An Extension of Vose's Markov Chain Model for Genetic Algorithms

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ABSTRACT

The paper presents an extension of Vose's Markov chain model for genetic algorithm (GA). The model contains not only standard genetic operators such as mutation and crossover but also two new operators – translation to the left/right and permutation of bits. The presented model can be used for finding the transition matrices and for the investigation of asymptotic properties by using Markov transition functions. The ergodity of the Markov chain describing the GA with new operators, translation to the left/right and permutation, is shown. The model is specialized for a case of Bentley's GA. For this GA the ergodity of the Markov chains and the asymptotic correctness in the probabilistic sense are shown. To model other aspects of the Bentley's GA (effective fitness, total transmission probability) the microscopic Exact Poli GP Schema Theory for Subtree-Swapping Crossover is used.

Categories and Subject Descriptors

G.3 [**Probability And Statistics**]: Markov processes, I.2.8 **[Artificial Intelligence**]: Problem Solving, Control Methods, and Search – *Genetic Algorithms*

General Terms

Theory

Keywords

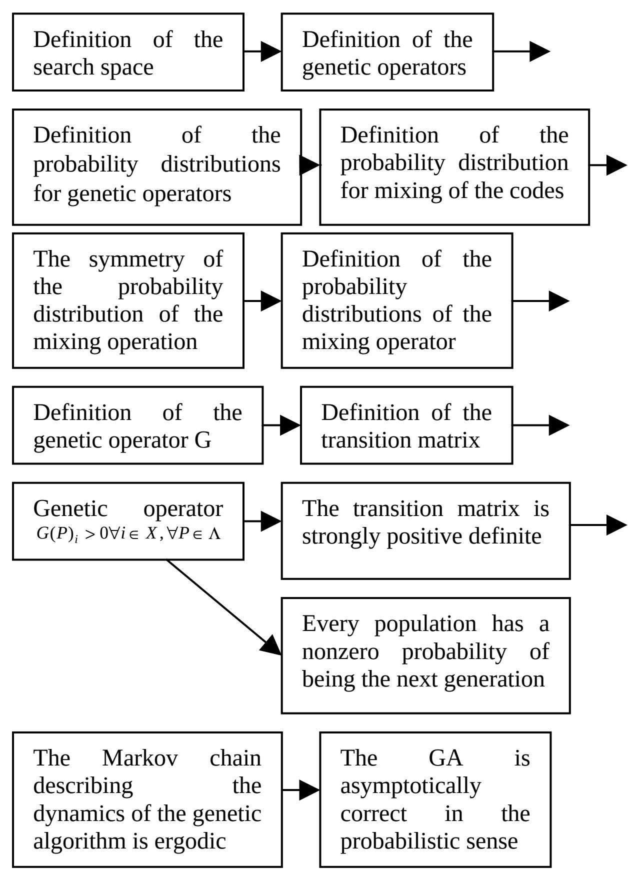
genetic algorithms, Theory, Markov chain model, ergodity, asymptotic correctness, Bentley's GA, Schema Theory

1. INTRODUCTION

There are many works based on Schema Theory [3] explaining how the GAs work. The other approach to the formalization of GA is based on the microscopic Markov chain models, such as Vose's model of SGA [5]. The Markov chain model enables the investigation of asymptotic properties and transition matrices. The only one known Markov chain model for more complicated evolutionary algorithm – GP with homologous crossover, without mutation - is presented in [4] by Poli. The paper presents an extension of Vose's Markov chain model for SGA to GA with new operators – translation to the left/right and permutation of bits. The model is specialized for the case of Bentley's GA [1] which is used to generate 3D-solids designs in a CAD system. Next, the microscopic Exact Poli GP Schema Theory for Subtree-Swapping Crossovers is applied to the Bentley's GA to calculate the effective fitness and the total transmission probability for a fixed-size-and-shape schema under hierarchical crossover.

2. EXTENSION OF THE VOSE'S MARKOV CHAIN MODEL

The extension of Vose's Theory for SGA consists of adding new operators: translation to the left/right and permutation. The genetic algorithm is modeled with a homogeneous Markov chain. The ergodity of the Markov chain and the asymptotic correctness in the probabilistic sense of genetic algorithm were shown. The sketch of the proof is shown on the figure. In the model the search



space X is defined as a set of all possible binary strings of length c. Mutation and crossover operators and their probability

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ditributions are defined as for SGA in [5]. Translation to the right consists of creating the code $z \in X$ from the code $x \in X$ by using the translation mask $u \in X$ as follows:

$z = x \circ u$ where: $x = (x_0, x_1, ..., x_{c-1}); u = (u_0, u_1, ..., u_{c-1})$

$$z = (z_0, z_1 ..., z_{c-1})\ ;\ z_w = \begin{cases} 0 \forall w < c - left(u) \\ x_{w-c+left(u)} \forall w \geq c - left(u) \end{cases}$$

$left(u) : X \rightarrow \{0,1,...,c-1\}$

$$left(u) = \begin{cases} c \text{ for } u = \mathbf{0} \\ \min\{a \in \{0,1,...,c-1\} : u_a = 1\} \end{cases} ;\ \underbrace{\mathbf{0} = (0,...,0)}_{c \text{ times}}$$

Translation to the left consists of creating the code $z \in X$ from the code $x \in X$ by using the translation mask $u \in X$ according to the rule: $z = x \hat{\circ} u$ where

$$z = (z_0, z_1, ..., z_{c-1}) : z_w = \begin{cases} 0 \forall w \geq left(u) \\ x_{w+c-left(u)} \forall w < left(u) \end{cases}$$

The transposition operator consists of creating the code $z \in X$ from the code $x \in X$ by using the transposition mask $v \in X$ according to the rule: $z = x \bullet v$ where

$$z_w = \begin{cases} x_w \text{ for } w = \mathbf{0} \\ x_{\min\{a \in \{0,...,c-1\}: v_a = 1\}} \text{ for } w = \max\{a \in \{0,...,c-1\} : v_a = 1\} \\ x_{\max\{a \in \{0,...,c-1\}: v_a = 1\}} \text{ for } w = \min\{a \in \{0,...,c-1\} : v_a = 1\} \\ x_w \text{ in the other case} \end{cases}$$

The probability distribution $move_x$ of the result of the translation to the right of the code x is given by the following formula:

$move_x(\{z\}) = \sum_{u \in X} \gamma_u [x \circ u = z]$ where

$$\gamma_u = \begin{cases} \frac{1}{c} \text{ if } (\mathbf{1}, u) = 1 \\ 0 \text{ in the other case} \end{cases}$$

The probability distribution $\overline{move_x}$ of the result of the translation to the left of the code x is given by the following formula:

$\overline{move_x}(\{z\}) = \sum_{u \in X} \gamma_u [x \hat{\circ} u = z]$

The probability distribution $transp_x$ of the result of the transposition of the code x is given by the following formula:

$transp_v(\{z\}) = \sum_{v \in X} \alpha_v [x \bullet v = z]$ where

$$\alpha_x = \begin{Bmatrix} \binom{c}{2} + 1 & \forall v : (v, \mathbf{1}) \in \{0,2\} \\ 0 \text{ in the other case} & \end{Bmatrix}$$

3. APPLICATION OF THE EXTENDED MARKOV CHAIN MODEL TO BENTLEY'S GA

The extended Markov chain Model is applied for the Bentley's GA under the assumption, that crossover points are numbers of primitives. The mutation of groups of alleles is modeled as composition of permutation and translation to the left/right operators. The probability distribution mut_x of the result of mutation of alleles of the code x is defined as in [5]. The probability distribution $cross_{x,y}$ is given by the following formula:

$cross_{x,y}(\{z\}) = \sum_{t \in X} \frac{\eta_t + \eta_{\bar{t}}}{2} [(x \otimes t) \oplus (\bar{t} \otimes y) = z]$ with crossover type

$$type_t = \begin{Bmatrix} \frac{1}{n-1} \text{ if } (\exists l \in (0, npq) : t = 2^{l-1}) \wedge (\exists u \in (0, npq) : (1, t) = upq) \\ 0 \text{ in the other case} \end{Bmatrix}$$

4. EXACT MICROSCOPIC SCHEMA THEORY FOR BENTLEY'S GA

Theoretical results presented below are obtained on the basis of Microscopic Schema Theory for GP with Subtree-Swapping Crossover [3]. Hierarchical crossover is modeled as strongly typed crossover [2]. For hierarchical crossover probability of choosing nodes, coded in the Node Reference System as $(d_1, i_1), (d_2, i_2)$, in h_1, h_2 is equal to:

$$p_{scgp}(d_1, i_1, d_2, i_2 \mid h_1, h_2) = \frac{CM(d_1, i_1, d_2, i_2, h_1, h_2)}{\sum_{D_1=0}^{\max\_depth(h_1)} \sum_{D_2=0}^{\max\_depth(h_2)} \sum_{I_1=0}^{pq} \sum_{I_2=0}^{pq} CM(D_1, I_1, D_2, I_2, h_1, h_2)}$$

where *CM* returns 1 when nodes are in the same class and 0 in the other case. The total transmission probability and the effective fitness for a fixed-size-and-shape schema H under hierarchical crossover and no mutation can be easily calculated on the base of the theorems presented in [3].

5. CONCLUSIONS

The paper has presented the extension of Vose's Markov chain model for GA with mutation, crossover, translation to the left/right, and permutation of bits. The ergodity of the Markov chain describing the GA has been shown. The presented theory has been applied to the Bentley's GA. The ergodity of the Markov chain and the asymptotic correctness in the probabilistic sense for the Bentley's GA has been shown. Also the microscopic Exact Poli GP Schema Theory for Subtree-Swapping Crossovers has been applied to the Bentley's GA to calculate the effective fitness and the total transmission probability for a fixed-size-and-shape schema under hierarchical crossover.

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