Flight Midcourse Guidance Control Based On Genetic Algorithm

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ABSTRACT
An advanced flight midcourse guidance law based on genetic algorithm (GA) is proposed. The proposed midcourse guidance formulation minimizes the flight time and maximizes the terminal energy subject to a terminal intercept condition. GA is used to search the optimal attack angle for the flight trajectory. By combining GA and singular perturbation technique (SPT), the optimal flight guidance law is obtained consequently. SPT is applied to approximate the terminal flight time. Meanwhile, the paper completely eliminates the need for solving two-point boundary-value problems (TBPVP), which is too complex for derivation and implementation. The simulation results show that the resulting guidance law is near-optimal and the proposed method is valid. Especially, the GA guidance law can apply to intercept the maneuver targets successfully.

Categories & Subject Descriptors:
G.1.6–Optimization; I.2.8–Problem Solving, Control Methods and Search; J.2–Physical Science and Engineering.

General Terms:
Design, Algorithms

Keywords:
Genetic algorithm; Flight midcourse guidance control; Singular perturbation technique

1. INTRODUCTION
Flight guidance control problems in general and interception of airborne targets in particular, have been an important area of research on technologies supporting anti-ballistic missile defense in the last decade. Flight guidance control schemes of early generations applied proportional navigation guidance (PNG) or its modifications. Later, modern guidance laws were developed based on a linearized kinematical model and a linear quadratic optimal formulation (Zarchan,1990, Cottrel, 1971). Recently, various solutions such as feedback linearization, [1] sliding model control, [2] differential games, [3] predictive control [4] and neural networks [5] etc. are introduced into this field. However, a straightforward solution for the nonlinear optimal guidance law using the conventional methods may lead to a two-point boundary-value problem (TPBVP) which involves iterations of forward integration of the state equation and backward integration of the adjoint equation until the solution converges. Not only does the TPBVP involve substantial computation, but its convergence properties can be poor; furthermore, it is very sensitive to the initial conditions and noise disturbances.

The great potential of genetic algorithm (GA) has long been recognized in combination optimization and nonlinear control. GA uses search procedures based on the mechanism of natural genetics, which combines a Darwinian survival-of-the-fitness strategy to eliminate unfit characteristics and uses random information exchange. In addition, the GA optimization course involves little exterior information, and the fitness function is taken as its only optimization criteria. The singular perturbation technique (SPT) has been shown to produce good approximations to the optimal control for a variety of problems. The chief advantage of this approach is that it involves only the solutions to a set of coupled nonlinear equations and the noniterative integration of quadratures. This approach can be applied to the problem of generating near-optimal solutions. To the best knowledge of the authors, combining GA with SPT to applying to flight guidance control has not yet been performed.

The main work included in this paper begins with the nonlinear model of the flight guidance control, so the errors caused by linearization are avoided. In this paper, the flight trajectory of target is firstly divided into N elements to transform the optimal guidance problem to N flight trajectory optimization problems, and a linear combination of flight time and terminal specific energy is taken as the performance index of the optimization problems. Then, an advanced midcourse guidance law based on genetic algorithm (GA) is developed. The proposed midcourse guidance formulation minimizes flight time and maximizes the residual energy subject to a terminal intercept condition. GA is used to search the optimal attack angle for the flight trajectory and the SPT is applied to approximate the terminal flight time of

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the midcourse phase. Through these methods, the paper completely eliminates the need to solve two-point boundary-value problems which is too complex for derivation and implementation. The simulation results show that the resulting guidance law is near-optimal and the proposed method is valid and near-optimal.

This paper is organized as follows. Section 2 gives the detailed representation of the mathematical model of the BVR missile including the force analysis and motion analysis, section 3 develops the optimal guidance law based on the singular perturbation technique and genetic algorithm, section 4 verifies the performance of the proposed optimal guidance law through simulation on a BVR missile, and the final section presents conclusions and future work to be done.

2. MATHEMATICAL MODEL

The midcourse phase of the flight is defined as the period following launch until seeker lock-on is realized. The main purpose of the midcourse guidance is to navigate a missile so that is may operate in optimal conditions in regard to missile performance and relative geometry against a target when seeker lock-on is achieved.

Fig. 1 shows the force analysis of a beyond-vision -range (BVR) missile and the symbols used in this paper.

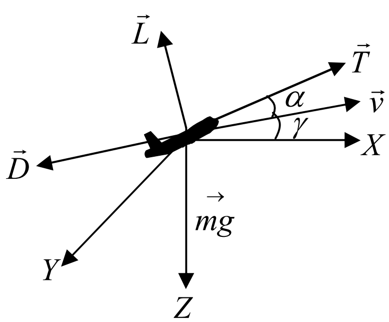

Fig. 1 The force analysis of the missile

For simplicity, the motions of the BVR missiles are constrained within X-Z plane. As for the 3-D motion of the BVR missile, we can also study them in the 2-D space through proper coordinate transformations. [6] During the midcourse flight, the BVR missile is modeled as a point mass. The state variables are the velocity of the BVR missiles v , the flight path angle γ , and the position (x,h) of the BVR missile in the inertial space, where, h denotes the flight altitude. Then the state equations of the BVR missile are expressed as follows.

$$\dot{\gamma} = \frac{(L + T\sin\alpha)}{mv} - \frac{g\cos\gamma}{v} \tag{1}$$

$$\dot{v} = \frac{(T\cos\alpha - D)}{m} - g\sin\gamma \tag{2}$$

$$\dot{x} = v\cos\gamma \tag{3}$$

$$\dot{h} = v\sin\gamma \tag{4}$$

$$\begin{cases} L = 1/2\rho v^2 s C_L & C_L = C_{L\alpha}(\alpha - \alpha_0) \\ D = 1/2\rho v^2 s C_D & C_D = C_{D0} + KC_L^{\ 2} \end{cases} \tag{5}$$

The aerodynamic derivatives $C_{L\alpha}$, C_{D0} and k are given as functions of the Mach number M, which are given in Table1.

Table 1 Fundamental data for BVR missile [7]

M	0.2	0.8	0.93	1.05	1.3	1.6	2.4	3.5	5.0
C_{D0}	0.242	0.211	0.255	0.406	0.444	0.370	0.254	0.190	0.149
k	0.110	0.135	0.134	0.108	0.108	0.116	0.120	0.134	0.153
$C_{L\alpha}$	9.31	7.65	7.73	9.74	10.03	9.28	9.02	8.02	7.16

The mass m and the thrust T of the BVR missile are given as functions of time t , which are shown in Fig. 2.

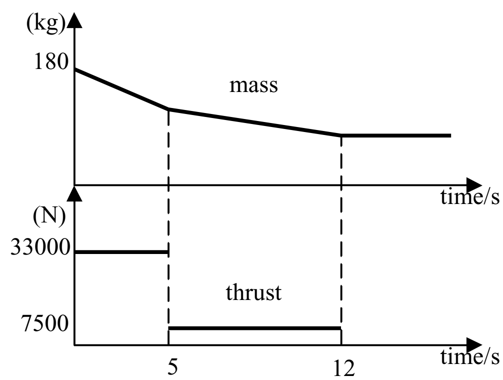

Fig. 2 Time history of missile mass and thrust

In our study, the attack angle of the BVR missiles is treated as control variable, which should satisfy the inequality constraint:

$$\alpha_{\min} \le \alpha \le \alpha_{\max} \tag{6}$$

During the midcourse guidance, tactical consideration dictates short flight time as the basic performance index. In addition, the energy of the BVR missiles must be conserved during the midcourse phase so that sufficient energy is available for terminal engagement of intelligent targets. So, the optimal performance index is represent as

$$J = -\zeta E(t_f) + (1-\zeta)\int_0^{t_f} dt \tag{7}$$

where, t_f and $E(t_f)$ denote the final time and the terminal energy of the midcourse phase respectively. The weighting factor ζ enables the tradeoff between flight time and terminal energy. This factor is constrained as

$$0 \le \zeta \le 1 \tag{8}$$

3. OPTIMAL GUIDANCE LAW BASED ON GENETIC ALGORITHM

3.1 Representation of Problem

In this paper, after the trajectory of the target is divided into N elements, the flight guidance control problem is changed into N nonlinear trajectory optimization problems. As for the *i*th problem, we can take use of GA to search the optimal trajectory, and the singular perturbation technique is applied to approximate the final time t_{fi} $(i=1,\cdots,N)$ of each problem.

The flight trajectory of the target is guaranteed by the ground support system through filtering and identification and transmitted to the missile during the midcourse guidance. A given target trajectory is divided into N elements which are represented as E_iE_{i+1} $(i=1,2,\cdots,N)$ (as shown in Fig. 3).

As for the target position $\dot{E}_i$, we can compute the orientation angle φ_i of the target and the slant range R_i between the target and the missile through the Eqs.9 and 10.

$$\phi_i = \arctan\left(\frac{h_{ti}-h_{mi}}{x_{ti}-x_{mi}}\right) \tag{9}$$

$$R_i = \sqrt{(h_{ti}-h_{mi})^2+(x_{ti}-x_{mi})^2} \tag{10}$$

According to φ_i and R_i, we can get the approximate expression of the final time from the missile position P_i to the target position $\dot{E}_i$, where θ_{ti} is the course angle of the target, v_{ti} is the target velocity, the subscripts *m* and *t* denote missile and target respectively.

The final time t_{fi} is derived from zero-order singular perturbation technique. [8] We assume the target flies at constant speed in the interval from $\dot{E}_i$ to E_{i+1}, $(i=1,2,...,N)$. So, there must be a shortest course for interception of the target, and the final time is the time used by the interceptor. The derivation process of t_{fi} is as follows. The symbols employed in this section are shown in Fig.3

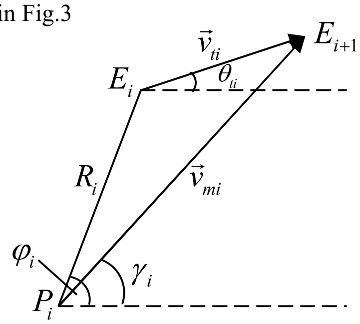

Fig. 3 Geometry of estimating final time t_{fi}

By projecting R_i and v_{ti} to $\overrightarrow{P_iE_{i+1}}$, we can obtain the expression of the final flight time,

$$t_{fi} = \frac{R_i\cos(\phi_i-\gamma_i)}{v_{mi}-v_{ti}\cos(\gamma_i-\theta_{ti})} \tag{11}$$

In order to insure the validity of the homing guidance, we must conserve sufficient energy for terminal engagement of intelligent targets. In other words, the energy consumed in the midcourse phase should be minimal. We represent the consumed energy by the terms of the power of missile $\dot{E}_i$,

$$\dot{E}_i = \frac{(T_i\cos(\alpha_i)-D_i)\cdot v_{mi}}{m_i g} \tag{12}$$

where, D_i is the drag force on the missile, which is represented in Eq. 13

$$D_i = \frac{1}{2}\rho v_{mi}{}^2 s C_L \tag{13}$$

Then, we can transfer the performance index function in Eq. 7 to the following expression,

$$\min J$$

$$J = \zeta\dot{E}_i + (1-\zeta)\int_0^{t_{fi}} dt \tag{14}$$

where, the factor ζ is constrained by Eq. 8.

3.2 GA Optimization

3.2.1 Coding scheme of chromosome

Binary coding scheme is usually adopted in normal GA for its simplicity and good feasibility. But the defect of Hamming cliff limits its application. In order to achieve satisfactory nonlinear optimization effect, we adopt length-variable coding scheme. [9]

3.2.2 Construction of fitness function

In GA, the fitness function is the only motivation for the nonlinear optimization. The value of the fitness function should not be negative. According to the performance index function J in Eq. 14, we construct the fitness function as shown in Eq. 17

$$f_i(x) = \begin{cases} C_{i\max} - \zeta\dot{E}_i + (1-\zeta)\int_0^{t_{fi}} dt, & \zeta\dot{E}_i + (1-\zeta)\int_0^{t_{fi}} dt < C_{i\max} \\ 0, & \text{F M T F} \end{cases} \quad 15)$$

where, i denotes the ith trajectory optimization problem, $C_{i\max}$ is the maximum of $\zeta\dot{E}_i + (1-\zeta)\int_0^{t_{fi}} dt$. So the minimum of performance J is equivalent to the maximum of the fitness function $f_i(x)$.

3.2.3 Genetic operations

In this paper, the attack angle of the missile α is treated as the control variable, which is constrained by Eq. 6. Using GA, the initial population of the attack angle α_i is prepared at random, which contains m individuals. The generation is updated in the following process.

1) Generation of parents: m parent individuals are initially selected at random.

2) Calculation of fitness: As for each individual α_i of v_{mi}, the variable γ_i and are gained through Eqs.1 and 2. Then according

to Eq.16, the fitness function of each individual in the population is calculated.

3) Selection: According to fitness of each individual, the finest chromosomes are selected from the roulette selection which determines a fine chromosome at random according to the specified probability based on the ranking.

4) Crossover and mutation operation: in order to insure the diversity of the population, the crossover and mutation operation are performed according to the specified probability.

5) Through step 3) and 4), the new generation is determined consequently. The fitness of each individual in the new generation is calculated. If the finest solution does not converge, return to the step 3).

6) If the finest solution converges or the maximum times of iteration arrives, we believe the optimal attack angle α_i is gained, the optimal v_{mi} and γ_i are gained, too. Meanwhile, the position of the missile at next time instant $E_{i+1}(x_{m,i+1}, h_{m,i+1})$ is calculated through Eqs.3 and 4.

To enhance intercept performance against maneuvering targets, we repeat the previous optimization process on each element of the target trajectory, using the latest available information about the target and missiles states. Finally, we can gain the optimal guidance law for the midcourse phase. In this paper, the variables between nodal values are interpolated using a third order spline function, and the 4th order Runge-Kutta algorithm is used as the integration scheme.

The flow chart of the whole optimization process is shown in Fig. 4.

4. SIMULATION RESULTS

The proposed optimal guidance law based on GA is tested using a BVR missile simulation. The mass m and the thrust T of the BVR missile are shown in Fig. 2.

About the flight trajectory of the target, we consider two types of conditions. One is that the target flies at constant speed following the horizontal trajectory, which is described by (a) and (b). Another is that the target possesses the horizontal or vertical maneuvering ability, which is described by (c) and (d). The latter can avoid the attack from firepower. The flight trajectories used in this paper are as the following.

(a) The target intrudes at speed of 200m/s from the distance of 40km at the height of 8km.

(b) The target escapes at speed of 200m/s from the distance of 20km at the height of 8km.

(c) The target intrudes at speed of 200m/s from the distance of 40km at the height of 8km. When the distance between the target and missile decreases to 20 km, the target begins to maneuver with the vertical overloading of 1.5g.

(d) The target intrudes at speed of 200m/s from the distance of 40km at the height of 8km. Meanwhile, the target maneuvers with the horizontal overloading of 1.5g.

We apply the proposed guidance law to intercept or pursuit the above-mentioned targets respectively. In GA, the population size m is 20, the probability of crossover, P_c , is set to 0.45, and the probability of mutation, P_m , is set to 0.06.

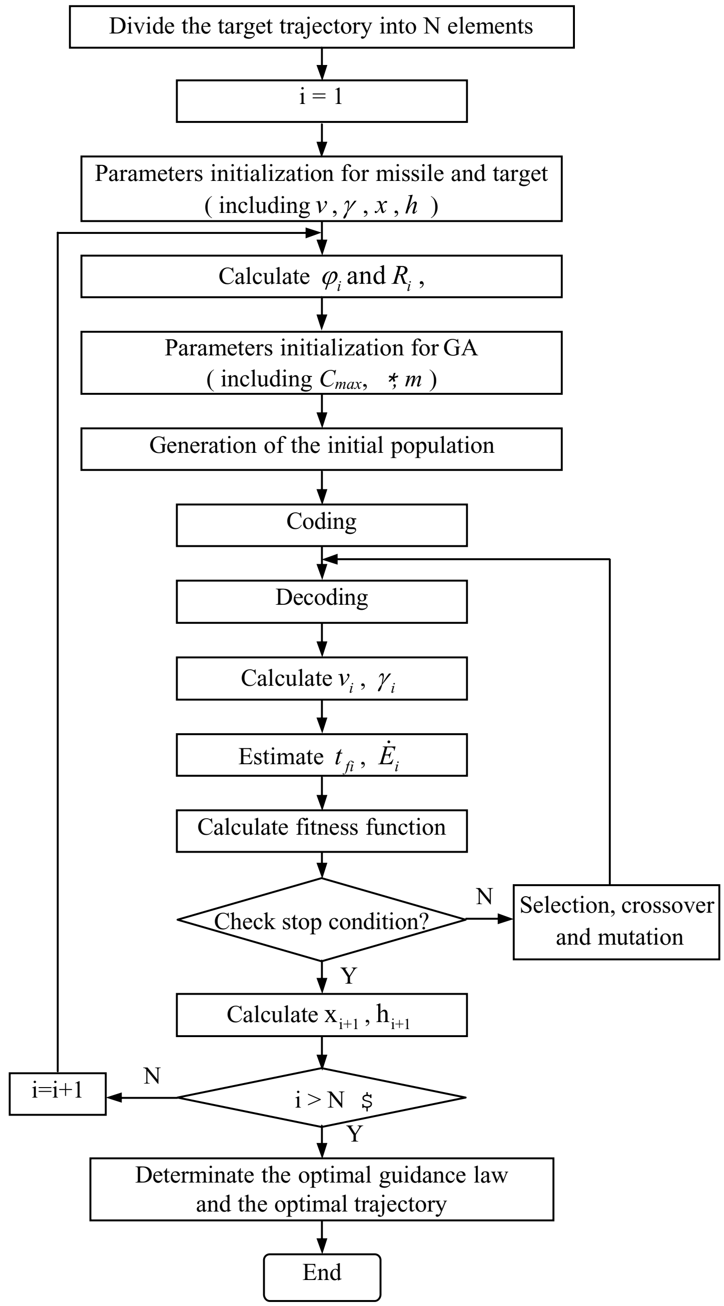

Fig. 4 The flowchart of the proposed scheme

The optimal flight trajectories of the missile are shown in Fig. 5, where the figures (a-d) describe the above- mentioned four kinds of targets. The miss distance and final time of the missile corresponding to the four kinds of targets are given in table 2.

As shown in Fig. 5 and table 2, the miss distance meets the requirement of performance index, and the target is intercepted by the missile successfully, which is guided by GA guidance law.

The validity of the proposed scheme is confirmed. Moreover, the proposed GA guidance law can intercept or pursuit the maneuver target, which is superior to the conventional proportional guidance law.

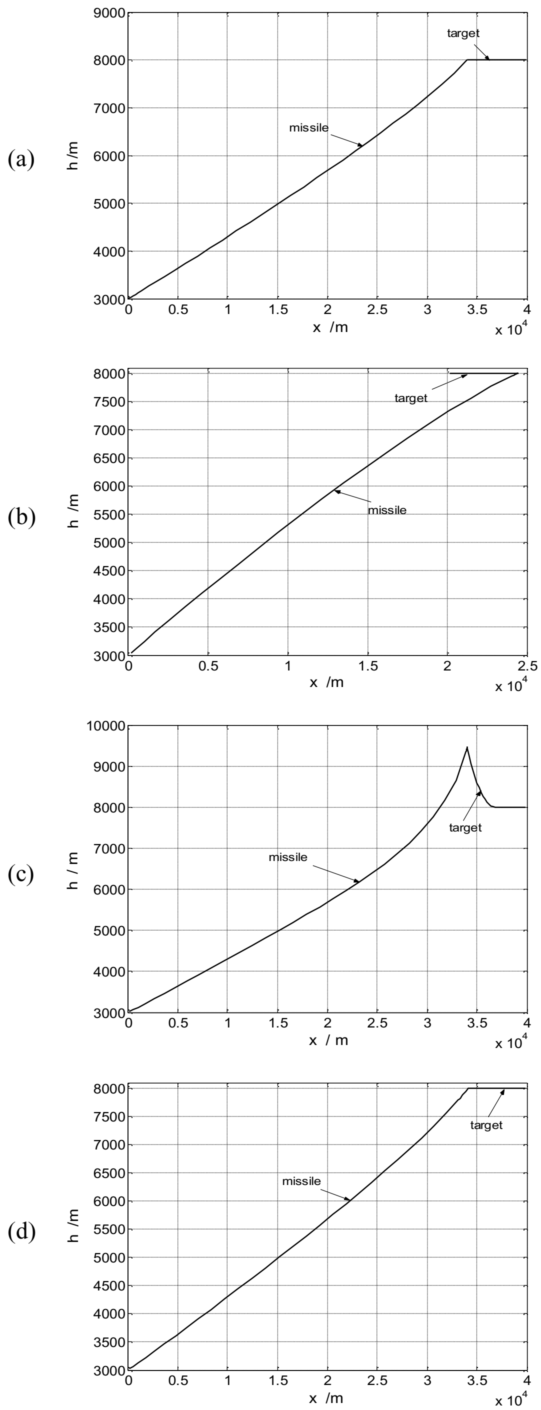

Fig. 5 The optimal flight trajectory of the missile and the target

Table 2 Simulation results on different targets

Index	Target 1	Target 2	Target 3	Target 4
Miss Distance /m	0.14	1.23	2.24	1.59
Final time /s	30.10	22.31	30.18	26.25

The energy consumption and the final time variation of the intruding target with constant speed are showed in Fig. 6. It is found that the missile can capture the target in shorter time and fewer energy consumption. That is to say, GA guidance law can intercept the targets quickly and save more energy for the homing guidance. So the proposed guidance law is a kind of near optimal control law. Above all, the GA guidance law can avoid solving the two-point boundary-value problem.

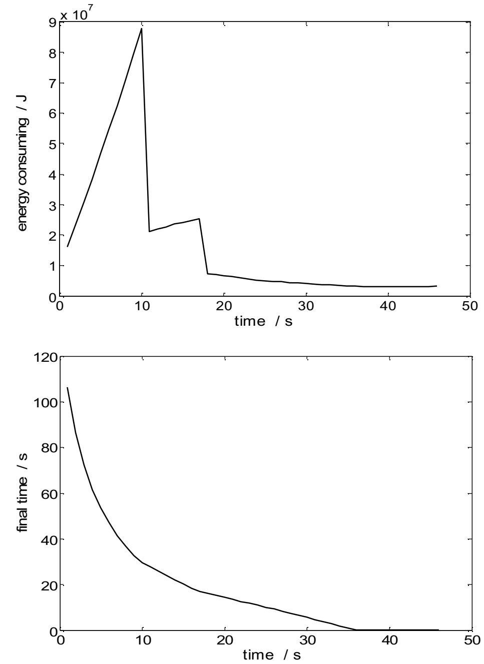

Fig. 6 The energy consumption and the final time of the intruding target with constant speed

5. CONCLUSION

In this paper, we transform the midcourse guidance problem to N nonlinear trajectory optimization problems and develop a midcourse guidance law based on genetic algorithm. Simulation results show that the proposed optimal guidance law is valid and near optimal. Above all, it is not necessary to solve the two-point boundary-value problem which is rather complex to solve.

As for the online implementation, we will adopt the table look-up method based on the results of this paper. Another method will be developed to improve the convergence speed of GA by bettering the generation method of initial population according to immune principle. [10] The above-mentioned two suggestions are our future work.

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