Improving Parallel Ordering of Sparse Matrices using Genetic Algorithms

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1 PROBLEM DEFINITION

Consider a system of linear equations

$$Ax = b,$$

where A is an $n \times n$ sparse symmetric and positive definite matrix, b is a given vector, and x is the unknown vector to be solved. In the direct solution of such linear systems, A is usually first decomposed into LL^T, which is known as Cholesky factorization, where L is lower triangular (Heath 1991).

Researchers have found that in parallel computation of the Cholesky factor of a sparse matrix, pivots of no data dependency can be eliminated simultaneously, and the order of the equations has great impact on exploiting the parallel pivoting (Heath 1991). The problem of finding good ordering of the sparse set of equations appropriate for parallel factorization is so called *parallel pivoting* or *parallel ordering* problem. For general graph, the ordering problem of minimizing the elimination steps is known to be *NP*-hard (Heath 1991). Our purpose is, given an ordering generated by some heuristic method to improve the ordering quality, i.e., lower the number of elimination steps.

2 THE HYBRID GA

Our method is a modification of Goldgerg's simple genetic algorithm, characterized by a novel hybrid of two crossover operators. We choose a sequence representation: a chromosome $\chi = x_1 x_2 \dots x_n$ corresponds to an ordering of the matrix A. In the literature, there are various sequencing crossovers for the order-based GAs. We tested those mentioned in Starweather's work (Starkweather 1993) and found that *position crossover* (POX) has the best performance. But the convergence rate is too slowly. To allievate this problem, we adapted Blanton Jr. and Wainwright's *merge operator* (Blanton Jr. 1993) in the present work. Toward the purpose of our work, the precedence vector is initially set to the solution obtained by some heuristic method and replaced by the best chromosome of the current population subsequently. We called this modified merge operator as *adaptive merge crossover* (AMX), which is applied every tenth generation in our hybrid GA while POX is performed for other times.

Table 1: Performance of the hyprid GA in BCSPWR04.

Xover	MDD+JK	Best	Gen
POX	38	26	300
AMX	38	31	8
POX+AMX	38	28	130

A preliminary test was performed on the power network matrices from Harwell-Boeing collection. For simplicity, we only show the result for BCSPWR04 in Table 1; similar phenomena were observed for other matrices. The column 'MDD+JK' denotes the number of parallel elimination steps obtained by minimum degree followed by Jess and Kees method (Heath 1991). The column 'Best' denotes the best obtained by each crossovers, and 'Gen' denotes the minimum generation to reach the best for each crossovers. In this test, the maximum generation, crossover rate, mutation rate, population size are set to 1000, 0.6, 0.1, and 100, respectively, and the turnament selection and swapping mutation are used. As the result shows, this hybrid would balance both on exploring as well as exlpoiting to reach a better solution.

References

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