

Automated Re-Invention of Six Patented Optical Lens Systems using Genetic Programming

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ABSTRACT

This paper describes how genetic programming was used as an invention machine to automatically synthesize complete designs for six optical lens systems that duplicated the functionality of previously patented lens systems. The automatic synthesis was done “from scratch”—that is, without starting from a pre-existing good design and without pre-specifying the number of lenses, the physical layout of the lenses, the numerical parameters of the lenses, or the non-numerical parameters of the lenses. One of the six genetically evolved lens systems infringed a previously issued patent; three contained many of the essential features of the patents, without infringing; and the others were non-infringing novel designs that duplicated (or improved upon) the performance specifications contained in the patents. One of the six patents was issued in the 21st-century. The six designs were created in a substantially similar and routine way, suggesting that the approach used may have widespread utility. The genetically evolved designs are instances of human-competitive results produced by genetic programming in the field of optical design.

Categories and Subject Descriptors

G.1.6—Global Optimization; I.2.2—Automatic Programming Program Synthesis; I.2.8—Control Methods and Search; J.2—Physics

General Terms: Design, algorithms

Keywords: Genetic programming, automated design, optical lens system, patents, human-competitive results, invention machine

1 INTRODUCTION

An optical lens system is an arrangement of refractive or reflective materials that manipulate light [15].

Optical design is more of an art than a science. As Warren J. Smith states in *Modern Optical Engineering* [15, page 393]:

“There is no ‘direct’ method of optical design for original systems; that is, there is no sure procedure that will lead (without foreknowledge) from a set of performance specifications to a suitable design.”

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An existing design is frequently the starting point of optical design by humans and by conventional optical optimization software. Accordingly, Smith [15] and others in the field have compiled thousands of historically important and useful designs (many previously patented) as starting points.

A complete design for a classical optical lens system encompasses numerous decisions, including the choice of the system’s topology (that is, the number of lenses and their physical layout), choices for numerical parameters, and choices for non-numerical parameters.

The layout decisions required to define a lens system include the sequential arrangement of lenses between the object and the image, decisions as to whether consecutive lenses touch or are separated by air, the nature of the mathematical expressions defining the curvature of each lens surface (traditionally spherical, but nowadays often aspherical), and the locations and sizes of the field and aperture stops that determine the field of view and the maximum illumination of the image, respectively.

The numerical choices include the thickness of each lens and the separation (if any) between lens surfaces, the numerical coefficients for the mathematical expressions defining the curvature of each surface (which, in turn, implies whether each is concave, convex, or flat), and the aperture (semi-diameter) of each surface.

The non-numerical choices include the type of glass (or other material) for each lens. Each type of glass has various properties of interest to optical designers, notably including the index of refraction, n (which varies by wavelength); the Abbe number, V ; and the cost. Choices of glass are typically drawn from a standard glass catalog.

This paper describes how genetic programming can be used to automatically create a complete design for an optical lens system. The automatic synthesis is done “from scratch”—that is, without starting from a pre-existing good design and without pre-specifying the number of lenses, the physical layout of the lenses, the numerical parameters of the lenses, or the non-numerical parameters of the lenses.

Section 2 mentions previous work. Section 3 provides background on the design of optical lens systems. Section 4 discusses the preparatory steps used to apply genetic programming to optical systems. Section 5 presents the results. Section 6 is the conclusion.

2 PREVIOUS WORK

Genetic algorithms have been extensively used for optimizing the choices of parameters of optical systems with a pre-specified layout and pre-specified number of lenses, as listed in Jarmo

Alander's *An Indexed Bibliography of Genetic Algorithms in Optics and Image Processing* [1].

In a noteworthy paper, Beaulieu, Gagné, and Parizeau [3] used genetic programming to “re-engineer” the design of a four-lens system (itself produced by a run of the genetic algorithm) and thereby create an improvement over the best design produced by 11 human teams in a design competition held at the 1990 International Lens Design Conference. Their approach used functions that incrementally adjusted (additively or multiplicatively) the distance between lens surfaces, radius of curvature of lens surfaces, and stop location values.

Our group recently used genetic programming to create an optical lens system “from scratch”—without starting from a pre-existing good design and without pre-specifying the number of lenses, the physical layout of the lenses, or the numerical parameters of the lenses [2].

3 BACKGROUND ON THE DESIGN OF OPTICAL LENS SYSTEMS

A classical lens system is conventionally specified by a table called a *prescription* (or, if the system is being analyzed by modern-day optical simulation software, a *lens file*). Table 1 shows a prescription for the patented Tackaberry-Muller lens system [16] of figure 1. See a general textbook on optics [15] or our recently published paper [2] for a detailed explanation of a prescription (such as table 1).

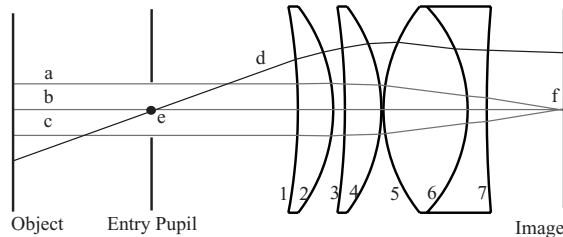


Figure 1 The Tackaberry-Muller lens system

Table 1 Lens file for Tackaberry-Muller system

Surface	Distance	Radius	Material	Aperture
Object	10^{10}	flat	air	
Entry pupil	0.88	flat	air	0.18
1	0.21900	-3.5236	BK7	0.62
2	0.07280	-1.0527	air	0.62
3	0.22500	-4.4072	BK7	0.62
4	0.01360	-1.0704	air	0.62
5	0.52100	1.02491	BK7	0.62
6	0.11800	-0.9349	SF61	0.62
7	0.47485	7.94281	air	0.62
Image		flat		

Once a classical optical system is specified by means of its prescription (lens file), many of its optical properties can be calculated by tracing the path of light rays of various wavelengths through the system. Ray-tracing analysis by hand is extremely time-consuming. Ray tracing is typically performed nowadays by optical simulation software (e.g., OSLO, Zemax,

Code V, KOJAC). Figure 2 shows three (of the many) conventional curves presenting characteristics of interest to optical designers, including distortion (figure 2a), astigmatism (figure 2b), and chromatic aberration (figure 2c) for the Tackaberry-Muller system.

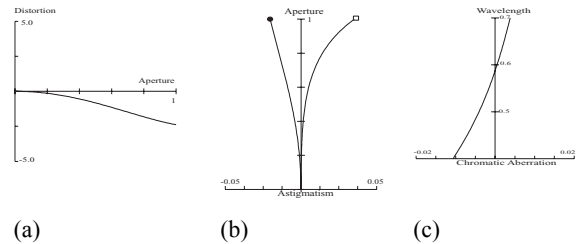


Figure 2 Distortion, astigmatism, and chromatic aberration of the Tackaberry-Muller system

Figure 3 shows the on-axis ray intercept diagram (figure 3a) and the full-field ray intercept diagram (figure 3b) for the Tackaberry-Muller system. Each of these diagrams has two parts, with the diagram for the meridional plane on the left and the sagittal on the right. In addition, a similar pair of figures is typically derived for a partial-field ray intercept.

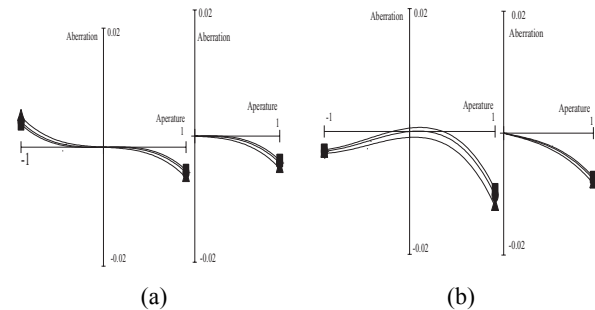


Figure 3 On-axis and full-field ray intercept diagrams for the Tackaberry-Muller system

4 PREPARATORY STEPS

We now describe our representation scheme (including our developmental process and function set), the fitness calculation for one of the six lens systems discussed herein (the others being similar), and the two domain-specific mutation operations that we used. Other details are in [2] and the technical report (available on the web) cited therein.

4.1 Representation Scheme

The widely-used and well-established format for optical prescriptions (and lens files for optical analysis software) suggests the developmental process [17, 5] for representing candidate individuals in the population.

Our developmental representation employs a turtle similar to the turtle used in Lindenmayer systems [11], the LOGO programming language, and our previous work using genetic programming to synthesize geometric patterns [8] and antennas [4]. The turtle starts at the point (point e in figures 1 and 4) where the system’s main axis (line b in figures 1 and 4) intersects with the entry pupil surface.

The three-argument SS (“spherical surface”) function causes the turtle to do three things at its starting point (and each subsequent

point to which the turtle moves). First, it inserts a spherical surface with a specified radius of curvature at the turtle's present location. Second, the SS function moves the turtle to the right by a specified distance along the system's main axis. Third, the SS function fills the space to the right of the just-added surface with a specified type of material.

Figure 4 shows the result of the insertion of spherical surface 1 (with a radius of curvature of -3.52361). After inserting this surface, the turtle moves the specified distance of 0.219 from its starting point g to point h along axis line b . The "BK7" in the figure indicates that glass of type BK7 (a commercially available crown glass) will be used to fill the space between g and whatever surface is inserted at point h (by the turtle's next step). These actions by the turtle reflect the information contained on row "1" of the prescription (table 1).

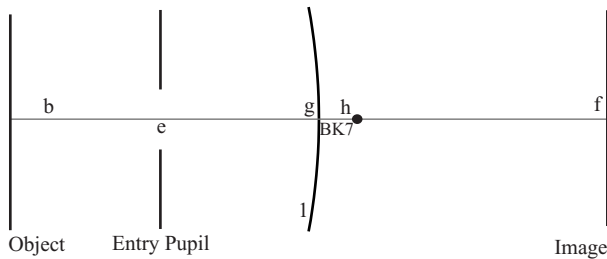


Figure 4 Turtle moves from g to h and inserts surface 1

Figure 5 shows the result of the insertion by the turtle of spherical surface 2 (with a radius of curvature of -1.05274). After inserting this surface, the turtle moves the specified distance of 0.07280 from point h to i . Surfaces 1 and 2 together define a lens of thickness 0.219 of BK7 glass. Note that the material inserted to the right of surface 2 is air. That is, air will be used to fill the space between h and whatever surface is inserted at point i (by the turtle's next step). These actions reflect row "2" of the prescription (table 1).

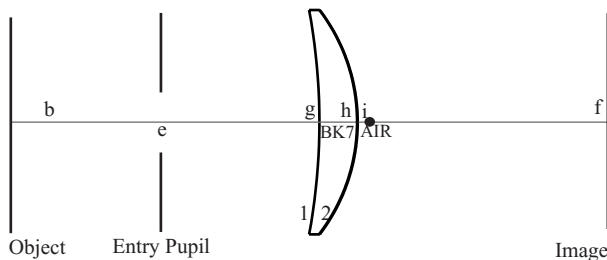


Figure 5 Turtle moves from h to i and inserts surface 2

Radius and distance values are each established by value-setting subtrees of the SS function consisting of a single perturbable numerical value. The material is established by a value-setting subtree consisting of a terminal identifying the type of material (e.g., air, BK7).

The turtle can similarly create a free-standing second lens (surfaces 3 and 4 in figure 1) and an additional lens (surfaces 5 and 6 in figure 1). The doublet lens (surfaces 6 and 7 of figure 1) is then created by inserting SK4 glass (instead of air) to the right of surface 6.

The two-argument PROGN2 function is a connective function that first executes its first argument and then executes its second argument.

A constrained syntactic structure specifies how the functions and terminals may be combined in a program tree. The constrained syntactic structure enforces the use of one terminal set (containing perturbable numerical values) for each value-setting subtree that establishes the numerical value for thickness and radius of curvature; another terminal set for establishing the type of material (containing symbolic values changeable by our glass mutation operation described below); and another function set for all other parts of the program tree. The constrained syntactic structure further specifies that the top-most function in a program tree is a PROGN2 function (to avoid trees that construct only a single surface).

The object surface (OBJ), image surface (IMS) and entry pupil (EP) together constitute a *test fixture* that is directly analogous to the test fixtures used in connection with the automatic synthesis of electrical circuits by means of genetic programming [9, 10].

4.2 Fitness Measure

The fitness measure used in the field of optical design (whether by evolutionary search or human design) is multi-objective in the sense that it contains numerous elements.

We developed our own lens analysis simulator based on KOJAC, a public-domain educational software package for optical ray tracing originally authored by Olivier Scherler and currently maintained by Olivier Ripoll, to evaluate the performance of candidate lens systems. We wrote code to use the ray traces produced by KOJAC to compute relevant optical characteristics and additionally wrote code for the image analysis. We embedded all of this code in our genetic programming system operating on our Beowulf cluster computer. We used a commercially available software package (OSLO from Lambda Research) running on a single workstation for post-run validation of final results.

The fitness measure begins by analyzing an axial and chief ray trace in order to derive aberration and paraxial coefficients (and assigns a fatally high penalty value if either ray trace fails). Fitness is incremented by the weighted sum of the deviations between the behavior of the candidate individual and the target values of various performance measures. In particular, fitness is incremented by the sum of 1,000 times each of the following aberration deviations: spherical aberration, astigmatism, distortion, coma, axial chromatic, lateral chromatic; 100 times the absolute deviation of effective focal length (EFL) from target; 100 times the absolute deviation of back focal length (BFL) from target (but only if it is less than target); and 25 times the absolute deviation of Petzval radius from target (but only if it is less than target). The weights were chosen to approximately equalize the influence of each of the above types of deviations in a manner similar to our recent work with automatic circuit systems involving multi-objective fitness measures.

Then, a 17×17 grid is overlaid on the entry pupil and a ray is shot through the corner defining each grid position contained inside the entry pupil.

Figures 6a, 6b, and 6c are gray scale versions of a three-color spot diagram for the Tackaberry-Muller system. The rays from figure 6 are traced for each of three wavelengths (486, 588 and 656 nm) and projected through the system onto the image plane. Figure 6a shows the trace from the axial point. Figure 6b shows the trace from the 70% of the field of view (FOV). Figure 6c shows the full FOV performance. Figure 6d shows the

modulation transfer function performance of the Tackaberry-Muller system in the tangential and sagittal planes of each of the FOV positions.

The spot diagram measures the deviation resulting from the compound error of the chosen lens aberration contributions. An ideal diffraction limit spot size (corresponding to the minimum spot size that can be discernable when diffraction effects are taken into account) is determined for the system and the root mean square (RMS) error for the ray intercept deviations is calculated. Fitness is incremented by 200, 340 and 400 times the difference between the target RMS error for the axial, 70% FOV and full FOV, respectively. The increasing penalty multiplier reflects the increasing difficulty in attaining the desired performance. The modulation transfer functions measure the contrast and resolution differences between the object and the image. Each curve is sampled at 30 increments of 10 cycles/mm across the target system and that modulation efficiency is defined as the target to meet.

After an individual reaches a specified satisfactory level for all of the foregoing requirements, individuals are evaluated for parsimony. The parsimony penalty is the sum of 100 times the number of lenses, 100 times the number of different types of glass used, the width of the lens system (its footprint), and (optionally, but not used for the work in this paper) the cost of the glass (found in the glass catalog).

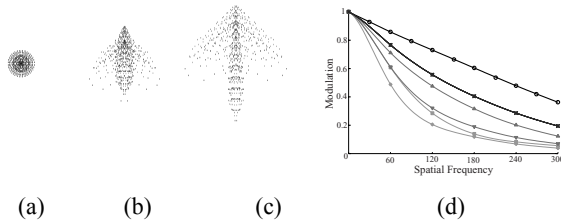


Figure 6 Fitness measure

For the specific problem discussed herein, each candidate lens system is first examined to see if it is “all air.” If so, the individual receives a fatally high penalty value of fitness. In addition, a significant (but not fatal) infeasibility penalty is applied if the back focal length (BFL) is negative (meaning the image is inside the lens system, instead of being to the right), or if the final surface is not air (meaning that a flat glass surface touches the image surface).

4.3 Toroidal Mutation for Curvatures

Because a flat surface can be viewed as a spherical surface with a very large positive or negative radius of curvature, our mutation operation operates in a toroidal way when it is applied to a terminal representing the radius of curvature.

4.4 Glass Mutation

In real-world situations, the optical designer usually chooses one of a relatively small number of commercially available types of glass (such as those found in the Schott catalog). Accordingly, we used a domain-specific mutation operation that permits mutation only to a “nearby” type of glass (in the multidimensional property space) in the chosen catalog.

4.5 Control Parameters

The population size was approximately 75,000 (500 individuals per node and slightly more or less than 150 nodes for each of the

six problems discussed in this paper). Because there is only one active function (SS), all of its arguments are terminals, and the glass mutation operator is unusually important, we performed crossover, numerical mutation, and glass mutation at 30% probability each, with reproduction at 9% and tree mutation at 1%.

5 RESULTS

We applied the above techniques to six patented eyepieces:

- the 1940 Konig patent [7],
- the 1953 Ludewig patent [12],
- the 1958 Tackaberry-Muller patent [16],
- the 1968 Scidmore patent [14],
- the 1985 Nagler patent [13], and
- the 2000 Koizumi-Watanabe patent [6].

Each patent states the inventor’s high-level design goals, a detailed specification of the invention, one or more independent claims (usually with an associated prescription) and a diagram of the optical lens system. We present a complete set of information for all six patented lens systems in our detailed technical report available on the web [14]. We show a diagram for all six patented lens systems and a diagram for all six genetically evolved systems in this paper. However, because of space limitations, we present only a sampling of the other information contained in the technical report in this paper. For example, we show the prescription for the patented system only for the Tackaberry-Muller (see table 1) and we briefly describe the fitness measure used in the run of genetic programming only for the Tackaberry-Muller system (see section 4.2). We show the prescription for two of the six genetically evolved results.

As will be seen, one of the six genetically evolved lens systems infringed a previously issued patent; one was very close to infringing (the one small numerical difference apparently being due to the fact that the genetically evolved individual was an improvement on the design goals stated in the patent); three contained many of the essential features of the patents without infringing; and the others were non-infringing novel designs that duplicated (or improved upon) the performance specifications contained in the patents.

5.1 Ludewig Eyepiece

Figure 7 shows the patented Ludewig lens system [12].

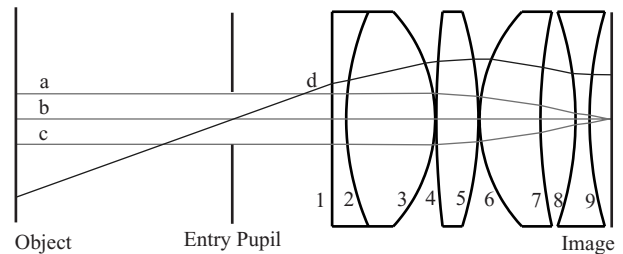


Figure 7 Ludewig patent

Figure 8 shows the best-of-run lens system created by our run of genetic programming in generation 257 on the Ludewig problem and table 2 shows the prescription (lens file) for this best-of-run individual. As can be seen, the genetically evolved system is similar to the patented Ludewig system in that it has the same number of lenses, surfaces, and multi-part lenses (one doublet

and three singlet lenses); however, the genetically evolved system is slightly different in detail.

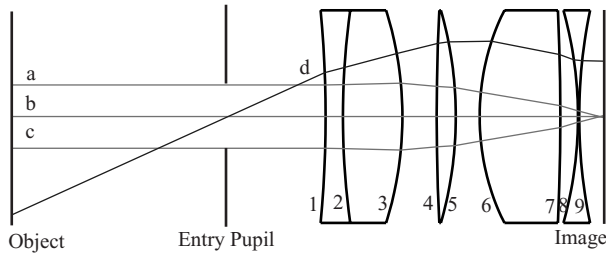


Figure 8 Best-of-run lens system from generation 257 for the Ludewig problem

Table 2 Lens file for the best-of-run individual from generation 257 for the Ludewig problem

Surface	Distance	Radius	Material	Aperture
OBJ	10^{10}	flat	air	
EP	0.7805	flat	air	0.2
2	0.13795	-5.57235	SF58	0.67183
3	0.47498	3.878076	LASFN31	0.67183
4	0.27378	-1.79665	air	0.67183
5	0.15277	10.6411	SK51	0.67183
6	0.1905	-1.95551	air	0.67183
7	0.64411	1.240546	LAK23	0.67183
8	0.14101	-11.7773	air	0.67183
9	0.01	-1.99635	LASF36A	0.67183
10	0.19280	2.640905	air	0.67183
IMS		flat		0.67183

Table 3 compares nine characteristics of the best-of-run individual from generation 257 on the Ludewig problem with those of the patented lens system. As can be seen in table 3, the best-of-run individual is a slight improvement over the patented Ludewig system [12] in our tested field of view (smaller values being better).

Table 3 Characteristics of the best-of-run individual from generation 257 for the Ludewig problem

	Ludewig	Evolved
Spherical Aberration	-0.005591	-0.004989
Coma	-0.003188	-0.002905
Astigmatism	0.004033	0.0038897
Petzval	-0.005012	-0.004795
Distortion	-0.01207	-0.0109947
Distortion Percentage	2.74	2.39
Max Distortion Percentage	2.74	2.39
Axial Chromatic	-0.001494	-0.001404
Lateral Chromatic	-0.005518	-0.005523

5.2 Koizumi-Watanabe Eyepiece

Figure 9 shows the patented Koizumi-Watanabe lens system [6].

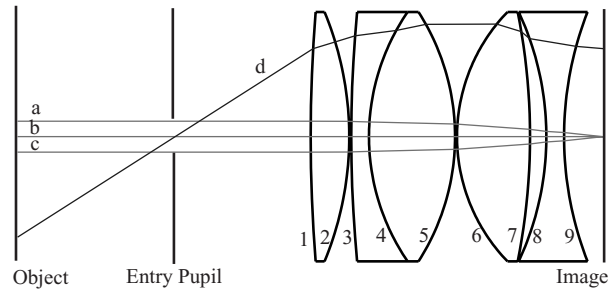


Figure 9 Koizumi-Watanabe patent

Figure 10 shows the best-of-run lens system created by our run of genetic programming in generation 295 on the Koizumi-Watanabe problem. As can be seen, the genetically evolved lens system has the same number of lenses as the patented system.

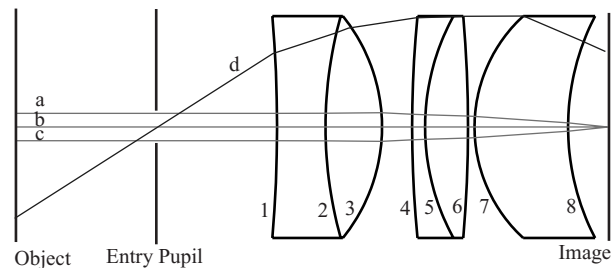


Figure 10 Best-of-run lens system from generation 295 for the Koizumi-Watanabe problem

Table 4 shows the inventors' design goals, as stated in the Koizumi-Watanabe patent. The design goals in this problem emphasized achieving a specific performance of aberration correction while maintaining a distortion level less than 6% (the perceptible level when in use) with at least a 55° wide field of view. An additional objective was minimizing the number of lenses. The evolved individual maintains the aberration corrections at the specified field of view with slightly less distortion. The genetically evolved individual here satisfies the performance and design goals of the problem, using a different topology. That is, genetic programming invented a lens system that duplicated the functionality of the patented invention in a novel way (i.e., genetic programming "engineered around" the patent).

Table 4 Design goals of the Koizumi-Watanabe patent

Koizumi-Watanabe patent	Genetically evolved
Wide field of view (specifically, greater than 55 degrees)	The field of view extends to 65 degrees
Less than 6% distortion through full field of view	5.78% is the max distortion
Less than six lens elements	The evolved individual has five lenses
Low aberration contributions	Evolved individual has performance equivalent to patented system

5.3 Nagler Eyepiece

Figure 11 shows the patented Nagler lens system [13]. This system has four groups of lenses.

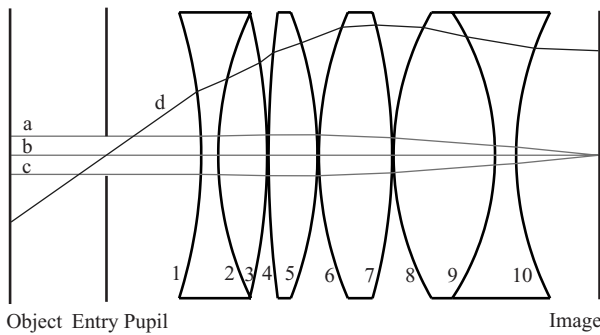


Figure 11 Nagler patent

Figure 12 shows the best-of-run lens system created by our run of genetic programming in generation 300 on the Nagler problem. This system has two groups of lenses.

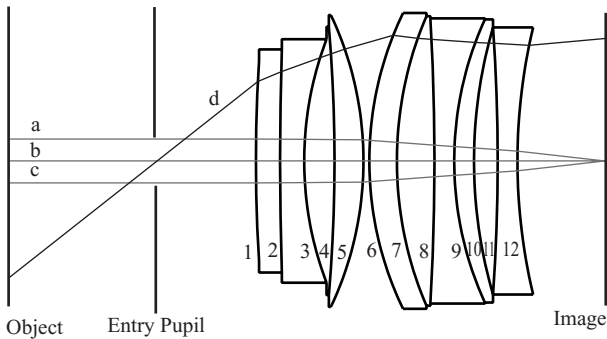


Figure 12 Best-of-run lens system from generation 300 for the Nagler problem

Table 5 shows the inventor’s design goals, as stated in the patent. The Nagler patent focused on a specific performance (low astigmatism) at an ultra-wide field of view. The genetically evolved individual used a very different topology to achieve the specified field of view with performance levels comparable to that of the patented system. The patent specifies a four-group solution consisting of a 2-1-1-2 lens configuration. As seen in figure 12, the evolved individual uses four additional lenses but compacted into a two-group solution (with four lenses in one group and six in the other).

Table 5 Design goals of the Nagler patent

Nagler Patent	Genetically evolved
Very-Wide field of view (>70 degrees)	Field of view extends to 70 degrees
Reduced Astigmatism at the edge of the field	Lens system has very low astigmatism in image
Low aberration contributions	Evolved individual has equivalent performance to embodiment
Compact six lens system	10-lenses

5.4 Scidmore Eyepiece

Figure 13 shows the patented Scidmore lens system [14].

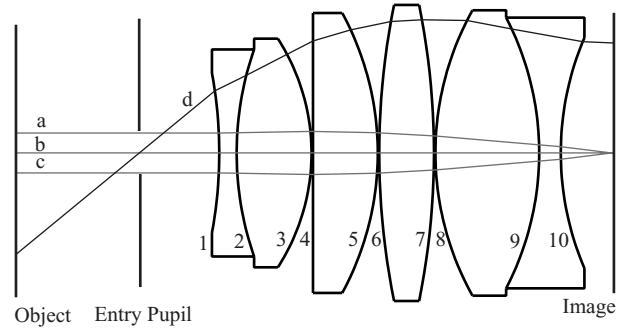


Figure 13 Scidmore patent

Figure 14 shows the best-of-run lens system created by our run of genetic programming in generation 299 on the Scidmore problem.

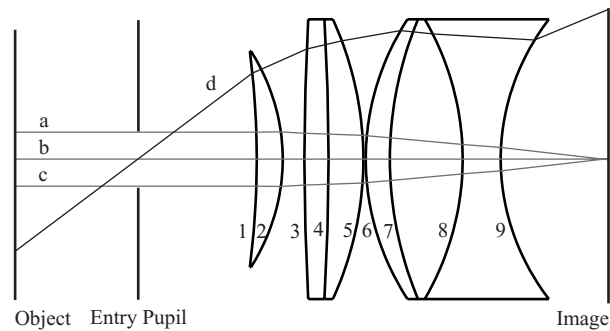


Figure 14 Best-of-run lens system from generation 299 for the Scidmore problem

Table 6 shows the inventor’s design goals, as stated in the Scidmore patent. The focus was on achieving a minimum acceptable performance level over a wide field-of-view and minimizing the surface count for manufacturing purposes.

Table 6 Design goals of the Scidmore patent

Scidmore Patent	Genetically evolved
Very-Wide field of view (>80 degrees)	Field of view extends to 80 degrees
Minimize lens elements (six lens elements in patent)	Evolved individual has six lens elements
Low aberration contributions	Evolved individual has equivalent performance to embodiment

The evolved individual achieved the field-of-view specification as well as performance equivalent to that of the patented lens system. As can be seen in figure 14, the evolved individual also uses one less lens group.

5.5 Konig and Tackaberry-Muller Eyepiece

The Tackaberry-Muller patent [16] cites the 1940 Konig patent [7] and is a special case of it. The patented Tackaberry-Muller system is shown in figure 1.

Figure 15 shows the best-of-run lens system created by our run of genetic programming in generation 490 on the Tackaberry-Muller problem.

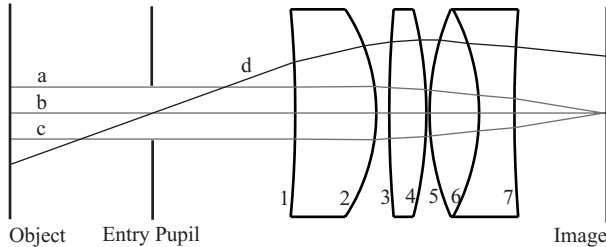


Figure 15 Best-of-run individual from generation 490 for the Tackaberry-Muller problem

The only difference (a slightly out-of-range radius of curvature) between claim 1 of the Tackaberry-Muller patent [16] and the features of the best-of-run lens system from generation 490 is apparently due to the improved performance of the genetically evolved design compared to the patented design [2].

Table 7 shows the lens file for the best-of-run individual.

Table 7 Lens file for best-of-run individual from generation 490 for the Tackaberry-Muller problem

Surface	Distance	Radius	Material	Aperture
OBJ	10^{10}	flat	Air	
EP	0.88	flat	Air	0.18
1	0.50022	-7.2605	SSK3	0.62
2	0.07953	-1.16406	Air	0.62
3	0.2298	7.5438	SSK3	0.62
4	0.02	-2.5807	Air	0.62
5	0.30591	1.6321	LAK16A	0.62
6	0.2176	-1.3486	SF58	0.62
7	0.5706	8.7356	Air	0.62
IMS		flat		

As can be seen in table 8, the best-of-run individual is a slight improvement over the design in the Tackaberry-Muller patent [16], using our fitness measure employing an exact ray trace of the marginal and chief rays for aberration calculations.

Table 9 shows that the best-of-run lens system from generation 490 infringes claim 1 of the Konig patent [7].

In summary, one of the six genetically evolved lens systems infringed the previously issued patent and three contained many of the essential features of the patents without infringing. One of the six patents was issued in the 21st-century.

Table 9 Comparison of evolved system to Konig patent

Claim 1 of Konig patent	Evolved optical system
“An optical system for telescope eyepieces, comprising a front, a medial and a rear element, said elements being convergent and axially spaced by air,”	The evolved individual contains three convergent elements (two single lenses and one doublet lens) and they are separated by air.
“the sum of the distances apart of said elements being at most one-third of the focal length of said system,”	The focal length of the evolved system is 0.9958 and the sum of the distances is 0.099531 (approximately 1/10).
“said rear element being a single lens, the numerical value of the curvature of the rear surface of said lens being smaller than the numerical value of the refractive power of said lens,”	The curvature of the rear element (the lens defined by surfaces 1 and 2) is 0.1377 (i.e., $1/7.260474245$) and its refractive power is 0.443518 (computed by the standard textbook formula).
“said medial element consisting of at least one lens and at most two lenses,”	The medial element (the lens defined by surfaces 3 and 4) is a single lens.
“said front element consisting of at least one lens,”	The front element (surfaces 5, 6, and 7) consists of two lenses.
“the front lens of said medial element and that lens of said front element which faces this front lens of said medial element being convergent,”	These two lenses (namely the medial lens defined by surfaces 3 and 4 and the portion of the doublet defined by surfaces 5 and 6) are both convergent.
“at least one optically effective surface of one of said two convergent lenses being a cemented surface,”	The doublet lens defined by surfaces 5, 6, and 7 has a common surface 6.
“the refractive power of one cemented surface of said two convergent lenses amounting to at least eleven twentieths of the algebraic sum of the refractive powers of all cemented surfaces of said convergent lenses,”	The refractive power of the specified surface is 0.13652. It is also the only common surface and hence amounts to the entire sum described.
“the numerical value of last said sum being greater than one twelfth of the sum of the numerical values of the curvatures of those surfaces of said convergent lenses which face each other.”	It is equal to 0.136492 which is greater than one twelfth.

One of the eight criteria presented in [9] for saying that an automatically created result is “human-competitive” is

“The result was patented as an invention in the past, is an improvement over a patented invention, or would qualify today as a patentable new invention.”

Based on this definition, we claim that the six genetically evolved results in this paper are instances of “human-competitive” results produced by genetic programming.

Table 8 Characteristics of the best-of-run individual from generation 490 for the Tackaberry-Muller patent

	Tackaberry -Muller	Evolved
Spherical aberration	-0.003999	-0.003103
Coma	-0.002829	-0.002496
Astigmatism	0.002817	0.002788
Petzval	-0.006505	-0.006354
Distortion	-0.009907	-0.009244
Distortion percentage	2.4166	1.8344
Maximum distortion percentage	2.4166	1.8344
Axial chromatic	-0.001122	-0.0007371
Lateral chromatic	-0.002213	-0.001904

6 CONCLUSION

This paper described how genetic programming was used to automatically synthesize complete designs for optical lens systems. The automated synthesis was done “from scratch”—that is, without starting from a pre-existing good design and without pre-specifying the number of lenses, the physical layout of the lenses, the numerical parameters of the lenses, or the non-numerical parameters of the lenses. The evolved lens systems duplicated the functionality (or infringed) six patented lens systems, including one 21st-century patent.

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