Practical Modeling of Simple Genetic Algorithm, via deterministic paths, by Absorbing Markov Chains.

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Abstract

A practical dynamical model of an efficient Simple Genetic Algorithm is presented, introducing in the matrix of the Nix and Vose Markov model a practical postulate related to the schema theorem, that induces deterministic correction factors in the matrix, through Heaviside´s unitary step function. This alteration permits SGA to evolve by efficient deterministic channels. The model simulates the real behaviour of an efficient SGA. The Markov chain is transformed into an absorbing Markov chain. Using the absorbing theory, the expected waiting time, EWT, is computed easily in any situation. For the case of maximum uncertainty, it is obtained an expression for EWT that improves from the standard Nix and Vose model, in relation to the experimental data. Through the deterministic paths, the steady state is obtained when the absorbing state, global optimum, is reached. To emphasize that, with this practical improvement, the theoretical unification of the model of Nix and Vose for the SGA with the general model of the evolutionary algorithms with elitism is procured.

1 FROM NIX-VOSE TO ABSORBING MARKOV MODELS

This approach emerges from the excessive theoretical values (1) obtained for the EWT(J) in problems from the actual world. Below, it is presented a summary of the definitions and results that develop the absorbent model from the Nix and Vose model (2). It is used the usual nomenclature for the genetic and search parameters:

Practical Postulate: *Let z(t) be the sampling state at step t, and <F(z(t))> the average fitness of such state. Then, the only transitions permitted in an efficient S.G.A. are those that accomplish:* $\langle F(z(t+1))\rangle \geq \langle F(z(t))\rangle$. *The probabilities of transition states of a greater fitness to a smaller one are forbidden*

The incorporation of this correction factor in the model can be added directly in the elements $q_{i,j}$, using the unitary function of Heaviside, resulting:

$$a_{i,j}^{n} = \frac{q_{i,j}^{n} U_{<F(j)>} < F(i) >}{\sum_{j} a_{i,j}^{n}}$$

In the new matrix [**A**], it is introduced the states ordering by average fitness (3). With this choice [**A**] becomes a regular or canonical absorbing matrix. It is easy to find the steady state, limit matrix $[\mathbf{A}]^{\infty}$, as the optimum state; the absorbing state.

Explicit EWT in the absorbing model in the maximum uncertainty case, with states of equal initial probability

$$EWT(J) \approx ln\left(a \cdot \frac{(r+n-1)!}{(r-1)! \cdot n!} - a\right);\ r >> \text{n};\ a = 1.782$$

As conclusions, elitism is a sufficient, but not necessary, condition for the theoretical convergence,. Also, the appearance and retaining of the meta-stable states is harmful for the convergence in a moderate and finite time. Via deterministic paths, it is achieved the theoretical unification of the models of the elitist EA´s (4) and SGA. In practice, both models are equivalent.

References

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