Comparison of Evolutionary Multiobjective Optimization with Reference Solution-Based Single-Objective Approach

Hisao Ishibuchi
Graduate School of Engineering
Osaka Prefecture University
Sakai, Osaka 599-8531, Japan
+81-72-254-9350
hisaoi@cs.osakafu-u.ac.jp

Kaname Narukawa
Graduate School of Engineering
Osaka Prefecture University
Sakai, Osaka 599-8531, Japan
+81-72-254-9351
kaname@ci.cs.osakafu-u.ac.jp

ABSTRACT
In this paper, we demonstrate advantages and disadvantages of an evolutionary multiobjective optimization (EMO) approach in comparison with a reference solution-based single-objective approach through computational experiments on multiobjective 0/1 knapsack problems. The main characteristic feature of the EMO approach is that no *a priori* information about the decision maker's preference is assumed. The EMO approach tries to find well-distributed trade-off solutions with a wide range of objective values as many as possible. A final solution is supposed to be chosen from the obtained trade-off solutions by the decision maker. On the other hand, the reference solution-based approach utilizes the information about the decision maker's preference in the form of a reference solution. We examine whether the EMO approach can find good trade-off solutions close to an arbitrarily given reference solution. Experimental results show that good solutions are not always obtained by the EMO approach. We also examine where the reference solution-based approach can find many trade-off solutions around the given reference solution. Experimental results show that many trade-off solutions can not be obtained even when an archive population of non-dominated solutions is stored in the reference solution-based approach. Based on these observations, we suggest a hybrid approach.

Categories and Subject Descriptors
I.2.8 [**Artificial Intelligence**]: Problem Solving, Control Methods, and Search – *Heuristic Methods.*

General Terms
Algorithms.

Keywords
Evolutionary multiobjective optimization (EMO), genetic algorithms, decision maker's preference, reference solutions, diversity-preserving strategies, NSGA-II.

1. INTRODUCTION
Evolutionary multiobjective optimization (EMO) is a very active research area in the field of evolutionary computation. Since Schaffer's pioneering work [10], various EMO algorithms have been proposed in the literature (e.g., see [1], [2]). Those EMO algorithms are designed to find a large number of well-distributed trade-off solutions with a wide range of objective values. Usually no *a priori* information about the decision maker's preference is utilized when EMO algorithms search for trade-off solutions. A final solution is supposed to be chosen from the obtained trade-off solutions by the decision maker. The EMO approach is called an ideal multiobjective optimization procedure in Deb [2]. It is implicitly assumed that the choice of a final solution from the obtained trade-off solutions is much easier for the decision maker than the elicitation of his/her preference in advance. On the other hand, multiobjective optimization problems can be handled in the framework of single-objective optimization when the information about the decision maker's preference is available in advance. For example, when a reference solution is given as an ideal solution in the objective space, the distance from the reference solution can be used as a single objective function to be minimized (while it does not necessarily reflect the decision maker's preference truly).

It is essential for the success of the EMO approach to find a large number of well-distributed trade-off solutions with a wide range of objective values. In this paper, we examine whether EMO algorithms can find good trade-off solutions close to an arbitrarily given reference solution through computational experiments on a multiobjective 0/1 knapsack problem. In each trial of our computational experiment, first a reference solution is generated in the objective space. Then a single-objective genetic algorithm is used to minimize the distance from the reference solution. An EMO algorithm is also used to find a large number of well-distributed trade-off solutions without utilizing the information about the reference solution. Finally the obtained solution by the single-objective genetic algorithm is compared with the obtained trade-off solutions by the EMO algorithm. While we mainly use the NSGA-II algorithm [3] as a representative EMO algorithm, we also examine the performance of SPEA [11], M-PAES [9], and MOGLS [7], [8]. It is visually shown that these state-of-the-art EMO and memetic EMO algorithms can not always find good trade-off solutions close to an arbitrarily given reference solution.

It is not always easy (usually very difficult) for the decision maker to specify a reference solution in advance. Thus the specified reference solution does not always reflect the decision maker's preference truly in the reference solution-based single-objective approach. Moreover the distance from the preference solution does not necessarily reflect the decision maker's preference relation, either. The decision maker may want to see multiple trade-off solutions around the (tentatively specified) reference solution. From this viewpoint, we also examine whether multiple trade-off solutions can be obtained by the reference solution-based single-objective approach. An archive population of non-dominated solutions is stored separately from the standard main population in our single-objective genetic algorithm. A similarity-based mating scheme of Ishibuchi & Shibata [5], [6] is also incorporated in our single-objective genetic algorithm to increase the diversity of solutions in the objective space. Experimental results show that our single-objective genetic algorithm can not find multiple trade-off solutions with a wide range of objective values around the reference solution even when the archive population and the mating scheme are incorporated. Experimental results also show that the obtained trade-off solutions are misleading because some of them are not close to the Pareto front. In order to find a large number of good trade-off solutions around the reference solution, we combine the single-objective fitness function defined by the distance from the reference solution with the NSGA-II algorithm. More specifically, the single-objective fitness function is used to choose parent solutions for recombination in our hybrid algorithm while the fitness evaluation mechanism in the original NSGA-II algorithm is still used for generation update. Experimental results show that a large number of good trade-off solutions around the reference solution are obtained by our hybrid algorithm.

2. TWO APPROACHES

2.1 Evolutionary Multiobjective Optimization

A k-objective maximization problem can be written as follows:

$$\text{Maximize } \mathbf{f}(\mathbf{x}) = (f_1(\mathbf{x}), \ ..., \ f_k(\mathbf{x})) \text{ subject to } \mathbf{x} \in \mathbf{X}, \tag{1}$$

where $\mathbf{f}(\mathbf{x})$ is the k-dimensional objective vector, $\mathbf{x}$ is the decision vector, and $\mathbf{X}$ is the feasible region in the decision space. When the following relation holds between two solutions $\mathbf{x}$ and $\mathbf{y}$, $\mathbf{x}$ is said to be dominated by $\mathbf{y}$ (i.e., $\mathbf{y}$ is better than $\mathbf{x}$):

$$\forall i, \ f_i(\mathbf{x}) \le f_i(\mathbf{y}) \quad \text{and} \quad \exists j, \ f_j(\mathbf{x}) < f_j(\mathbf{y}). \tag{2}$$

When there is no feasible solution in $\mathbf{X}$ that dominates $\mathbf{x}$, $\mathbf{x}$ is referred to as a Pareto-optimal solution. The set of objective vectors corresponding to all Pareto-optimal solutions is referred to as Pareto front. The task of EMO algorithms is to find a large number of well-distributed Pareto-optimal solutions with a wide range of objective values over the Pareto front. It is, however, unpractical to search for true Pareto-optimal solutions of large-scale multiobjective combinatorial optimization problems. In this case, EMO algorithms search for near Pareto-optimal solutions. A set of non-dominated solutions among examined ones during the execution of an EMO algorithm is presented to the decision maker.

In the EMO approach, a single final solution is determined by the following two-step procedure, which is referred to as an ideal multiobjective optimization procedure in Deb [2]:

Step 1: Find well-distributed trade-off solutions with a wide range of objective values using an EMO algorithm.
Step 2: Choose one of the obtained trade-off solutions.

In the second step, the decision maker is supposed to choose one of the obtained trade-off solutions based on his/her preference. It should be noted that no *a priori* information about the decision maker's preference is utilized in the first step.

The success of the EMO approach depends on the quality of the obtained trade-off solutions in the first step. Each trade-off solution should be close to the Pareto front. Moreover, the trade-off solutions should have a large diversity. Thus there are two sub-goals in the design of EMO algorithms: to improve the convergence of solutions to the Pareto front and to increase the diversity of solutions. Recently developed EMO algorithms usually use Pareto ranking, elitism and diversity-preserving mechanisms to improve both the convergence and the diversity.

2.2 Reference Solution-Based Approach

When a reference solution $\mathbf{x}^*$ is available from the decision maker, we can formulate the following single-objective minimization problem from the k-objective maximization problem in (1):

$$\text{Minimize } d(\mathbf{f}(\mathbf{x}), \ \mathbf{f}(\mathbf{x}^*)) \text{ subject to } \mathbf{x} \in \mathbf{X}, \tag{3}$$

where $d(\ )$ measures the distance between the decision vector $\mathbf{x}$ and the reference solution $\mathbf{x}^*$ in the objective space. In this paper, we use the Euclidean distance:

$$d(\mathbf{f}(\mathbf{x}), \ \mathbf{f}(\mathbf{x}^*)) = \sqrt{|\ f_1(\mathbf{x}) - f_1(\mathbf{x}^*)\ |^2 + \ \cdots + |\ f_k(\mathbf{x}) - f_k(\mathbf{x}^*)\ |^2}\ . \tag{4}$$

A single final solution of the k-objective optimization problem in (1) is obtained by the following procedure, which is referred to as a reference solution-based single-objective approach in this paper:

Step 1: Specify a reference solution $\mathbf{x}^*$ in the objective space.
Step 2: Find an optimal or near optimal solution of the single-objective optimization problem in (3).

The success of this approach depends on whether the given reference solution truly reflects the decision maker's preference or not. It is, however, not always easy (usually very difficult) for the decision maker to specify a reference solution for a multiobjective optimization problem in advance.

3. COMPARISON OF TWO APPROACHES

3.1 Motivations

The above-mentioned two approaches use different ideas to obtain a single final solution. It would be interesting to compare the two approaches through computational experiments from the viewpoint of the quality of the obtained final solution. In each trial of our computational experiment, we assume that a reference solution is given. The performance of the EMO approach is evaluated by examining whether EMO algorithms can find good trade-off solutions close to an arbitrarily given reference solution. Of course, no information about the given reference solution is utilized when EMO algorithms search for trade-off solutions.

On the other hand, the success of the reference solution-based approach depends on the given reference solution itself. Since it is usually very difficult for the decision maker to specify a reference solution in advance, we assume that the given reference solution does not truly reflect the decision maker's preference. The actual ideal solution for the decision maker may be different from the given reference solution. In this situation, it is desirable to present a number of good trade-off solutions around the given reference solution to the decision maker. The obtained trade-off solutions may give the decision maker some information about the trade-off structure in the multiobjective optimization problem, which in turn facilitates the specification of another reference solution closer to his/her actual ideal solution . Alternatively, the decision maker can choose one of the obtained trade-off solutions. The chosen solution is not necessarily the best solution in terms of the distance from the given reference solution. We evaluate the reference solution-based approach by examining whether a number of good trade-off solutions with a wide range of objective values can be obtained around the given reference solution.

3.2 Settings of Computational Experiments

In order to visually demonstrate advantages and disadvantages of each approach, we use a two-objective 500-item 0/1 knapsack problem of Zitzler & Thiele [11]. We choose this test problem for the following reasons: (i) this is a relatively large combinatorial optimization problem with the search space size of 2^{500}, and (ii) true Pareto-optimal solutions are known for this problem [11].

A multiobjective 0/1 knapsack problem with k knapsacks (i.e., k objectives and k constraints) and n items in Zitzler & Thiele [11] can be written as follows:

$$\text{Maximize } \mathbf{f}(\mathbf{x}) = (f_1(\mathbf{x}),\ f_2(\mathbf{x}),\ ...,\ f_k(\mathbf{x}))\,, \tag{5}$$

$$\text{subject to } \sum_{j=1}^{n} w_{ij} x_j \le c_i\,,\quad i = 1, 2, ..., k\,, \tag{6}$$

$$\text{where } f_i(\mathbf{x}) = \sum_{j=1}^{n} p_{ij} x_j\,,\quad i = 1, 2, ..., k\,. \tag{7}$$

In this formulation, $\mathbf{x}$ is an n-dimensional binary vector, p_{ij} is the profit of item j according to knapsack i, w_{ij} is the weight of item j according to knapsack i, and c_i is the capacity of knapsack i. Each solution $\mathbf{x}$ is handled as a binary string of length n.

When an EMO algorithm is applied to the multiobjective 0/1 knapsack problem in (5)-(7), genetic operations often generate infeasible solutions that do not satisfy the constraint conditions in (6). We use a repair method based on a maximum profit/weight ratio as Zitzler & Thiele [11] in all EMO algorithms in this paper. When an infeasible solution is generated, a feasible solution is created by removing items (i.e., by changing the corresponding values in the binary string $\mathbf{x}$ from 1 to 0) in the ascending order of the maximum profit/weight ratio defined as follows:

$$q_j = \max\{p_{ij} / w_{ij} \mid i = 1, 2, ..., k\}\,,\quad j = 1, 2, ..., n\,. \tag{8}$$

On the other hand, we use the following weighted profit/weight ratio in the reference solution-based single-objective approach:

$$q_j = \sum_{i=1}^{k} a_i p_{ij} \Big/ \sum_{i=1}^{k} w_{ij}\,,\quad j = 1, 2, ..., n\,, \tag{9}$$

where a_i can be viewed as the importance of the i-th objective. We will explain the specification of a_i later in this paper.

In our computational experiments, first a single-objective genetic algorithm is used for independently optimizing each objective of the two-objective 500-item knapsack problem. That is, the single-objective genetic algorithm is applied to two single-objective 0/1 knapsack problems, each of which has one of the two objectives and the same constraint conditions as the original two-objective problem. The weighted profit/weight ratio repair is used in the two single-objective 0/1 knapsack problems where the weight vector $(a_1,\ a_2)$ is specified as (1, 0) and (0, 1), respectively.

Let us denote the two solutions as $\mathbf{x}^1$ and $\mathbf{x}^2$ where $\mathbf{x}^i$ is the obtained solution by maximizing the i-th objective. Using the two solutions, we normalize the objective space as follows:

$$[f_1(\mathbf{x}^2),\ f_1(\mathbf{x}^1)] \times [f_2(\mathbf{x}^1),\ f_2(\mathbf{x}^2)] \Rightarrow [0,\ 2] \times [0,\ 2]\,. \tag{10}$$

This normalization is illustrated in Figure 1 where the obtained two solutions $\mathbf{x}^1$ and $\mathbf{x}^2$ are denoted by open circles. The locations of these solutions in the objective space are transformed by Eq. (10) to (2, 0) and (0, 2) in the normalized objective space as shown in Figure 1. We use this normalization in order to systematically specify reference solutions in the normalized objective space. In computational experiments, we examine the following five specifications of the reference solution in the normalized objective space: (4, 0), (3, 1), (2, 2), (1, 3), (0, 4). Each specification of the reference solution in the normalized objective space is shown by a closed circle in Figure 1. It should be noted that each element of the reference solution in the normalized objective space can be viewed as being the relative importance of the corresponding objective. Thus we use the reference solution as the weight vector $(a_1,\ a_2)$ in the weighted profit/weight ratio repair of the reference solution-based approach. The Euclidean distance from the reference solution is measured in the normalized objective space in the reference solution-based approach. It should be noted that the specification mechanism of the reference solution in the normalized objective space can be easily extended to the case of k objectives ($k > 2$) using the following conditions:

$$\hat{f}_i(\mathbf{x}^*) \in \{0,\ 1,\ 2,\ ...,\ 2k\}\,,\quad i = 1,\ 2,\ ...,\ k\,, \tag{11}$$

$$\hat{f}_1(\mathbf{x}^*) + \hat{f}_2(\mathbf{x}^*) + \cdots + \hat{f}_k(\mathbf{x}^*) = 2k\,, \tag{12}$$

where $\mathbf{x}^*$ is the reference solution and $\hat{f}_i(\mathbf{x}^*)$ is the normalized reference objective value of the i-th objective.

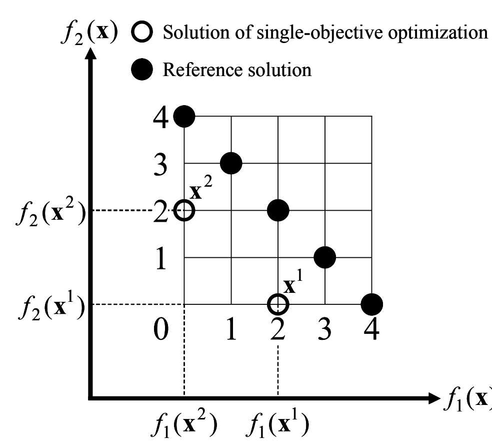

Figure 1. Normalization of the objective space and the five reference solutions used in our computational experiments.

We use the total number of generations as the stopping condition in both the EMO and reference solution-based approaches. Let N be the total number of generations in the EMO approach. That is, an EMO algorithm is terminated after the N-th generation. On the other hand, each execution of the single-objective genetic algorithm is terminated after the $N/3$-th generation in the reference solution-based approach. This is because the single-objective genetic algorithm is invoked three times in a single trial of the reference solution-based approach for the two-objective test problem (i.e., two times for the individual optimization of each objective and once for the distance minimization from the reference solution). We use the following genetic operations in both the EMO and reference solution-based approaches:

One point crossover with the probability of 0.8,
Bit-flip mutation with the probability of 0.002 (i.e., 1/500).

3.3 Performance of EMO Approach

We apply the NSGA-II algorithm of Deb et al. [3] to the two-objective 500-item 0/1 knapsack problem with the following parameter specifications:

Population size: 300,
Stopping condition: 1200 generations.

Thus 360,000 solutions are examined during the execution of the NSGA-II algorithm. We examine the average performance over independent 100 runs. In Figure 2, we show the 50% attainment surface [4] over the 100 runs, a typical solution set by a single run, and the Pareto front.

On the other hand, the single-objective genetic algorithm is used in the reference solution-based approach with the same population size and the stopping condition of 400 generations. As we have explained in the previous subsection, the five specifications of the reference solution are examined in the reference solution-based approach. For each specification, the average result is calculated over 100 independent runs in the objective space. In Figure 3, we show the average objective vector corresponding to each specification of the reference solution. The Pareto front is also shown in Figure 3 for comparison.

From the comparison between Figure 2 and Figure 3, we can see that good trade-off solutions for the decision maker were obtained by the NSGA-II algorithm only when the reference solution in the normalized objective space was (2, 2); see Figure 1. In the other four cases, good trade-off solutions for the decision maker were not obtained. In these cases, the decision maker has to choose a final solution that is not close to the reference solution. This means that the EMO approach does not work well in these cases.

We also examine the use of SPEA [11], M-PAES [9], and MOGLS [7], [8] in the EMO approach instead of the NSGA-II algorithm. We use the same computation load in each algorithm (i.e., examination of 360,000 solutions). Parameter specifications in each algorithm are summarized in Table 1. The average performance of each algorithm is calculated over 100 independent runs. The obtained 50% attainment surface by each algorithm is shown in Figure 4. From the comparison between Figure 3 and Figure 4, we can see that no algorithms in Figure 4 (and also in Figure 2) were comparable to the reference solution-based approach in terms of the proximity of obtained trade-off solutions to an arbitrarily given reference solution.

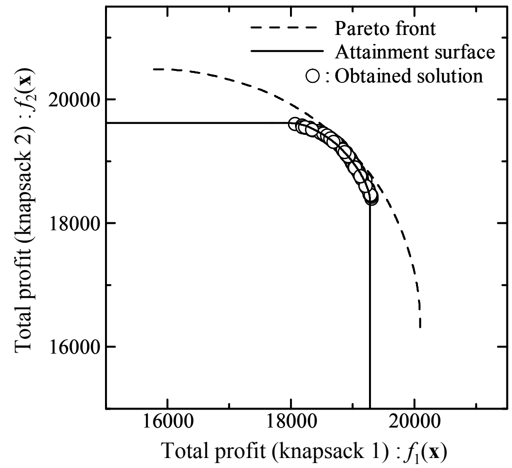

Figure 2. Experimental results by the NSGA-II algorithm.

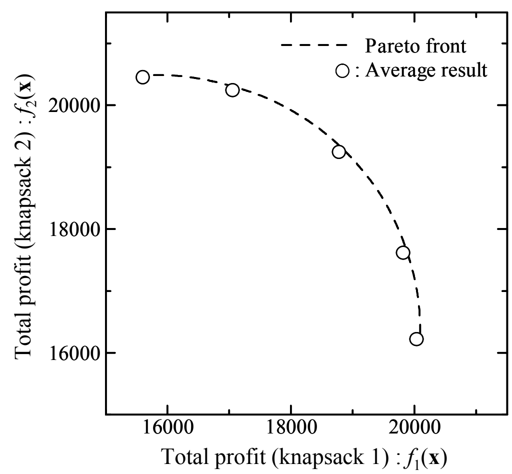

Figure 3. Experimental results by the reference solution-based approach.

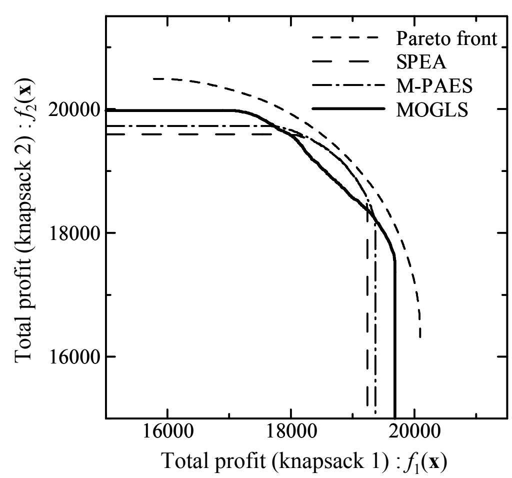

Figure 4. Experimental results by other EMO algorithms.

Table 1. Parameter specifications.

	SPEA	M-PAES	MOGLS
Population size	300	60	6000
Initial population size	300	60	300
Archive population size	75	Unlimited	Unlimited
Local archive population size	N.A.	240	N.A.

In Table 2, we compare the average CPU time of each algorithm. All algorithms were implemented in C and executed on a PC with a Pentium IV 3.2 GHz processor under the same computational load in terms of the total number of examined solutions. We can see from Table 2 that the reference solution-based approach has an advantage over many EMO algorithms in terms of CPU time. This is because EMO algorithms in general need more CPU time for evaluating each solution than single-objective algorithms.

Table 2. Average CPU time (Seconds).

Reference solution	EMO Approach			
	NSGA-II	SPEA	M-PAES	MOGLS
12.0	35.2	11.1	23.2	26.4

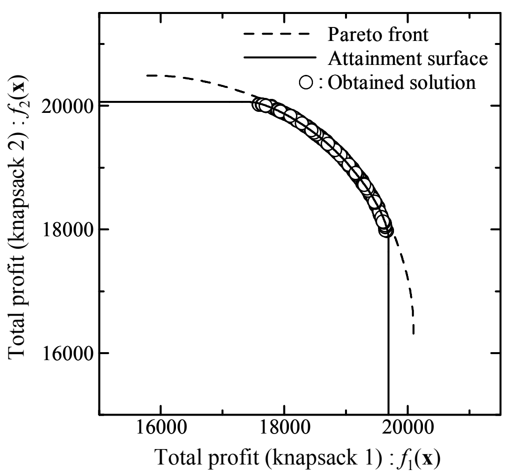

Figure 5. Experimental results by the NSGA-II algorithm with 10 times larger computational load than the other experiments.

We further examine the performance of the NSGA-II algorithm under much more computational load. More specifically, we continue the execution of the NSGA-II algorithm until 12000 generations (10 times larger computational load than the above-mentioned computational experiments). The 50% attainment surface over 100 independent runs and a typical solution set by a single run are shown in Figure 5 together with the Pareto front. From the comparison between Figure 3 and Figure 5, we can see that the NSGA-II algorithm could not still find good trade-off solutions for an arbitrarily given reference solution.

From Figure 2 and Figure 5, one may think that an additional diversity-increasing mechanism seems to be needed in the NSGA-II algorithm. We incorporate a similarity-based mating scheme of Ishibuchi & Shibata [5], [6] into the NSGA-II algorithm. Their mating scheme is illustrated in Figure 6. First the standard binary tournament scheme is iterated for choosing α candidates for the first parent (i.e., Parent A) and β candidates for the second parent (i.e., Parent B). Next the average solution of the α candidates is calculated in the objective space. Then the most dissimilar solution from the average solution is selected as Parent A among the α candidates. Finally the most similar solution to Parent A is chosen as Parent B from the β candidates. The similarity between two solutions is measured by the Euclidean distance in the objective space.

Experimental results with the stopping condition of 1200 generations are shown in Figure 7 (50% attainment surface over 100 independent runs and a typical solution set by a single run) where two specifications are examined in the mating scheme: (α, β) = (5, 5), (10, 10). While the mating scheme increased the diversity of solutions in Figure 7 from Figure 2 by the original NSGA-II algorithm, the EMO approach still could not find good trade-off solutions close to an arbitrarily given reference solution.

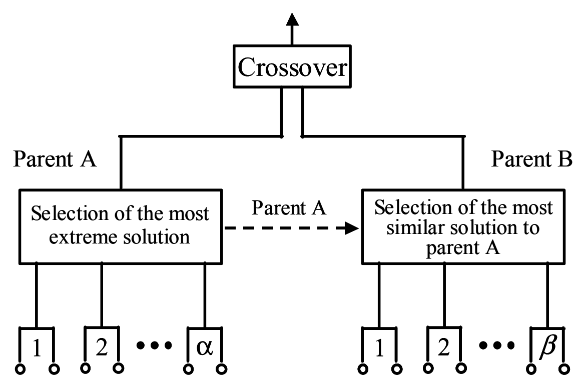

Figure 6. Similarity-based mating scheme in [5], [6].

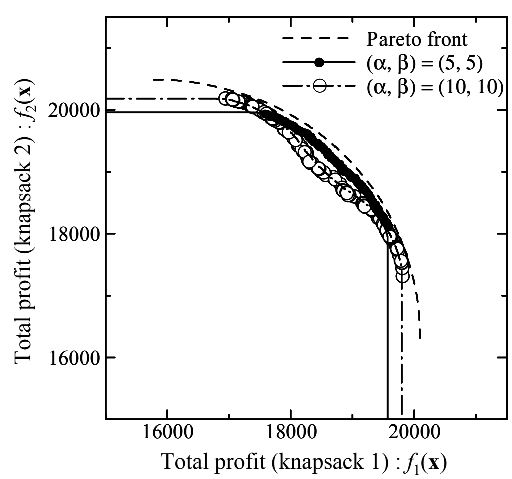

Figure 7. Experimental results using the mating scheme.

3.4 Performance of Reference Solution-Based Approach

As shown in Figure 3 and Table 2, we can efficiently find a good trade-off solution close to an arbitrarily given reference solution using the reference solution-based approach. The main difficulty in this approach is that the given reference solution does not always reflect the decision maker's preference truly. Thus the decision maker may want to see multiple trade-off solutions around the reference solution.

From this viewpoint, we examine whether the reference solution-based approach can find multiple trade-off solutions around the reference solution. We combine an archive population with the single-objective genetic algorithm that minimizes the distance from the reference solution. Non-dominated solutions among the examined ones are stored in the archive population with no size limitation during the execution of the single-objective genetic algorithm. The role of the archive population is just to store non-dominated solutions. We calculate the 50% attainment surface over 100 independent runs for each specification of the reference solution. Experimental results are shown in Figure 8 together with a typical solution set by a single run for each specification of the reference solution. From this figure, we can see that the obtained trade-off solutions did not have a large diversity.

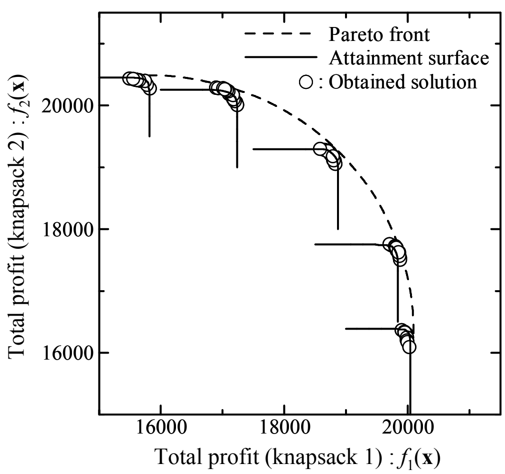

Figure 8. Experimental results by the reference solution-based approach with the archive population.

We also incorporate the above-mentioned similarity-based mating scheme into the single-objective genetic algorithm with the archive population. Experimental results are shown in Figure 9 where the two parameters in the mating scheme are specified as (α, β) = (10, 10). From the comparison between Figure 8 and Figure 9, we can see that the diversity of the obtained trade-off solutions was increased by the mating scheme. Experimental results in Figure 9, however, may give incorrect information about the trade-off structure to the decision maker. This is because some of the obtained trade-off solutions are far from the Pareto front. It should be noted in Figure 9 (and Figure 8) that the reference solution-based approach presents a single solution set to the decision maker corresponding to the given reference solution while we simultaneously show five solution sets corresponding to the five specifications of the reference solution in Figure 9.

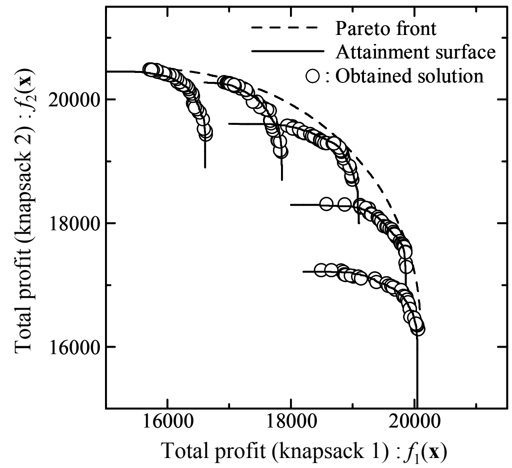

Figure 9. Experimental results by the reference solution-based approach with the archive population and the mating scheme.

4. HYBRID APPROACH

Before suggesting a hybrid version of the EMO and reference solution-based approach, we illustrate each approach again using the two-objective 500-item 0/1 knapsack problem. For illustration purpose, let us assume that the reference solution is incorrectly given as (16000, 21000) in the objective space by the decision maker while his/her true ideal solution is (17000, 21000). These two solutions are shown in Figure 10 as G and T, respectively.

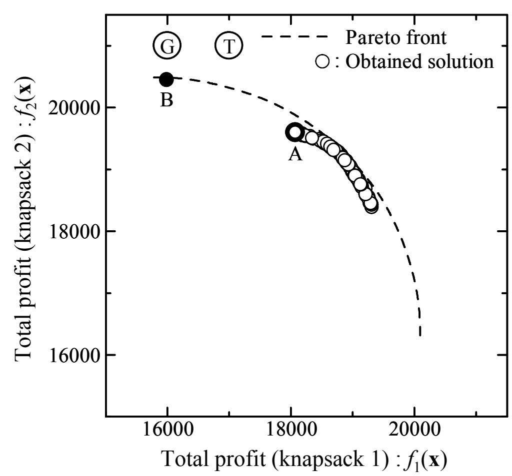

Figure 10. The given reference solution G, the true ideal solution T, the final solution A by the EMO approach, and the final solution B by the reference solution-based approach.

First we apply the original NSGA-II algorithm to the two-objective knapsack problem without using the information about the given reference solution G. The NSGA-II algorithm is executed for 1200 generations in the same manner as the above-mentioned computational experiments. Obtained trade-off solutions are shown in Figure 10 by open circles together with the given reference solution G and the true ideal solution T. In this case, the decision maker may choose the solution A in Figure 10. We can see that the finally obtained solution A by the EMO approach is not close to the true ideal solution T or the given reference solution G. This is because the NSGA-II algorithm could not find a variety of trade-off solutions with a wide range of objective values over the entire Pareto front in Figure 10. That is, the obtained trade-off solutions by the NSGA-II algorithm in Figure 10 do not approximate the entire Pareto front.

Next we apply the reference solution-based approach to the two-objective knapsack problem using the information about the given reference solution G (16000, 21000) in the same manner as the previous computational experiments in Subsection 3.4. That is, first each objective is independently optimized by a single-objective genetic algorithm with the stopping condition of 400 generations. Then the objective space and the given reference solution is normalized using the optimal solution with respect to each objective. The normalized reference solution is used in a single-objective genetic algorithm where the distance from the normalized reference solution is minimized. The normalized reference solution is also used as the weight vector in the repair procedure in the same manner as the previous computational experiments in Subsection 3.4. The obtained solution is shown in Figure 10 by the closed circle B. While the obtained solution B is close to the given reference solution G, it is not close to the true ideal solution T.

The illustrative computational experiments in Figure 10 clearly demonstrate the disadvantage of each approach. The EMO approach could not efficiently find a variety of good trade-off solutions that approximate the entire Pareto front of a large-scale multiobjective combinatorial optimization problem. On the other hand, the reference solution-based approach could not work well in the case where the given reference solution does not correctly reflect the decision maker's true preference. One research direction to resolve this situation is the improvement of the search ability of EMO algorithms used in the EMO approach. There exist a large number of studies along this research direction in the literature (e.g., see [1], [2]). The similarity-based mating scheme is an example of such studies.

It is, however, very difficult for EMO algorithms to find a variety of good trade-off solutions with a wide range of objective values that approximate the entire Pareto front in the case of large-scale multiobjective combinatorial optimization problems. In this case, it may be required to efficiently utilize the information about the decision maker's preference in EMO algorithms. Here we suggest a hybrid version of the EMO and reference solution-based approaches. The EMO algorithm in our hybrid approach is the same as the NSGA-II algorithm except for its parent selection mechanism. We use the distance from the reference solution as the fitness function in the selection phase of parent solutions together with the similarity-based mating scheme. That is, candidate solutions in the similarity-based mating scheme are chosen by iterating the binary tournament selection scheme where the distance from the reference solution is used to evaluate each solution. The fitness evaluation mechanism of the original NSGA-II algorithm is still used in the generation update phase. This means that the next population is constructed from the current population and the offspring population using Parent ranking and a crowding measure exactly in the same manner as the NSGA-II algorithm.

We apply the hybrid approach to the knapsack problem in the same manner as the reference solution-based approach in Figure 10. That is, the modified NSGA-II algorithm is executed for 400 generations using the normalized reference solution. The two parameters in the similarity-based mating scheme are specified as $(\alpha, \beta) = (10, 10)$. An obtained solution set by a single run is shown in Figure 11. We can see from Figure 11 that a large number of good trade-off solutions (i.e., trade-off solutions close to the Pareto front) were obtained around the given reference solution G. It should be noted that some of them are close to the true ideal solution T. When the obtained trade-off solutions in Figure 11 are presented to the decision maker, he/she may realize that the given reference solution does not correctly reflect his/her true preference. In this case, some solution (e.g., Solution C) close to the true ideal solution T may be chosen from the obtained trade-off solutions by the decision maker. Solution C in Figure 11 is much closer to the true ideal solution T than Solution A by the EMO approach and Solution B by the reference solution-based approach in Figure 10. As shown in Figure 11, the decision maker can choose a final solution from the obtained trade-off solutions according to his/her preference. Moreover the obtained trade-off solution set gives the decision maker some information about the trade-off structure around the given reference solution G. Such information may help the decision maker to correctly understand the multiobjective optimization problem at hand.

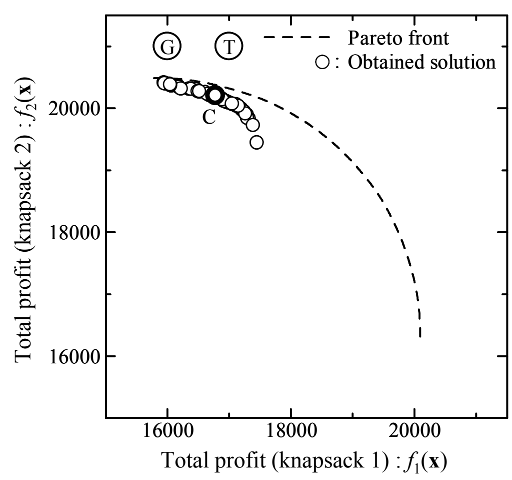

Figure 11. Experimental results by the hybrid approach.

The performance of the hybrid approach is also examined in the same manner as the reference solution-based approach in Subsection 3.4. That is, the modified NSGA-II algorithm in the hybrid approach is executed for each of the five specifications of the reference solution. The 50% attainment surface over 100 independent runs and a typical solution set by a single run are shown in Figure 12 for each specification of the reference solution together with the Pareto front. From Figure 12, we can see that a lager number of good trade-off solutions were obtained by the hybrid approach. From the comparison between Figure 2 by the original NSGA-II algorithm with 1200 generations in the EMO approach and Figure 12 by the modified NSGA-II algorithm with 400 generations in the hybrid approach, we can see that the hybrid approach can efficiently utilize the information from the decision maker about the reference solution.

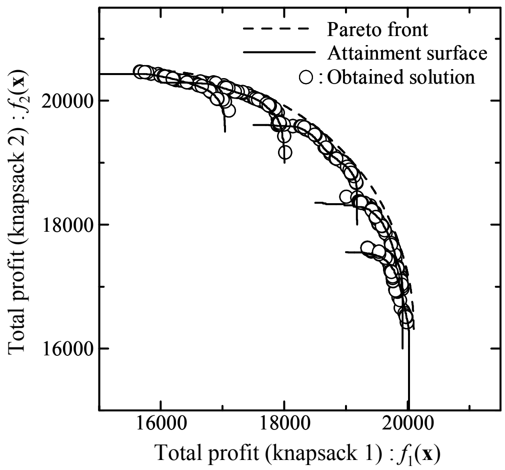

Figure 12. Experimental results by the hybrid approach for each of the five specifications of the reference solution.

5. CONCLUDING REMARKS

In this paper, first we compared the evolutionary multiobjective optimization (EMO) approach with the reference solution-based approach to multiobjective optimization problems. Through computational experiments, we visually demonstrated advantages and disadvantages of each approach. One difficulty in the EMO approach was that a variety of non-dominated solutions with a wide range of objective values over the entire Pareto front were not always found for large-scale multiobjective combinatorial optimization problems. That is, obtained non-dominated solutions did not always approximate the entire Pareto front well. As a result, good trade-off solutions for the decision maker were not always found. This difficulty was demonstrated through computational experiments where five specifications of the reference solution were examined. Experimental results showed that the EMO approach could not always find good trade-off solutions close to an arbitrarily given reference solution. On the other hand, multiple trade-off solutions were not found around the given reference solution by the reference solution-based approach. Thus this approach did not work well when the given reference solution did not appropriately reflect the decision maker's preference. Based on these observations, we suggested a hybrid version of the two approaches. It was demonstrated that a large number of good trade-off solutions were obtained around the given reference solution by the hybrid approach.

6. ACKNOWLEDGMENTS

The authors would like to thank Ms. Shiori Kaige and Dr. Yusuke Nojima for their help with computational experiments. This work was partially supported by Kayamori Foundation of Information Science Advancement, and Japan Society for the Promotion of Science (JSPS) through Grand-in-Aid for Scientific Research (B): KAKENHI (14380194).

7. REFERENCES

[1] Coello Coello, C. A., Van Veldhuizen, D. A., and Lamont, G. B. *Evolutionary Algorithms for Solving Multi-Objective Problems*. Kluwer Academic Publishers, Boston, MA, 2002.

[2] Deb, K. *Multi-Objective Optimization Using Evolutionary Algorithms*. John Wiley & Sons, Chichester, 2001.

[3] Deb, K., Pratap, A., Agarwal, S., and Meyarivan, T. A fast and elitist multiobjective genetic algorithm: NSGA-II. *IEEE Trans. on Evolutionary Computation 6,* 2 (April 2002) 182-197.

[4] Fonseca, C. M., and Fleming, P. J. On the performance assessment and comparison of stochastic multiobjective optimizers. *Lecture Notes in Computer Science 114: Parallel Problem Solving from Nature - PPSN IV*, Springer, Berlin (September 1996) 584-593.

[5] Ishibuchi, H., and Shibata, Y. A similarity-based mating scheme for evolutionary multiobjective optimization. *Lecture Notes in Computer Science 2723: Genetic and Evolutionary Computation - GECCO 2003*, Springer, Berlin (July 2003) 1065-1076.

[6] Ishibuchi, H., and Shibata, Y. Mating scheme for controlling the diversity-convergence balance for multiobjective optimization. *Lecture Notes in Computer Science 3102: Genetic and Evolutionary Computation - GECCO 2004*, Springer, Berlin (June 2004) 1259-1271.

[7] Jaszkiewicz, A. Genetic local search for multi-objective combinatorial optimization. *European Journal of Operational Research 137*, 1 (February 2002) 50-71.

[8] Jaszkiewicz, A. On the performance of multiple-objective genetic local search on the 0/1 knapsack problem - A comparative experiment. *IEEE Trans. on Evolutionary Computation 6*, 4 (August 2002) 402-412.

[9] Knowles, J. D., and Corne, D. W. M-PAES: A memetic algorithm for multiobjective optimization. *Proc. of 2000 Congress on Evolutionary Computation* (San Diego, CA, July 16-19, 2000) 325-332.

[10] Schaffer, J. D. Multiple objective optimization with vector evaluated genetic algorithms. *Proc. of 1st International Conference on Genetic Algorithms and Their Applications* (Pittsburgh, PA, July 24-26, 1985) 93-100.

[11] Zitzler, E. and Thiele, L. Multiobjective evolutionary algorithms: A comparative case study and the strength Pareto approach. *IEEE Trans. on Evolutionary Computation 3*, 4 (November 1999) 257-271.