Water Distribution Systems Optimal Design Using Cross Entropy

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ABSTRACT
This paper describes the methodology and application of Cross Entropy (CE) to the optimal design problem of a water distribution system (WDS). The CE method is a new powerful evolutionary iterative technique based on the concept of rare events (or the Kullback-Leibler distance measure of information). The optimal design problem of a WDS is to find its component characteristics (e.g., pipe diameters, pump heads and maximum power) which minimize its capital and operational costs such that the system hydraulic laws are maintained (i.e., Kirchoff's Laws No. 1 and 2), and constraints on quantities and pressures at the consumer nodes are fulfilled. The CE methodology is demonstrated using a well known bench-mark problem reported in the WDSs research literature, reaching the best solution already obtained and suppressing the computational effort required to achieving it.

Categories and Subject Descriptors
J.2 [**Computer Applications**]: Physical Science and Engineering – *engineering.* J.6 [**Computer Applications**]: Computer-Aided Engineering – *computer –aided design (CAD).* G.3 [**Mathematics of Computing**]: Probability and Statistics – *probabilistic algorithms, random number generator.*

General Terms
Algorithms, Performance, Design.

Keywords
Combinatorial Optimization, Cross-Entropy, Water Distribution Systems, Optimal Design, Water Resources.

1. INTRODUCTION
A WDS is a collection of hydraulic control elements connected together to convey quantities of water from sources to consumers. The behavior of a WDS is governed by: (1) the physical laws which describe flow and quality distributions; (2) the consumer's demands; and (3) the system layout. The problem of WDS optimal design attracted numerous papers over the last four decades, concentrating on two main schemes: the linear programming gradient (LPG) approach introduced by Alperovits and Shamir [1] in which an "inner" linear programming problem is solved for a fixed set of flows in the pipes, while the flows are altered at an "outer" problem using a gradient type scheme; and the general genetic algorithm (GA) approach [e.g., Savic and Walters, 4]. This paper describes the methodology and application of CE [6,7] to the optimal design problem of a WDS.

GECCO'05, June 25-29, 2005, Washington, DC, USA.
ACM 1-59593-010-8/05/0006.

The CE method utilizes a supplementary random mechanism which translates the combinatorial optimization problem (COP) into an associate stochastic problem (ASP) by randomizing the original deterministic problem. The CE algorithm involves two main stages in each of the ASP iterations: (1) generation of a random sample data according to the supplementary random mechanism and the calculation of the associated objective function, and (2) updating the parameters of the ASP on the basis of the data of stage (1) in the direction of solutions improvements. The CE algorithm employs a discrete distribution which converges to unities and zeros, where the associated unit elements of the final distribution uniquely define the optimal decision variables of the problem in hand.

2. CE METHOD FOR OPTIMIZATION
The method utilizes its rational from simulation techniques for the estimation of rare events probabilities:

$$\text{Estimate:} \qquad \mathbf{P}_p(S(x) \geq \gamma) \tag{1}$$

where x is a random vector with probability distribution p on the set χ, S is a performance function on χ and γ is some real value. Next, the CE method is employed as an optimization method:

$$\text{Determine:} \qquad \max_{x \in \chi} S(x) \tag{2}$$

The corresponding state at which the maximum is reached is indicated by:

$$\max_{x \in \chi} S(x) = S(x^*) = \gamma^* \tag{3}$$

Instead of finding the optimal solutions x^* to a particular problem directly, the CE method aims to find the most favorable sampling distribution p^*. This distribution is considered to be optimal if only optimal (or near optimal) solutions can be generated from it. The CE process entails the following general iterative procedure:

1. Choosing an initial reference vector $\hat{p}_0$ in the ASP with some components $P(X = x_i) = p_i, \quad i = 1,...,m$. Set $t = 1$.
2. Generate N sample vectors X_i, $i = 1,...N$ using a predefined random mechanism with $\mathrm{p} = \hat{\mathrm{p}}_{t-1}$ and compute the value of γ_t to be the $(1-\rho)$ evaluation of $S(X)$, where ρ is a parameter in the range of: $10^{-2} \leq \rho \leq 10^{-1}$.

3. For a fixed γ_{t-1} obtain the probability distribution vector $\hat{p}_t$ from the solution of: $\max_p \frac{1}{N}\sum_{i=1}^{N} I_{(S(x_i)\geq\gamma)} \ln f(x_i, p)$.
4. Smooth the probability distribution vector using a smoothing parameter $0.3 \leq \alpha \leq 0.9$ through: $\hat{p}_t = \alpha \cdot \hat{p}_t + (1-\alpha)\cdot \hat{p}_{t-1}$.
5. Check if stopping criterion is met, for example: if for some $t \geq d$ $\hat{\gamma}_t = \hat{\gamma}_{t-1} = ... = \hat{\gamma}_{t-d}$ stop; Set $t = T$ - final iteration and $\hat{\gamma}_t$ as the estimate of γ^*; otherwise - set $t = t+1$.

3. NETWORK DESIGN PROBLEM

The physical behavior of a WDS is governed by a set of linear and non-linear equations, which includes energy and mass conservation equations and the head loss formulas. In addition, a set of constraints are defined which consist of design constraints, minimum pressure requirements and the delivery of the prescribed demand flows. Out of the many different possible designs that can be randomly generated by the CE algorithm, only few can actually form the design of the network. Given that non-feasible solutions can not constitute the design of the network, only the feasible solutions are candidates. This requires the definition of supplementary mechanisms that can distinguish the non-feasible solutions from the feasible ones, and guarantee that only the available pipe diameters can uniquely be chosen for each pipe. These are incorporated in the main CE algorithm for solving the optimal WDS design problems. The CE method for minimal cost design is demonstrated on the two-looped network (Figure 1) introduced by Alperovits and Shamir [1], which was used intensively as a bench-mark problem.

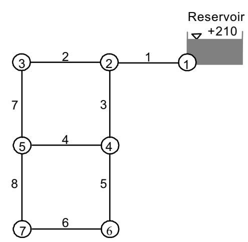

Figure 1. Two Looped Network

The layout of the network can be viewed as a graph $G(V,E)$, with the full system data in [1]. Each one of the eight pipes in the network can have one of 14 commercially available pipe diameters with its corresponding unit costs. The entire set of available diameters is incorporated by adding fictitious nodes into the system with every fictitious node representing its corresponding pipe diameter, randomizing the network at the nodes. The selection of the diameters is represented by the partition of the nodes into two subsets $V = \{V_1, V_2\}$, where $\{V_1\}$ holds the non-selected nodes and $\{V_2\}$ holds the selected nodes. The partition vector is then associated with the reference parameter, which is defined as a random vector X with independent components and the discrete probability distribution of $P(X = x_i) = p_i, \ \forall i$.

4. NUMERICAL RESULTS

The CE optimization method was applied to three well explored bench-mark water distribution systems: the Two Looped network [1], the Hanoi network [2], and the New York Tunnels system [5]. The CE algorithm developed herein is demonstrated on the two-looped problem. The CE parameters used, were: sample size of $N = 4480$, $\rho = 0.01$ and $\alpha = 0.7$. Defining the converging criterion for the probabilities as $p_i < 10^{-5}$ or $1 - p_i < 10^{-5}$ the average number of iterations until convergence was 8. The number of the objective function evaluations was approximately 15,000 whereas when using GA it was around 10^6 [4]. The optimal solution found was $419,000 which coincides with the best solution reported using GA and AC [3,4] and is shown in Table 1. The probabilities were initialized to be: $p_i = 1/14$. As the algorithm evolves the probability distribution approaches to unities and zeros as shown in Figure 2.

Table 1. Two-Looped Network – Optimal Design

Link	1	2	3	4	5	6	7	8
Diameter [inch]	18	10	16	4	16	10	10	1

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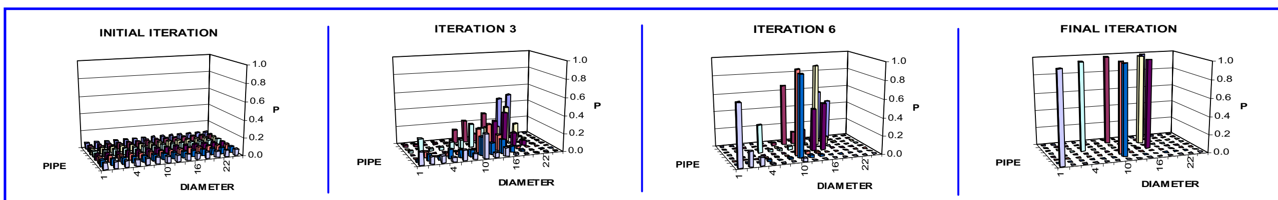

Figure 2: Updating probabilities