vee (y \in \xi)\} \tag{2}$$

where Ξ is the set of basic formae.

A recombination operator is said to be transmitting iff $\{z | X(x, y, z) > 0\} \subseteq \Gamma_{\{x,y\}}$. Now, let $\phi : \mathcal{S} \to \mathcal{R}^+$ be the target function (minimization is assumed). DOR is a transmitting recombination operator for which:

$$\mathrm{DOR}(x, y, z) > 0 \Rightarrow \forall w \in \Gamma_{\{x,y\}} : \phi(w) \geq \phi(z) \tag{3}$$

Thus, no other solution in the dynastic potential is better than any solution generated by DOR. According to this definition, the use of DOR implies performing an exhaustive search in a small subset of the solution space. Such an exhaustive search can be efficiently done by means of a subordinate A*-like mechanism.

DOR uses optimistic estimations $\hat{\phi}(\Psi)$ of the fitness of partially specified solutions Ψ (i.e., $\forall z \in \Psi : \hat{\phi}(\Psi) \leq \phi(z)$) for directing the search to promising regions. These solutions are incrementally constructed using the formae to which any of the parents belong. More precisely, let $\Psi_0^1 = \mathcal{S}$. Subsequently,

$$\Psi_{i+1}^{2j} = \Psi_i^j \cap \Sigma(\Psi_i^j, x), \text{ and} \tag{4}$$

$$\Psi_{i+1}^{2j+1} = \Psi_i^j \cap \Sigma(\Psi_i^j, y) \tag{5}$$

are considered. Whenever $\bar{\phi} < \hat{\phi}(\Psi)$ (where $\bar{\phi}$ is the fitness of the best-so-far solution generated during this process), the macro-forma Ψ is closed (i.e., discarded), hence pruning dynastically suboptimal solutions. Otherwise, the process is repeated for open macro-formae. Each $\Sigma(\Psi, w)$ is termed a *construction unit*. These construction units are defined as

$$\Sigma(\Psi, w) = \cap_{1 \leq i \leq g} \xi_{j_i}, w \in \xi_{j_i}, \tag{6}$$

and their structure depends on the problem considered. The parameter g is called the *granularity* of the representation. It can be seen that the size of the set of solutions in which DOR searches is $O(2^{n/g})$, where n is the dimensionality of the representation.

The minimal value of g for a given representation is termed the *basic* granularity (e.g., $g = 1$ when the representation is orthogonal). If the computational complexity of DOR is too high for this basic granularity, g can be increased so as to make DOR combine larger portions of the ancestors.

Experimental results on the Brachystochrone design problem and the Rosenbrock function show a nearly-linear relation between the granularity of the representation and the reduction of the computational effort. Furthermore, it is shown that intermediate granularity values are better since low g is computationally prohibitive and high g reduces the chances for information interchange during recombination. This is verified on orthogonal and non-orthogonal separable representations exhibitting epistasis [Cotta et al., 1999].

References

[Cotta et al., 1999] Cotta, C., Alba, E., and Troya, J. M. (1999). Improving the scalability of dynastically optimal recombination by tuning representation granularity. Technical Report LCC-ITI 99/06, Dept. Lenguajes y Ciencias de la Computación, University of Málaga.